DYNAMIC ANALYSIS OF TALL STRUCTURES SUBJECTED TO EARTHQUAKE MOTION

by

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Introduction

The central problem regarding the destroying action of earthquakes is reduced lastly to the determination of seismic solicitations to which buildings are subjected. The knowledge of seismic forces allows a rational, antiseismic projection of each construction type.

In Roumania which is a country subjected to intense earthquakes, such problems have presented especially during last years, a special importance. Because of a lack of preoccupation in this direction in the past, the strong earthquake of november,1o-th, 194o /2/ has caused almost in the whole country huge damages.

In this study we intend to state an expeditious computation method concerning the appreciation of fisplacements, velocities, accelerations or inertial forces which develop within the resistance structure of a high building after a seismic disturbance.

On the basis of the proposed computation method is the criterion of structure rigidity which plays a principal role with regard to the modification of the dynamic response/18/, /29/. We may assert that the rigidity of a building leads its behaviour when subjected to the solicitations of an earthquake. This rigidity has a dynamic and total character, depending on the direction in which acts the seismic wave. Therefore, of dynamic point of view, the rigidity of a building may be completely precised by means of the fundamental vibration period or frequency for which we propose, in this paper, several direct computation expressions.

For the practical computation of seismic forces of greatest importance is the determination of the dynamic structure response. The dynamic response of a structure (relative displacement with reference to the ground; relative velocity, absolute acceleration, inertial force or basis shear force) actioned by loads caused by earthquakes, results principially

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from the condition of realizing a general dynamic equilibrum between the static-dynamic characteristics of the structure and the dynamic properties of the seismic wave and the medium through which the latter propagates.

In the following lines we shall refer to the dynamic behaviour of elastic multistory structures, in order to state: the static dynamic characteristics of the structure (gamma of the proper vibration-frequencies or -periods,totality of the normal oscillation modes, critical damping factors, value and distribution of mass on the building hight) and the dynamic properties of the seismic wave and of the medium through which it propagates (absorbtion energy, spectrum intensity, magnitude, physic-mechanical properties of soil, a.s.o.)

The study of these problems allows to determine the variation of the spectrum seismic coefficients, i.d.of the relationship between the seismic coefficients of calculation and the rigidity of building expressed by the period. The computation of seismic forces, based upon this proceeding receives a scientifical basis, as compared to the arbitrary methods of the unique seismic coefficients (refer to the modified Mercalli scale) which are independent of the intrinsical features of the structure.

II. DYNAMIC RESPONSE OF ELASTIC STRUCTURES SUBJECTED TO SEISMIC DISTURBANCES

In the common antiseismic computation we consider that the solicitation produced by a seismic wave appears as an arbitrary motion which acts at the base of the building. The velocities and accelerations which take place within the structure elements depend upon the dispacement which of analytical point of view is considered as being solicited by structure.

Supposing that function Zo(t) is mathematically precised, it is necessary to determine the dynamic response of structure for these disturbances. Therefore, in the following lines we will study, only succintly, the forced oscillations of an elastic structure like that in fig.1, with n degrees of freedom (taking into account also the damping), when its grpund is subjected to an arbitrary displacement Zo(t) corresponding to the seismic motion.

The motion equation of type d'Alembert, corresponding to point k of the given system, may be written as:

$$M_k \frac{d^2[Z_k(t)+Z_o(t)]}{dt^2}+\beta_k\frac{dZ_k(t)}{dt}+\sum_{j=1}^{n} K_{kj} Z_j(t)=0 \qquad \text{(II-1)}$$

in which :

β_k - is the damping coefficient proportional to the velocity conformably to Voigt's hypothesis.

K_{kj} - represents the rigidity of point k of the system. It is defined as the force which must be applied in point k, so that point j might displace with an unit $Z_j = 1$

The equation(II-1) may be also written as :

$$M_k \ddot{Z}_k(t) + \beta_k \dot{Z}_k(t) + \sum_{j=1}^{n} K_{kj} Z_j(t) = -M_k \ddot{Z}_o(t) \qquad \text{(II-2)}$$

If such equations are written for all masses of elastic structure, one obtains an equation-system of order n, identic with that of the forced oscillations due to the disturbing forces of type $-M_k Zo(t)$

For the above equation, solutions are obtained by means of the method of the variable separation relative to the own functions of the system :

$$Z_k(t) = \sum_{i=1}^{n} Z_{ki} \Phi_i(t) \qquad \text{(II-3)}$$

where:

Φ_i - is a time function, corresponding to each vibration mode i, of the type of a generalized coordination. This function precises the deformation of the elastic system in time.

Taking into account the orthogonality property of the own vibration forms

$$\sum_{k=1}^{n} M_k Z_{ki} Z_{kj} = 0 \quad (i \neq j) \qquad \text{(II-4)}$$

and assuming that

$$\mathcal{E}_k = \frac{\beta_k}{2M_k} = \mathcal{E} \text{ (constant)}$$

is obtained a system of n independent equations relative to the normal modes of vibrations.

For the i mode, results the equation :

$$\sum_{k=1}^{n} M_k Z_{ki}^2 \left[\ddot{\Phi}_i(t) + 2\mathcal{E}\dot{\Phi}_i(t) + \omega_i^2 \Phi_i(t) \right] = -\sum_{k=1}^{n} M_k \ddot{Z}_o(t) Z_{ki} \qquad \text{(II-5)}$$

Instead of the $\mathcal{E}$ value we will introduce the critical damping factor γ_i which results from the logarithmic decrement of damping,

$$\delta_i = 2\pi\nu_i = \varepsilon T_i = \varepsilon \frac{2\pi}{\omega_i}$$

from which

$$\varepsilon = \nu_i \omega_i \qquad \text{(II-6)}$$

Taking into account (II-6) and $\sqrt{1-\nu_i^2} \simeq 1$,by integrating the equation(II-5) we obtain:

$$\Phi_i(t) = -\frac{c_i}{\omega_i}\int_0^t \ddot{Z}_o(\tau)\, e^{-\nu_i\omega_i(t-\tau)} \cdot \sin\omega_i(t-\tau)dt \qquad \text{(II-7)}$$

where:

$$c_i = \frac{\sum_k M_k Z_{ki}}{\sum_k M_k Z_{ki}^2}$$

In order to synthesize the expressions that will result, a new notation is introduced, calling <u>form coefficient</u> the expression

$$a_{ki} = c_i Z_{ki} \qquad \text{(II-8)}$$

By means of the relation (II-3) for the <u>i</u> vibration mode,results:

<u>d i s p l a c e m e n t s</u>

$$Z_k(t) = \frac{a_{ki}}{\omega_i}\int_0^t \ddot{Z}_o(\tau)\, e^{-\nu_i\omega_i(t-\tau)} \cdot \sin\omega_i(t-\tau)d\tau \qquad \text{(II-9)}$$

<u>v e l o c i t i e s</u>

$$\dot{Z}_k(t) = a_{ki}\int_0^t \ddot{Z}_o(\tau)\, e^{-\nu_i\omega_i(t-\tau)} \cdot \cos\omega_i(t-\tau)d\tau \qquad \text{(II-1o)}$$

<u>a c c e l e r a t i o n s</u>

$$\ddot{Z}_k(t) = \omega_i a_{ki}\int_0^t \ddot{Z}_o(\tau)\, e^{-\nu_i\omega_i(t-\tau)} \cdot \sin\omega_i(t-\tau)d\tau \qquad \text{(II-11)}$$

<u>i n e r t i a l   f o r c e s</u>

$$F_k(t) = M_k\omega_i a_{ki}\int_0^t \ddot{Z}_o(\tau)\, e^{-\nu_i\omega_i(t-\tau)} \cdot \sin\omega_i(t-\tau)d\tau \qquad \text{(II-12)}$$

<u>f u n d a m e n t a l   s h e a r   f o r c e</u>

$$F(t) = \sum_{k=1}^{n} M_k\omega_i a_{ki}\int_0^t Z_o(\tau)\, e^{-\nu_i\omega_i(t-\tau)} \cdot \sin\omega_i(t-\tau)d\tau \qquad \text{(II-13)}$$

For the seismic motion has an arbitrary propagation direction, and it may act in every direction and sense,we have neglected the sign in the above mentioned expressions. We stress that the accurate demonstration of these expressions forms the object of this paper/19/.

The total values of displacements,velocities,accelerations,inertia forces or of the fundamental shear force in point <u>k</u>, are obtained by summing the expressions (II-9), (II-10), (II-11), (II-12) and (II-13) relative to i (i= 1,2... n)

The knowledge of these expressions allows to perform the computations corresponding to that vibration mode we are interested in.

The form of the above mentioned relations allows to put into evidence the analytic separation of the static-dynamic characteristics of structure and the dynamic properties of earthquakes /8/,/16/,/17/.

Principially we may write the following equation in which

$$R_k(t) = R_{kI} \cdot R_{kII} \qquad \text{(II-14)}$$

in which

$R_k(t)$ - represents the dynamic response of structure corresponding to point k due to seismic disturbances.

R_{kI} - represents the influence of the static-dynamic characteristics, specific for each type of structure,on the behaviour of the building.

R_{kII} - represents the influence of the dynamic properties of the seismic wave and of the medium through which the latter propagates, on the behaviour of the building.

We shall stress that this problem can be formulated in various manners depending upon the way in which is assimilated the motion of the ground $Z_o(t)$, In general,only the appreciation of the influence of the dynamic properties of an earthquake may separate - principially - the different dynamic computation methods of seismic forces.

III. APPRECIATION OF THE OWN VIBRATION FREQUENCIES

Within the static-dynamic characteristics R_{kI} the periods or the own frequencies play an extremely important role. The accurate computation of these values from mathematical point of view, for structures with several freedom degrees,constitutes a difficult and laborious problem.

The cannonical system of equations corresponding to the free,undamped vibrations of an elastic structure may be obtained by applying the classic solving-methods known in the structure mechanics.

The totality of the own vibration frequencies is obtained by the annulation of the matrix, characteristic for the cannonical-linear and homogenous system thus obtained :

$$\left|A-\lambda B\right|_1^n = 0 \qquad \text{(III-1)}$$

In the technical litterature, specific for the antiseismic computation,has been elaborated a whole series of more or less accurate methods for the estimation of the proper oscillation frequencies. All these methods, based generally on the principles of theoretic mechanics,belong to the following researchers : Bernstein, Dunkerley,Goldberg, Holzer, Korchinsky, Newmark, Nieto, Porter, Ritz, Rayleigh, Rosenblueth, Salvadori, Vianello-Stodola, Zavriev a.s.o.

Avoiding the solution of an equation of superior degree resulting from the development of the (III-1) condition, without losing the accuracy of the obtained values, we shall present the ways by which one can get a simple and easy appliable expression for the fundamental oscillation frequency (ω_1). We propose too, an approximate formula for the estimation of the proper frequencies of upper degree (ω_i).

The definition of the ω_1 expression was performed by the delimitation of the interval within the limits of which it is involved, by lower and upper values. The expressions of these limit-values were obtained by means of energetic computation criteria.

With regard to the dynamic calculation of buildings formed of elastic frames using energetic principles,there are relatively few studies in the reference litterature. Among these we mention especially the works /1/,/3/,/4/, /8/ a.o.

1. <u>General computation hypothesis</u>

Considering that upon the building are acting disturbant loads of seismic type at the level of the foundation, following computation hypothesis can be introduced:

- the masses of the building are considered as being concentrated at the level of each story;
- in the first phase the building is considered as being founded on a stiff ground;
- the oscillations of the building are due only to the bending of the columns, the axial deformations of the latter being neglected;
- the oscillations are horizontal and completely plane;
- the resistance skeleton of the building consists of elastic frames with beams of infinite rigidity and elastic columns.

The last hypothesis is practically acceptable, especially in the case of common buildings of medium height.It appears in a series of studies, from which some have been mentioned in the bibliography /3/, /4/, /6/, /9/, /11/, /13/, /15/, /20/, /21/, /27/, /30/ and others too /7/.

In the dynamic study, the buildings consisting of such frames are called <u>shear buildings</u>.

2. Determination of the lower-limit of the fundamental frequency ω_L

The lower limit of the fundamental frequency ω_L was obtained by applying Dunkerley's procedure by means of which was written the relation

$$\frac{1}{\omega_{ex}^2} < \frac{1}{\omega_L^2} \qquad \text{(III-2)}$$

in which

$$\omega_L = \frac{1}{\sqrt{\sum_k^n \frac{1}{\omega_k^2}}} \qquad \text{(III-3)}$$

In this expression ω_k represents the exact value of the proper vibration frequency of the system shown in fig.2, loaded, this time, with only a single mass $M_k = Q_k/g$ at the level of each story k. Therefore, the frame in fig. 2 will ne decomposed into k systems with one degree of freedom (fig.3) to each corresponding a frequency ω_k which is to be accurately computed.

For the system in fig.2 with a single degree of freedom, the frequency is obtained by using the d'Alembert type equation

$$\omega_k^2 = \frac{g}{\overline{Z}_k} \qquad \text{(III-4)}$$

in which $\overline{Z}_k$ is the displacement of point k on the horizontal when the system is subjected only to the lateral Q_k force.

According to fig.3 it can be written :

$$\overline{Z}_k = \tilde{Z}_1 + \tilde{Z}_2 + \dots + \tilde{Z}_k = \sum_{j=1}^{k} \tilde{Z}_j$$

Considering that

$$\tilde{Z}_j = \frac{Q_j l_j^3}{12 m E I_j}$$

results finally

$$\overline{Z}_k = Z_0 \alpha_k \sum_{j=1}^{k} q_j \qquad \text{(III-5)}$$

where

$$Z_0 = \frac{Q_0 l_0^3}{12 m E I_0}$$

Substituting the expression(III-5) in (III-4) and coming back to (III-3) it results :

$$\omega_L^2 = \frac{g}{\sum_{k=1}^{n} \alpha_k \sum_{j=1}^{k} q_j} \qquad \text{(III-6)}$$

It is easy to see that the denominator of the relation (III-6) represents the static displacement Z_n^{ST} of the system shown in fig.2 loaded with all Q_k loads, that is

$$Z_n^{ST} = Z_0 \sum_{k=1}^{n} \alpha_k \sum_{j=1}^{k} q_j \qquad \text{(III-7)}$$

Based on the above data, formula (III-6) can be written:

$$\omega_L = \sqrt{\frac{g}{Z_n^{ST}}} \qquad \text{(III-8)}$$

This formula identical to Geiger's formula is known from the precise computation of the system with a single degree of freedom and it is sometimes recommended for an approximate use in systems with several degrees of freedom.

In conclusion, the lower limit value of the fundamental vibration frequency for frames of the type shown in fig. 2 will be :

$$\omega_L = \xi_L \sqrt{\frac{EI}{M_0 l_0^3}} \qquad \text{(III-9)}$$

in which

$$\xi_L = \sqrt{\frac{12m}{\alpha_1 q_1 + \alpha_2(q_1+q_2) + \cdots + \alpha_n(q_1+q_2+\cdots+q_n)}} \qquad \text{(III-10)}$$

<u>Consequently, the equation established for the lower limit of the fundamental vibration frequency, corresponding to elastic structures like those in fig.2, is identical to Geiger's formula applied to the same case.</u>

3. <u>Determination of the upper limit of the fundamental frequency</u> ω_u

The determination of the upper limit was done by Rayleigh's method concerning the principle of energy conservation in ideal elastic structures.

The values of the proper oscillation frequency corresponding to <u>i</u> mode is obtained with the help of equation:

$$\omega_i^2 = g \frac{\sum_{k=1}^{n} Q_k Z_k(i)}{\sum_{k=1}^{n} Q_k Z_k^2(i)} \qquad \text{(III-11)}$$

If $Z_k(i)$ represents the real elongations of the system in the normal mode <u>i</u>, the frequency value obtained by means of formula (III-11) is an exact one. But, because the $Z_k(i)$ variation is not known a priori, in most cases it is estimated according to the nature and particularities of the studied problem. In the following lines is considered only the fundamental vibration frequency.

Referring to fig.2, during oscillations corresponding to the first normal mode, the dynamic deformation of the $Z_k(i)$ system looks like that represented in the curve c. Because the variation of this curve can't be known a priori, it will be estimated in function of the real phenomenon, in two manners:

Hypothesis A: The dynamic deformation $Z_k(i)$ (curve c in fig.2) coincides with the static deformation of the Z_k^{ST} system (curve a, fig.2)

Hypothesis B: The dynamic deformation is assimilated to a linear variation (curve b, fig.2)

Hypothesis A

For the computation of the displacements on the horizontal of the structure in fig.2, subjected to the action of static applied Q_k forces, was deduced the following equation, given here in its final form :

$$Z_k^{ST} = Z_0 \sum_{j=1}^{k} q_j \sum_{r=j}^{n} \alpha_r \quad (j = 1,2,\dots k) \qquad \text{(III-12)}$$

Performing the corresponding transformations and considering (III-11),(III-12) one obtains the expression of the upper limit of the first circular oscillation frequency, ressembling, in form, to (III-9)

$$\omega_u = \xi_u \sqrt{\frac{EI}{M_0 l_0^3}} \qquad \text{(III-13)}$$

where

$$\xi_u = \sqrt{12m \frac{\sum_{k=1}^{n} \sum_{j=1}^{k} q_j \sum_{r=j}^{n} \alpha_r}{\sum_{k=1}^{n} \left[\sum_{j=1}^{k} q_j \sum_{r=j}^{n} \alpha_r \right]^2}} \qquad \text{(III-14)}$$

Hypothesis B

The variation of displacements of the structure on the horizontal was deduced as :

$$Z_k = Z_n^{ST} \frac{l_0}{L} \sum_{j=1}^{k} \lambda_j \qquad \text{(III-15)}$$

in which Z_n^{ST} represents the maximum displacement of the story n when structure is solicited by all the Q_k loads. The displacement is computed by means of formula (III-7)

Considering the practical case when the floors are disposed at equal distance ($\lambda_1 = \lambda_2 = \dots = 1$ and $L = n l_0$) one obtains the following expression for the upper limit of the fundamental frequency :

$$\bar{\omega}_U = \bar{\xi}_U \sqrt{\frac{EI}{M_0 l_0^3}} \qquad \text{(III-16)}$$

in which

$$\bar{\xi}_U = \sqrt{\frac{m.n}{(2n+1)\left[\alpha_1\bar{q}_1+\alpha_2(\bar{q}_1+\bar{q}_2)+\cdots+\alpha_n(\bar{q}_1+\bar{q}_2+\cdots+\bar{q}_n)\right]}} \qquad \text{(III-17)}$$

4. Delimitation of the existence interval of $\omega_{ex.}$ value

According to paragraphs 2 and 3 results :

$$\omega_L < \omega_{ex.} < (\omega_U , \bar{\omega}_U)$$

As noticed in their expressions, all ξ coefficients are only functions of the relative static-geometric characteristics of the structure.

By first referring to formulas (III-9) and (III-13) one obtains :

$$\omega_U = \eta\,\omega_L \qquad (\eta \geq 1)$$

and therefore

$$\eta = \frac{\xi_U}{\xi_L} \qquad \text{(III-18)}$$

Considering now (III-9) and (III-16) we have :

$$\bar{\omega}_U = \bar{\eta}\,\omega_L \qquad (\bar{\eta} \geq 1)$$

and thus :

$$\bar{\eta} = \frac{\bar{\xi}_U}{\xi_L} = \sqrt{\frac{3n}{2n+1}} \qquad \text{(III-19)}$$

In fig.4 are represented the $\xi(n)$ functions for the real situations encountered in practice. In the same figure is given also the variation of the interval η as a function of the number of stories, n .

The differences between ω_L and ω_U as compared to $\bar{\omega}_U$ may be computed in percentages as follows:

$$\varepsilon_L = \frac{\omega_{ex} - \omega_L}{\omega_{ex}} 100\%$$

$$\varepsilon_U = \frac{\omega_U - \omega_{ex}}{\omega_{ex}} 100\%$$

$$\bar{\varepsilon}_U = \frac{\bar{\omega} - \omega_{ex}}{\omega_{ex}} 100\%$$

The percentage differences between $\omega_U, \bar{\omega}_U$ and ω_L can also be obtained relative to ;

$$\varepsilon = \frac{\omega_U - \omega_L}{\omega_L} 100\% = (\eta - 1).100\%$$

$$\bar{\varepsilon} = \frac{\bar{\omega}_U - \omega_L}{\omega_L} 100\% = (\bar{\eta} - 1).100\%$$

Due to the fact that $\bar{\varepsilon}$ depends only of the number of stories, it was possible to tabulate it /16/. A limit may be observed of the difference between ω_L and $\bar{\omega}_U$

for buildings having a number of stories $n \geqslant 10$, at $\xi = 10\%$. For n=1, one obtains $\eta = \overline{\eta} = 1$ i.e. $\omega_L = \omega_{ex} = \omega_U = \overline{\omega}$, because we consider the case of a system with a single degree of freedom.

5. Consideration upon the settlement of the foundation soil

In case of a foundation resting on an elastic soil, owing to the settlement, the building rocks with an angle about an horizontal axis passing through point C (fig.5)

According to fig.5 the displacement in the direction of the beams (considered equal to the horizontal component of displacement when the vertical component is neglected) will be :

$$Z_k = Z_k^{ROT} + Z_k^{TRANS} \qquad \text{(III-20)}$$

in which

$$Z_k^{ROT} = \theta \sum_{j=1}^{k} l_j$$

represents the displacement at the level of each story, due to the rocking of the building about the O axis. In this equation it was admitted that $tg\,\theta \simeq \theta$ because the θ angle is sufficiently small.

In this case (III-12) and (III-14) become

$$Z_k = \theta \sum_{j=1}^{k} l_j + Z_o \sum_{j=1}^{k} q_j \sum_{r=j}^{n} \alpha_r \qquad \text{(III-21)}$$

and the formula (III-15)

$$Z_k = \theta \sum_{j=1}^{k} l_j + Z_n^{ST} \sum_{j=1}^{k} \lambda_j \frac{L_o}{L} = \left[\theta + Z_n^{ST} \frac{1}{L}\right] L_o \sum_{j=1}^{k} \lambda_j \qquad \text{(III-22)}$$

The limits of the fundamental frequency ω are obtained similaŕrly to the previous computations.

The value of θ angle may be determined theoretically in function of the dynamic coefficient of the elastic, non uniform compression of the ground or according to M.Biot in fuηction of the reactive elastic moment of the soil.

Extremely interesting opinions in connection with this problem are found in the works /9/,/24/,/25/,/28/ aso.

6. Expressions proposed for the direct computation of proper frequencies

Based on the equations of the lower limit (ω_L) the upper limit (ω_U), the fundamental frequency and on the other previous solutions /11-19/ we may do following remarks :

a) The hypothesis of beams of infinite rigidity is satisfactory for multistory frames whose relation between the building height (H) and the base length (B) is smaller than or equal to 5/27/ i.e. $H/B \leq 5$. We may consider that by this hypothesis the whole rigidity of the structure increasing, we approach the reality because the separating walls (non-portant) and even the floors contribute to increase the rigidity of the building on an horizontal direction. For the frames in which the relationship $EI_{beam}/EI_{column} > 2$ and the number of columns increases, this hypothesis may be considered as practically acceptable.

b) The ξ_u and $\bar{\xi}_u$ coefficients are ressembling each other and for common cases, they approach the value ξ_{ex} which is characteristic for the fundamental frequency, calculated by means of accurate analytic methods:

$$\omega_{ex} = \xi_{ex}\sqrt{\frac{EI_o}{M_o l_o^3}}$$

c) Generally, for common structures: $\xi_{ex} < \xi_u < \bar{\xi}_u$ and for those of pronounced rigidity $\xi_{ex} < \bar{\xi}_u < \xi_u$.

d) For buildings having about 8 stories, the $\xi_u, \bar{\xi}_u$ coefficients may be used, directly or a little reduced with 1-5% according to the number of the stories.

e) For buildings with 10 or more stories, one may use an average between ξ_L and ξ_u, or between ξ_L and $\bar{\xi}_u$ because when the number of stories increases, the ε_L error decreases, the exact value of frequency approaching the lower limit.

f) In order to determine a simple formula for the computation of the <u>fundamental frequency</u> (ω_1) as nearer as possible ~~to that resulting~~ when applying the exact analytical methods, taking into account the previous observations and on basis of a series of numerical calculations, it was proposed the following formula:

$$\omega_1 = \eta_1 \omega_L = \eta_1 \sqrt{\frac{g}{Z_n^{st}}} \qquad \text{(III-23)}$$

in which η_1 is a correction coefficient, arbitrary reduced from η /16/, /17/.

Considering /III-9/ and noting the product $\eta_1 \xi_L$ with ξ_1 the calculation formula <u>for</u> the <u>fundamental frequency</u> will be:

$$\omega_1 = \xi_1 \sqrt{\frac{EI_o}{M_o l_o^3}}$$

<u>The fundamental period</u> (T_1) of vibration being:

$$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{\xi_1}\sqrt{\frac{M_o l_o^3}{EI_o}}$$

in which

$$\xi_1 = \psi \cdot \sqrt{\frac{m}{\alpha_1 q_1 + \alpha_2 (q_1 + q_2) + \cdots + \alpha_n (q_1 + q_2 + \cdots + q_n)}} \qquad \text{(III-26)}$$

and ψ is a coefficient the variation of which is given in fig.6 in function of the number of stories.

Formula (III-24) gives approximations of 1% relative to those values obtained by exact methods. Relative to ξ_{ex} results the error

$$\varepsilon_1 = \frac{\xi_{ex} - \xi_1}{\xi_{ex}} 100\%$$

g) For the approximate computation of the circular frequencies of high order (ω_i) we propose the following relation which gives values more accurate to the real ones than other similar expressions

$$\omega_i = 1{,}3\, i\, \omega_1 \qquad (i = 2,3,\ldots) \qquad \text{(III-27)}$$

The oscillation periods will be

$$T_i = \frac{2\pi}{\omega_i} \simeq \frac{4.8}{i\,\omega_1} = \frac{T_1}{1.3i} \qquad \text{(III-28)}$$

h) Within the mentioned hypothesis, the expressions established in this paper may be considered (owing to their simple aspect and easy application) as being on the line of the formulations obtained in different works or even a little ahead of some of them.

T.TANIGUCHI : "General Principles of Aseismic Constructions" Tokyo, 1934	$T_1 = (0{,}07 \div 0{,}09)\, n$
U.S.Coast and Geodetic Survey : "Earthquake Investigations in California", 1934-1935	$T_1 = 0{,}1\, n$
JOINT COMMITTEE of the SAN FRANCISCO ASCE : "Lateral Forces of Earthquakes and Wind" Proc.ASCE, April 1951, vol.77, separate 66	$T_1 = 0{,}09 \frac{H}{\sqrt{B}}$
E.P.ULRICH and D.S.CARDER : "Vibrations of Structures" Proc. of the Symposium on the Earthquake Blast Effects on Structures " Los Angeles,Calif.1952	$T_1 = 0{,}0193\, H$
I.L.KORCHINSKY: "Oscillations of High Buildings" (in Russian),	$T_i = \frac{H}{150\, i}$
M.G.SALVADORI: "Earthquake Stresses in Shear Building" Proc.ASCE, 1953, vol.79, separate 177	$T_i = \frac{4}{2i-1} \sqrt{\frac{n \sum_{k=1}^{n} M_k}{K}}$

R.G.MERRITT and G.W.HOUSNER : "Effect of Foundation Compliance on Earthquake Stresses in Multistory Buildings" Bull.SSA, vol.44, no.4, Oct.1954

$$T_1 = \frac{\pi}{\sin\left(\frac{\pi}{4n+2}\right)} \sqrt{\frac{\sum_{k=1}^{n} M_k}{nK}}$$

EMILIO ROSENBLUETH: "Estructuras a prueba de temblores cimentadas sobre terreno firme" Inginieros Civiles Asociadas (ICA) no.17, México, 1954

$$T_1 = [0.09 \div 0.10](n+1)$$

in which $K = \frac{1}{H}\sum_{j=1}^{n} K_j l_j$ Represents the average value of the elastic constant between two stories

All presented formulae have been adapted to the MKS system.

i) In the case of buildings formed of frames the shape of which varies on the vertical and having at the same level the inertial moments of the columns of definite values, formula (III-24) may be also used but obly if introducing the average rigidity of each story.

$$k_r = \frac{1}{m}\sum_j k_{rj} \qquad \text{(III-29)}$$

We have used the notation :

$I_{rj} = k_{rj} I_0$ - inertial moment of the column on the r line and j column

Expression (III-29) resulted from the equivalency - of rigidity viewpoint - between every frame type and that represented in fig.2.

The proposed formulas are applicable also to trussed frames.

j) The exemplification of the proposed formulas is done on some frame types the characteristics of which are given in table 1. Table 2 contains the obtained results.

IV. NORMAL VIBRATION MODES

Once the frequencies of rree oscillations being known, the relative displacements Z_{ki} corresponding to each mode may be calculated by using the cannonical homogenous system of equations. Usually one chooses the initial value $Z_{1i} = 1$ according to which are resulting the other ordinatas of the respective mode.

An important problem is the determination of the coefficients of this cannonical equation system.

The equation corresponding to the degree of freedom $\underline{k}$ in the vibration mode, has the form :

$$b_{k1} Z_{1i} + b_{k2} Z_{2i} + \cdots + b^*_{kk} Z_{ki} + b_{kj} Z_{ji} + \cdots b_{kn} Z_{ni} = 0 \qquad \text{(IV-1)}$$

where

b_{kj} - represents the dynamic reaction in $\underline{k}$ relation when to $\underline{j}$ - relation was impressed an unitary, pulsatory displacement.

$b^*_{kj} = b_{kk} - M_k \omega_i^2$

We will give directly the expressions of these coefficients involved within the equations of type (IV-1). Their calculation is very simple and based on elements already known from the calculation of the proper frequencies.

$$b_{k-1,k} = -\frac{A_o}{q_{k-1,k}} \; ; \qquad b_{k+1,k} = -\frac{A_o}{q_{k,k+1}} \quad \left(A_o = \frac{12 m E I_o}{l_o^3}\right) \qquad \text{(IV-2)}$$

$$b_{kk} = -(b_{k-1,k} + b_{k+1,k})$$

Based on the reciprocity of the mechanic work, results $b_{kj} = b_{jk}$ and consequently :

$$b_{k-1,k} = b_{k,k-1} \; ; \quad b_{k+1,k} = b_{k,k+1} \qquad \text{(IV-3)}$$

Because the proper frequencies and mainly the upper ones are approximately determined, it follows naturally that for the calculation of the proper forms the system will not be absolutely verified. Therefore, the proper forms had to be corrected taking into account the orthogonality properties (II-4) in order to satisfy the (IV-1) system.

V. THE INFLUENCE OF DAMPING

According to expressions(II-9),(II-1o),(II-11),(II-12),(II-13) to the little increases of the critical damping factor corresponds a sensible decrease of the dynamic response of the structure. Therefore, the inertia forces will depend both upon the material of the building and of the adapted static and constructive system.

Important researches of a great practical value have been performed recently by L.S.Jacobsen/27/ at Stanford University. Most of the researchers of different countries consider that for the common buildings of concrete, the critical factor of damping has the value o,1. Housner, Hudson, Medvedev, Napetvaridze, Rosenblueth, Sorotkin and other numerous japanese investigators have an important contribution in this problem.

VI. SPECTRUM SEISMIC COEFFICIENTS

In order to simplify the practic computation of seismic forces, expression(II-12) was synthesized in the following manner:

$$F_k(i) = C(T_i) \cdot a_{ki} Q_k \qquad \text{(VI-1)}$$

in which

$$C(T_i) = \frac{1}{g} \cdot \frac{2\pi}{T_i} \int_0^t \ddot{Z}_0(\tau) e^{-\nu_i \omega_i (t-\tau)} \cdot \sin \omega_i (t-\tau) d\tau \qquad \text{(VI-2)}$$

represents the seismic spectrum coefficient of calculation. This coefficient varies with the period of the building and depends both on the intensity of earthquake and the nature of soil.

In order to estimate the $C(T_i)$ coefficient it is necessary to know the value of the integral appearing in the expression (VI-2) usually called the velocity spectrum (S_v). On the basis of real records of accelerations and of an adequat calculation by means of the analogous electronic calculator /8/ in the Institutes of California the investigators have succeeded to estimate the influence of earthquakes on buildings. Therefore, knowing the standard velocities spectrum the $C(T_i)$ coefficient may be defined as :

$$C(T_i) = K \frac{S_v}{T_i}$$

in which K is a coefficient which transforms the theoretical spectrum into a design spectrum.

In Soviet Union, the value of that integral was determined starting from a function of the $Z_0(t)$ displacements, which has resulted from an adequate interpretation of the seismographs recorded during different earthquakes/23/. This function assimilates the soil motion due to seismic action by summing a range of harmonic curves already known. The results arrived at, form the basis of the sovietic calculation norms in seismic region, SN-8, 1957.

But, we had to note that indifferently of the adopted method, the seismic coefficient $C(T_i)$ has an hyperbolic variation in M.A.Biot's and E.C.Robinson's sense, of form:

$$C(T_i) = \frac{\alpha}{T_i} \qquad \text{(VI-3)}$$

This variation is explainable because we know that the more flexible the buildings, the less reduced the seismic coefficiants, while the period T increases. It is also known, that of technical grounds, the seismic coefficients had to be limited both upper and lower limit. They depend also of the seismic zone, the nature of the soil through which passes the seismic wave and the physic-mechanical properties of the foundation-soil, by the phenomenon of interaction between soil and building.

If these problems are solved, the spectra may be automatically traced, with the only reserve that the coefficients limits, the upper $C^{u}(T_i)$ and the lower $C^{l}(T_i)$ correspond to security conditions. The respective figures may be precised also when studying the damages or destructions at the important buildings on probabilities basis.

The α coefficient has to respond to all these problems. For a certain seismic zone, the spectrum seismic coefficients may vary as in fig.7.

We have also to state the intervals in which are situated the various building types of rigidity viewpoint, i.e.

- rigid buildings $0 < T < T_1$
- semirigid buildings $T_1 < T < T_2$
- flexible buildings $T_2 < T$

In general $T_1 = (0{,}25 \div 0{,}45)$ sec. and $T_2 = (0{,}75 \div 1{,}0)$sec. The determination of absolute values is impossible and therefore these figures continue to be only informative.

We have to stress that the α parameter takes account of the soil through which passes the seismic wave. It may be increased in the case of weak soils (α') or reduced for solid soils. (α'').

The response spectrum plotted in fig.7 is a typic one; its practical application depending upon the accuratness with which are determined all the factors we have mentioned.

If this procedure is considered as a computation basis, the problem has an evident dynamic character and the building presents a precise individuality because it has a representative : the period.

The californian recommandation of 1951, 1956, 1958 and the sovietic ones in 1957 have handled a dynamic conception concerning the appreciation of seismic loads. Although it can't be spoken of a perfection, these prescriptions represent the most important applicative performances obtained up-to-day in the engineering seismology.

A similar conception will form - probably - the basis of the recommandation in México.

For Roumania, the author of this paper has proposed in 1959/18/ a similar proceeding concerning the estimation of seismic loads. The only reserve of this proposal consists in the fact that the α appreciation was a more empiric one, because of the lack of records and computers necessary for the interpretation of these records. We mention that all the figures presented in this standardization project of seismic forces may be justified of theoretic and practical viewpoint at the level of the most modern investigations, performed in this sense.

On this occasion are introduced isospectral curves for the estimation of the seismic coefficients which may characterise a zone of equal seismic intensity. Thus, the modified Mercalli-scale was given up because of its subjective character.

VII. CONCLUSIONS

The formulae established for the determination of periods, give a picture of the influence of the static-geometrical characteristics of structures upon the proper frequency, with help of the ξ coefficients. By using these formulae, is done a great economy of time and the control of the results is always ascertained.

The trial exposed in this paper like other analytical trials, remain still informative in spite of the exactness of the mathematic apparatus used. This is due to the fact that the earth motion after the earthquakes is extremely complicated having amplitudes, periods, accelerations and acting-directions very variable in time. It results as an imminent necessity a combination between the theoretical data and the experimental ones.

But it is known that the antiseismic security of buildings is not only a problem of calculation, but mainly of conception.

Without a precise knowledge - of physical viewpoint- of the destroying action of earthquakes and of the total behaviour of a building to earth motion, it will be difficult to assume that only by an abile manipulation of some mathematic developments it will be possible to conceive and to project a building resisting to earthquake.-

NOMENCLATURE

a_{ki} - form coefficient precising the dynamic response in point , corresponding to the normal vibration mode;

$C(T_i)$ - spectrum seismic coefficient;

E - Young's modulus of elasticity;

$F_k(i)$ - seismic force acting at k level corresponding to i vibration modulus;

g - gravity acceleration;

i - index precising the order of normal modes of circular frequencies and of the proper vibration periods;

$I_j = k_j I_o$ - inertia moment of beams at j level;

I_o - arbitrary inertia moment relative to which are reported the inertia moments of columns by means of k_j coefficient;

j, k, r - current points designing the position of stories, and of the masses M_j, M_k and M_r.

$l_j = \lambda_j l_o$ - height between floor j and j-1

l_o - arbitrary length relative to which are reported the distances between the stories by means of the λ_j coefficient;

m - number of the columns at the ground floor;

$M_k = \frac{Q_k}{g}$ - mass of the building load at the k-level;

n - number of the fixation points of masses equal to the number of stories and the number of the degrees of freedom of the building;

$Q_k = \alpha_k Q_o$ - gravity load concentrated at k level ;

Q_o - arbitrary gravity load, relative to which are reported the other loads by means of the coefficient;

$T_i = \frac{2\pi}{\omega_i}$ - proper vibration period of building corresponding to the normal mode i of vibration;

Z_{ki} - displacement of k point corresponding to the normal mode I ;

$Z_k(t)$ - real displacement of point k on the direction of oscillations (horizontal);

ξ_L, ξ_U, ξ_1 - adimensional factors which characterise the lower limit, the upper limit respectively the proposed value of the fundamental vibration frequency;

$\omega_L, \omega_U, \omega_1$ - fundamental frequency, lower limit, upper limit and the proposed one;

ω_i - circular vibration frequency corresponding to the normal mode i.

$q_j = \frac{\lambda_j^3}{k_j}$; $\bar{q}_j = \frac{1}{k_j}$

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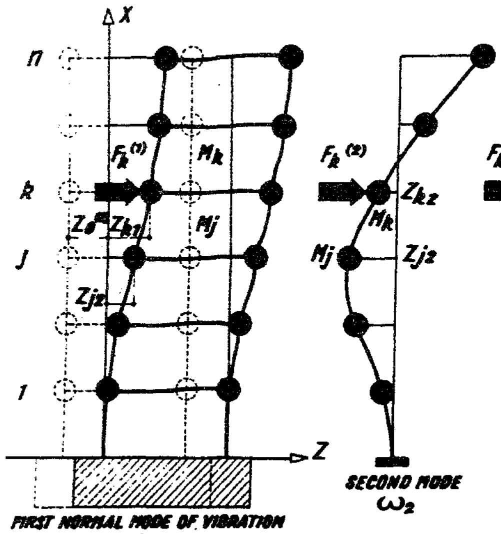

FIGURE 1

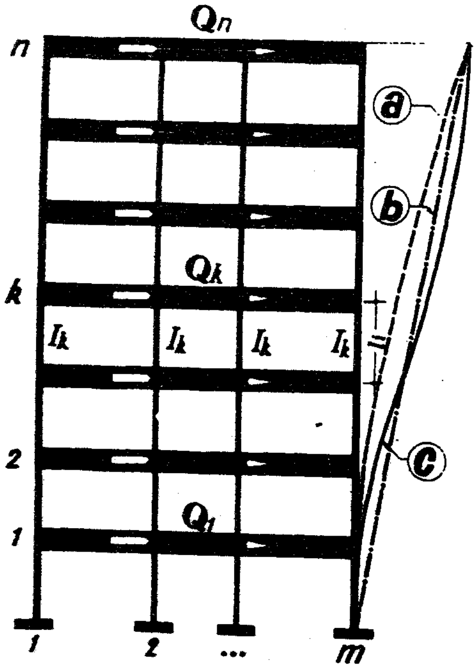

FIGURE 2

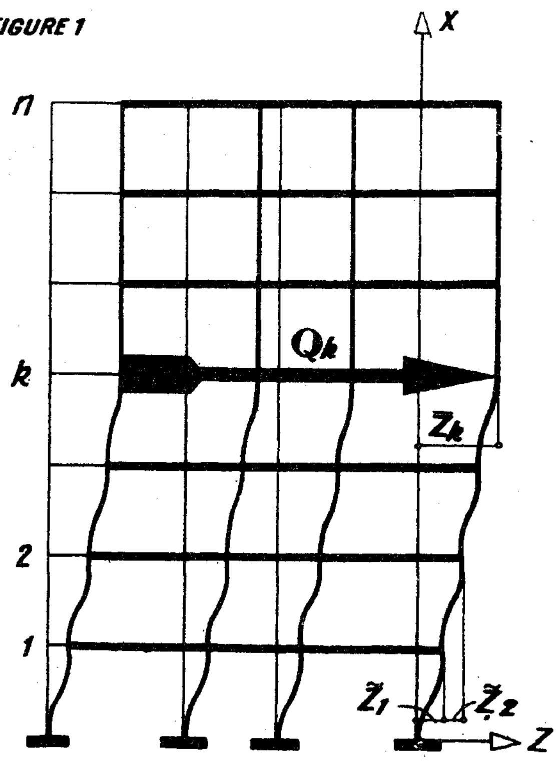

FIGURE 3

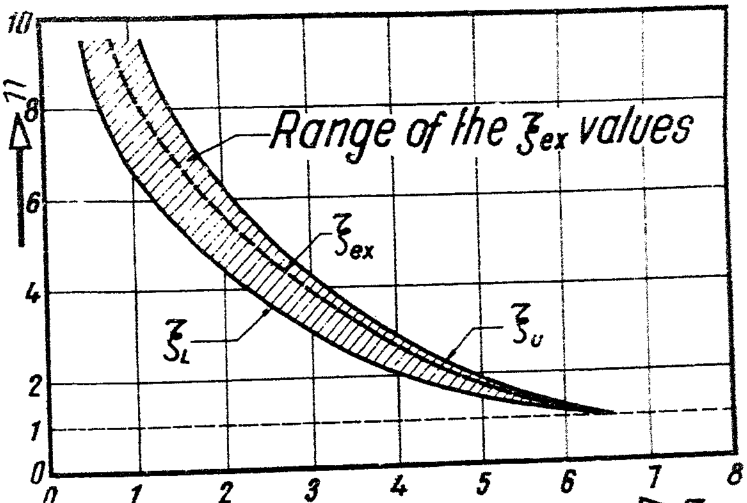

FIGURE 4

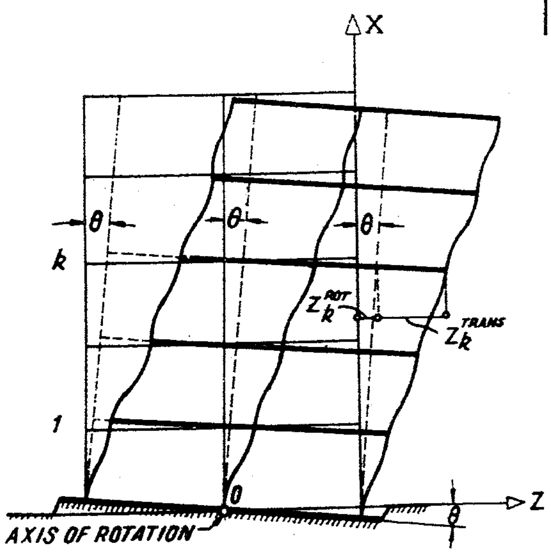

FIGURE 5

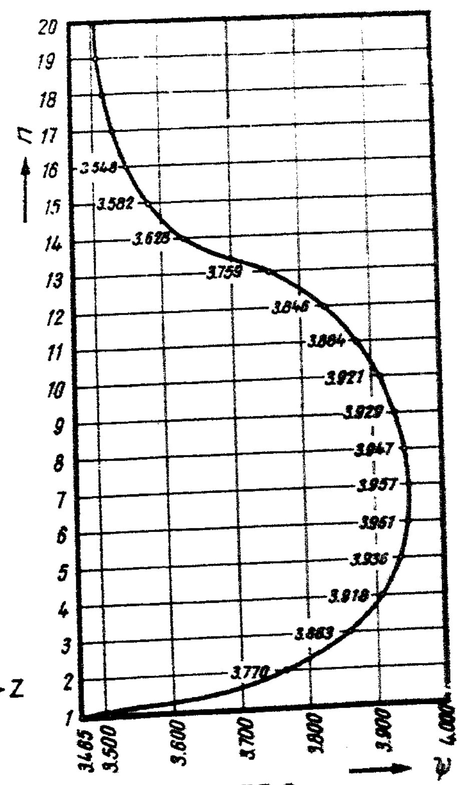

FIGURE 6

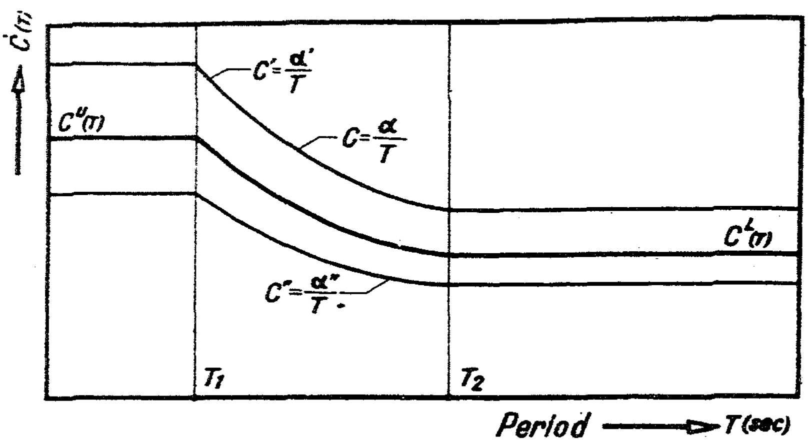

FIGURE 7

Table 1

BUILDING CHARACTERISTICS																		
m = 3	$\lambda_j = \frac{l_j}{l_o}$						$k_j = \frac{I_j}{I_o}$						$\alpha_j = \frac{Q_j}{Q_o}$					
n	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	k_1	k_2	k_3	k_4	k_5	k_6	α_1	α_2	α_3	α_4	α_5	α_6
1	1	—	—	—	—	—	1	—	—	—	—	—	1	—	—	—	—	—
2	1	1	—	—	—	—	1	0.6	—	—	—	—	1	1	—	—	—	—
3	1	1	1	—	—	—	1	1	0.6	—	—	—	1	1	1	—	—	—
4	1	1	1	1	—	—	1	1	0.6	—	—	—	1	1	1	1	—	—
5	1	1	1	1	1	—	1	1	0.6	0.6	0.4	—	1	1	1	1	1	—
6	1	1	1	1	1	1	1	1	0.6	0.6	0.4	0.4	1	1	1	1	1	1

Table 2

		$\mathfrak{J}$ COEFFICIENTS					DEVIATIONS				INTERVAL	
FRAME SHAPE	n	$\mathfrak{J}_L$	$\mathfrak{J}_{ex}$	$\mathfrak{J}_U$	$\mathfrak{J}_U$	$\mathfrak{J}_r$	ε_L	ε_U	ε_U	ε_1	η	η
	1	6	6	6	6	6	0	0	0	0	1	1
	2	3.131	3.408	3.410	3.432	3.410	8.13	0.06	0.71	-0.06	1.089	1.096
	3	2.323	2.593	2.605	2.635	2.591	10.41	0.34	1.62	+0.08	1.120	1.135
	4	1.732	1.957	1.965	2.000	1.959	11.49	0.41	2.19	-0.10	1.134	1.155
	5	1.347	1.539	1.585	1.573	1.532	11.90	0.47	2.87	-0.13	1.142	1.168
	6	1.083	1.248	1.260	1.286	1.247	12.42	0.96	3.05	+0.08	1.153	1.177