APPLICATION OF PROBABILITY METHODS FOR DESIGN THE EFFECT OF SEISMIC FORCES ON ENGINEERING STRUCTURES

By

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The present paper** is an attempt to apply for the design of structures for seismic forces probability methods, which are widely used nowadays in radio engineering and automatic regulation theory (1.2).

1. Seismic Acceleration of the Structure Foundation as a Random Process

It is well known that most strong-motion earthquakes are caused by tectonic processes in the earth-crust which are connected with ruptures of its continuity at a certain area.

A seismic wave occuring at each rupture, due to an unordered location of geological stratifications, undergoes on its way an infinite number of reflections and refractions and comes to the foundation of the structure as irregular waves which cause unordered random movement of the foundation.

Let us set a number of instruments in a seismic region under similar conditions and then observe some earthquake. The accelerograms recorded present some functions of time $F_1(t)$, $F_2(t)$ ...... $F_n(t)$. The random character of functions is displayed in the fact that though all instruments are identical, the values of functions at any definite moment of time t are different. Multiplicity of these functions describes seismic acceleration as a random process, each of the functions of the multiplicity is called its realization. Accelerograms of two typical earthquakes are given in Fig. 1 as examples of realizations.

In a general case seismic acceleration is a non-stationary random process. Let us assume that we know statistic characteristics of such process and dynamic characteristic of a structure, i.e. its response to a unit impulse k(t) which is called the impulse transfer function.

Here we use the correlative theory of random processes, i.e. take into account only those properties of the process that are determined by its average value

$$m(t) = \overline{F(t)} \qquad (1.1)$$

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and a correlative function

$$B(t,t') = \overline{F(t)F(t')} \tag{1.2}$$

The line above the expressions (1.1) and (1.2) indicates averaging according to multiplicity of realizations.

In this case the response of the structure to random action $Q_j(t)$ at zero initial conditions is:

$$x(t) = \int_0^t K(t-\tau)\,Q(\tau)\,d\tau \tag{1.3}$$

The average values of the response is:

$$\overline{x(t)} = \int_0^t K(t-\tau)\,\overline{Q(\tau)}\,d\tau \tag{1.4}$$

The correlative function is:

$$B_x(t,t') = \int_0^t\int_0^{t'} K(t-\tau)\,K(t'-\tau')\,B_Q(\tau,\tau')\,d\tau\,d\tau' \tag{1.5}$$

The mean-square value of the response is:

$$\overline{x^2(t)} = \int_0^t\int_0^t K(t-\tau)\,K(t-\tau')\,B_Q(\tau,\tau')\,d\tau\,d\tau' \tag{1.6}$$

where $Q_j = -m_j F(t)$, $F(t)$ is seismic acceleration of the structure foundation, m_j - mass at the point j.

Supposing that the response of the structure has normal distribution it is possible to determine the probability of large deviations from the average value according to the average and mean-square values of responses

$$P[|X-\bar{X}| > K\sigma] \approx \frac{e^{-\frac{1}{2}K^2}}{K\sqrt{2\pi}} \qquad K > 1 \tag{1.7}$$

where $\sigma = \left[\overline{(x-\bar{x})^2}\right]^{\frac{1}{2}}$ is standard deviation.

However, such way of evaluating the dynamic effect of seismic action is connected with intricate calculations and it is of little use in practice.

Let us consider the spectrum approach to the solution of this problem. From a great number of accelerograms recorded in various seismic regions we shall select for investigation only those accelerograms which within a certain interval have a fixed character upon the average (Fig.1). In this case seismic acceleration of the structure foundation may be considered approximately as a stationary ergodic random process. For such process the average $\overline{F(t)}$ and $\overline{F(t)F(t)}$, obtained by averaging the corresponding quantities of all realizations, may be substituted by the average values of the same quantities according to time. Consequently, while determining the average value of "m" and correlative function $B(t-t') = B(\tau)$ of a random process it is possible to use a single realization recorded for a sufficiently great period of time.

Let us establish normalized correlative functions for a number of accelerograms, each accelerogram being the realization of a stationary random process.

Normalized correlative functions $R(\tau)$ for five accelerograms recorded at different time in some seismic regions of the USA (3,4) are presented in Fig.2; the curves of formula

$$R(\tau) = e^{-\alpha|\tau|} \cos\beta\tau \qquad \begin{pmatrix} \alpha = 6 \div 8.5 \\ \beta = 16 \div 20 \end{pmatrix} \qquad (1.8)$$

approximately representing experimental correlative functions are drawn by dotted lines.

Fig. 2 makes clear that the curves of the type (1.8) satisfactorily coincide with the part of the empirical curves ($0 < \tau < 0.10 - 0.12$ sec) that is based on sufficiently reliable experimental data. The time of correlating seismic acceleration, i.e. the interval of time τ, within which noticeable statistic connection between the values of the random function ($R(\tau) > 0.05$) takes place, is approximately equal to 1.0 - 1.5 sec. Then it is not difficult to ascertain that for obtaining sufficient statistic data about seismic acceleration the duration of period T on the accelerogram should be chosen queal to 10-12 sec. At such interval any accelerograms may be considered in the first approximation as realizations of a stationary random process. For engineer evaluations such assumption is quite suitable.

For establishing a correlative function of seismic acceleration seismograms may be used. Let us take the given seismogram for a possible realization of a stationary random process and establish the normalized correlative function $R_d(\tau)$ for seismic displacement.

It is known that while differentiating a random process its correlative function is subjected to double differentiation according to τ. Let us approximately represent a normalized correlative function of a curve displacement as

$$R_d(\tau) = e^{-\alpha|\tau|} \cos\beta\tau$$

then for the correlative function of seismic acceleration we shall obtain the following:

$$R_a(\tau) = e^{-\alpha|\tau|}\left\{\left[(\alpha^2-\beta^2)^2-4\alpha^2\beta^2\right]\cos\beta\tau + 4\alpha\beta(\alpha^2-\beta^2)\sin\beta|\tau|\right\} \quad (1.9)$$

If the correlative function is known the well-known equations by Hinchin[(5)] are of help in calculating another statistic characteristic of seismic acceleration - its spectrum density.

$$S(\omega) = \lim_{T\to\infty} \frac{1}{2T} \left| F_T(i\omega) \right|^2 \qquad (1.10)$$

where $F_T(i\omega)$ is complex spectrum of acceleration.

It is assumed that the normalised correlative function $R(\tau)$ is (1.8), then its corresponding spectrum density is as follows:

$$S(\omega) = 2B(0)\int_0^\infty e^{-\alpha\tau} \cos\beta\tau \cos\omega\tau \, d\tau = 2B(0)\,\alpha \frac{\omega^2 + m^2}{\omega^4 + 2a\omega^2 + m^4}$$

$$\text{Where} \qquad m^2 = \alpha^2 + \beta^2 \; ; \quad a = \alpha^2 - \beta^2 ; \qquad (1.11)$$

In Fig.3 there are given normalised spectrum densities $S_H(\omega)$ calculated according to (1.11) for correlative functions in Fig.2.

2. Response of the System with a Single Degree of Freedom to Random Forces in Transition Regime

If $x(i\omega,t)$ is the transfer function of the system, i.e. its response to forces is $e^{i\omega t}$ and $S(\omega)$ is the spectrum density of a random force, then the mean-square value of the system response is as follows:

$$x^2(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(\omega) \left| x(i\omega, t) \right|^2 d\omega \qquad (2.1)$$

where $|x(i\omega,t)|^2$ is the square of the transfer function modulus.

Let us determine $|x(i\omega,t)|^2$ for the system with a single degree of freedom.

The equation of motion of system is

$$m_1 \ddot{x} + (u + iv)cx = e^{i\omega t} \qquad (2.2)$$

The attenuation is taken into account by means of the hypothesis set forth by E. S. Sorokin (6)

where

$$u = \frac{4 - \gamma^2}{4 + \gamma^2}, \quad v = \frac{4\gamma}{4 + \gamma^2}, \quad \gamma = \frac{\delta}{\pi},$$

δ is the logarithmic decrement of attenuation, $c = m_1\omega_1^2$ is the rigidity coefficient of the system and m_1 is its mass.

The general solution of the equation (2.2)

$$x(i\omega,t) = x_H(i\omega,t) + \Phi(i\omega)e^{i\omega t}\quad(A+Bi)e^{-n\omega_1 t}(\cos K\omega_1 t$$

$$+ i\sin K\omega_1 t) + m_1|\Phi(i\omega)|^2(-\omega^2 + u\omega_1^2 - iV\omega_1^2)(\cos\omega t + i\sin n\omega t) \quad (2.3)$$

$x_H(i\omega,t)$ is the general solution of the equality (2.2) without its right part. It is represented by the sum of two exponential functions; one of the functions corresponds to attenuating oscillations and the other - to the increasing oscillations. The increasing oscillations which do not satisfy the condition of physical realization of the problem are thrown off in the general solution.

$\Phi(i\omega)$ is the transfer function of the system (its stationary response to the force $e^{i\omega t}$)

$$K = \frac{1}{\sqrt{1 + \frac{\gamma^2}{4}}} \quad ; \quad n = \frac{\gamma}{2\sqrt{1 + \frac{\gamma^2}{4}}}$$

Complex arbitrary constants A and B are determined from initial conditions

$$t = 0,\ \operatorname{Re} x(i\omega,t) = \operatorname{Re}\dot{x}(i\omega,t) = 0;\ \text{Hence at}\ n \approx \frac{\gamma}{2}. \quad (2.4)$$

$V \approx \gamma$, $u \approx 1$ the square of the transfer function modulus is

$$|x(i\omega,t)|^2 = |\Phi(i\omega)|^2\left\{1 + e^{-\gamma\omega_1 t} - 2e^{-\frac{\gamma}{2}\omega_1 t}\left[\cos(\omega-\omega_1)t - \frac{\gamma}{2}\sin(\omega-\omega_1)t\right]\right\}$$

$$-2\gamma e^{-\frac{\gamma}{2}\omega_1 t}\frac{\left[\omega_1^4 - \omega_1^2\omega^2 - \omega_1\omega(\omega_1^2 - \omega^2)\right]}{m_1^2(\omega^4 - 2u\omega_1^2\omega^2 + \omega_1^4)^2}\ \sin(\omega-\omega_1)t \quad (2.5)$$

The mean-square value of the seismic force acting upon the system is

$$\overline{Q_1^2(t)} = m_1^2\,\overline{F^2(t)} = B_1(0) \quad (2.6)$$

The spectrum density of a random force is

$$S(\omega) = 2B_1(0)\,\alpha\ \frac{\omega^2 + m^2}{\omega^4 + 2a\omega^2 + m^4} \quad (2.7)$$

Substituting (2.5) and (2.7) to (2.1) we obtain the following:

$$\overline{X_1^2(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(\omega)\left|\chi(i\omega,t)\right|^2 d\omega$$

$$= \frac{2B(0)\alpha}{m_1^2}\Big\{(1+e^{-\gamma\omega_1 t})\Gamma_1 - 2e^{-\frac{\gamma}{2}\omega_1 t}(\cos\omega_1 t + \frac{\gamma}{2}\sin\omega_1 t)\Gamma_2$$

$$-2\gamma e^{-\frac{\gamma}{2}\omega_1 t}\left[\omega_1^2 \sin\omega_1 t\,\Gamma_3 + \omega_1\cos\omega_1 t\,\Gamma_4\right]\Big\}$$

$$\Gamma_1 = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{(\omega^2+m^2)\,d\omega}{(\omega^4-2u\omega_1^2\omega^2+\omega_1^4)(\omega^4+2a\omega^2+m^4)}$$

$$= \frac{\left[\frac{\omega_1^3}{\alpha}+\frac{1}{\gamma}(1+\gamma^2)\omega_1^2+2\alpha\omega_1+\frac{m^2}{\gamma}\right]}{2\omega_1^3\left[\omega_1^4+2\alpha\gamma\omega_1^3+(4\alpha^2+\gamma^2m^2-2m^2)\omega_1^2+2\alpha\gamma m^2\omega_1+m^4\right]}$$

$$\Gamma_2 = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{(\omega^2+m^2)\cos\omega t\,d\omega}{(\omega^4-2u\omega_1^2\omega^2+\omega_1^4)(\omega^4+2a\omega^2+m^4)}$$

$$= \frac{e^{-\alpha t}}{4\alpha\beta}\left[A(\beta\cos\beta t-\alpha\sin\beta t)+\frac{B}{m^2}(\beta\cos\beta t+\alpha\sin\beta t)\right]$$

$$+\frac{e^{-\frac{\gamma}{2}\omega_1 t}}{2\gamma\omega_1}\left[C(\cos\omega_1 t-\frac{\gamma}{2}\sin\omega_1 t)+\frac{D}{\omega_1^2}(\cos\omega_1 t+\frac{\gamma}{2}\sin\omega_1 t)\right] \qquad (2.10)$$

$$A = -C = \frac{2m^2(a+\omega_1^2)+\omega_1^4-m^4}{(\omega_1^4-m^4)^2+4(a+\omega_1^2)(m^4+a\omega_1^2)\omega_1^2}$$

$$B = \frac{-2m^4(a+\omega_1^2)A+m^2}{\omega^4-m^4}; \qquad D = \frac{2\omega_1^4(a+\omega_1^2)A-m^2}{\omega_1^4-m^4}$$

$$\Gamma_3 = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{(\omega^2-\omega_1^2)(\omega^2+m^2)\cos\omega t\,d\omega}{(\omega^4+2a\omega^2+m^4)(\omega^4-2u\omega_1^2\omega^2+\omega_1^4)^2}$$

$$= \frac{e^{-\alpha t}}{4\alpha\beta}\left[A_1(\beta\cos\beta t-\alpha\sin\beta t)+\frac{B_1}{m^2}(\beta\cos\beta t+\alpha\sin\beta t)\right]$$

$$+\frac{e^{-\frac{\gamma}{2}\omega_1 t}}{2\gamma\omega_1}\left[E_1(\cos\omega_1 t-\frac{\gamma}{2}\sin\omega_1 t)+\frac{F_1}{\omega_1^2}(\cos\omega_1 t+\frac{\gamma}{2}\sin\omega_1 t)\right]$$

$$+\frac{e^{-\frac{\gamma}{2}\omega_1 t}}{4\gamma^3\omega_1^3}\Big\{C_1\left[(1+\frac{\gamma}{2}\omega_1 t)\cos\omega_1 t-\frac{\gamma^3}{8}\sin\omega_1 t\right]$$

$$+\frac{D_1}{\omega_1^2}\left[(1+\frac{\gamma}{2}\omega_1 t)\cos\omega_1 t+\frac{\gamma^2}{2}\omega_1 t\sin\omega_1 t\right]\Big\} \qquad (2.11)$$

$$\Gamma_4 = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\omega(\omega^2-\omega_1^2)(\omega^2+m^2)\sin\omega t\, d\omega}{(\omega^4-2u\,\omega_1^2\omega^2+\omega_1^4)^2(\omega^4+2a\omega^2+m^4)}$$

$$= \frac{e^{-\alpha t}}{4\alpha\beta}\left[A_1(2\alpha\beta\cos\beta t - a\sin\beta t) + B_1\sin\beta t\right]$$

$$+ \frac{e^{-\frac{\gamma}{2}\omega_1 t}}{4\gamma^3\omega^4}\left\{C_1\left[\frac{\gamma^2}{4}\omega_1 t\cos\omega_1 t + (1+\frac{\gamma}{2}\omega_1 t)\sin\omega_1 t\right]\right.$$

$$\left. + \frac{D_1}{2\omega_1^2}\left[-\frac{\gamma^2}{2}\omega_1 t\cos\omega_1 t + (1+\frac{\gamma}{2}\omega_1 t)\sin\omega_1 t\right]\right\}$$

$$+ \frac{e^{-\frac{\gamma}{2}\omega_1 t}}{2\gamma}\left[E_1(\gamma\cos\omega_1 t + \sin\omega_1 t) + \frac{F_1}{\omega_1^2}\sin\omega_1 t\right] \qquad (2.12)$$

$$A_1 = -E_1 = \frac{\left[(\omega_1^4-m^4)^2 - 4m^4(\omega_1^2+a)^2\right](m^2-\omega_1^2-2a)}{\left[(\omega_1^4-m^4)^2+4a(\omega_1^2+a)(\omega_1^4-m^4) - 4m^2(m^2+\omega_1^2)(\omega_1^2+a)\right]\left[\omega_1^4-m^4+2a(\omega^2+a) + 4m^4(\omega_1^2+a)^2\right]^2}$$

$$B_1 = -\frac{m^2-\omega_1^2-2a-\left\{\left[\omega_1^4-m^4+4a(\omega_1^2+a)\right]^2-4m^4(\omega_1^2+a)^2\right\}A_1}{4(\omega_1^2+a)\left[\omega_1^4-m^4+2a(\omega_1^2+a)\right]}$$

$$D_1 = 1+\left[2\omega_1^4(\omega_1^2+a)-4m^4(\omega_1^2+a)+8a(\omega_1^2+a)^2\right]A_1$$
$$-\left[\omega_1^4-m^4+4(\omega_1^2+a)^2\right]B_1$$

$$C_1 = 2(a+\omega_1^2)B_1 - \left[\omega_1^4-m^4+4a(\omega_1^2+a)\right]A_1\;;$$

$$F_1 = 2(a+\omega_1^2)A_1 - B_1\;;$$

The mean-square value of the dynamical coefficient is as follows:

$$\overline{\xi_1^2(t)} = \frac{\overline{\chi_1^2(t)}\, m_1^2 \omega_1^4}{B_1(0)}$$

$$= 2\alpha\omega_1^4\Big[\left(1+e^{-\gamma\omega_1 t}\right)T_1 - 2e^{-\frac{\gamma}{2}\omega_1 t}\left(\cos\omega_1 t + \frac{\gamma}{2}\sin\omega_1 t\right)T_2$$
$$-2\gamma e^{-\frac{\gamma}{2}\omega_1 t}\left(\omega_1^2 \sin\omega_1 t\, T_3 + \omega_1 \cos\omega_1 t\, T_4\right)\Big] \qquad (2.13)$$

The diagrams of dynamical coefficients $\xi_1(t) = \sqrt{\overline{\xi_1^2(t)}}$ established for frequencies of free oscillations of the system $\omega_1 = 20.94$, 6.28 1/sec are presented in Fig.4. The diagrams show that for the system with the frequency $\omega_1 = 20.94$ 1/sec. (structures of average rigidity) already at $t > 3$ sec. the value of dynamical coefficient $\xi_1(t)$ is close to the value of ξ in stationary regime, i.e. at $t \to \infty$; for the frequency $\omega_1 = 6.28$ (flexible structures) $\xi_1(t)$ practically coincides with ξ_1 at $t > 8$ sec.

Since the duration of an earthquake averages 15-25 sec. and transition regime for real buildings and structures with frequencies $\omega_1 = 31.4 - 2.09$ 1/sec. attenuates already at t approximately equal to 10-12 sec., the action of seismic forces upon the structure may be considered in stationary regime.

The ξ_1 diagrams established by formula (2.13) at $t \to \infty$ for the case of spectrum densities $S_x(\omega)$ shown in Fig.3 (at $\gamma = 0.1$) are presented in Fig.5.

The same figure shows the curves β obtained by prof. J. L. Korchinsky while treating a series of seismograms of earthquakes which occurred in the USSR; the "a" curve is adopted in Standards and Codes for Building in Seismic Regions (CH-8-57). The β curves for the part $T > 0.3$ sec. closely resemble the ξ_1 curves.

3. The Action of Seismic Forces upon the System with Two and Three Degrees of Freedom

The mean-square value of the structure response to these forces can be easily determined if the spectrum density of the response is known.

$$\overline{\chi_1^2} = \overline{(\chi_{11}+\chi_{12})^2} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{x_1}(\omega)\, d\omega \qquad (3.1)$$

Let the expressions $\chi_{11}(i\omega)$, $\chi_{11}^*(i\omega)$, $\chi_{12}(i\omega)$ and $\chi_{12}^*(i\omega)$ be complex and complex conjugated spectrums of the structure response. The first index at $\chi(i\omega)$ is the number of the co-ordinate and the second is the number of force.

Then

$$\begin{aligned}\mathcal{X}_{11}(i\omega) &= \Phi_{11}(i\omega)\,Q_1(i\omega) = -m_1\Phi_{11}(i\omega)\,F(i\omega)\\ \mathcal{X}^*_{11}(i\omega) &= \Phi^*_{11}(i\omega)\,Q^*_1(i\omega) = -m_1\Phi^*_{11}(i\omega)\,F^*(i\omega)\\ \mathcal{X}_{12}(i\omega) &= \Phi_{12}(i\omega)\,Q_2(i\omega) = -m_2\Phi_{12}(i\omega)\,F(i\omega)\\ \mathcal{X}^*_{12}(i\omega) &= \Phi^*_{12}(i\omega)\,Q^*_2(i\omega) = -m_2\Phi^*_{12}(i\omega)\,F^*(i\omega)\end{aligned} \tag{3.2}$$

Here $\Phi_{11}(i\omega)$ and $\Phi_{12}(i\omega)$ are transfer functions for the response of point 1 to forces applied at points 1 and 2.

$$\Phi^*_{11}(i\omega) = \Phi_{11}(-i\omega) = \Phi_{11}(i\omega)\ ;\quad \Phi^*_{12}(i\omega) = \Phi_{12}(-i\omega) = \Phi_{12}(i\omega) \tag{3.3}$$

$F_1(i\omega)$, $F^*_1(i\omega)$, $F_2(i\omega)$ and $F^*_2(i\omega)$ are complex and complex conjugated spectra of seismic acceleration F(t).

Hence the spectrum density of the response is as follows:

$$S_{X_1}(\omega) = \lim_{T\to\infty}\frac{|\mathcal{X}_1(i\omega)|^2}{2T} = \lim_{T\to\infty}\frac{[\mathcal{X}_{11}(i\omega)+\mathcal{X}_{12}(i\omega)][\mathcal{X}^*_{11}(i\omega)+\mathcal{X}^*_{12}(i\omega)]}{2T}$$

$$= |\Phi_{11}(i\omega)|^2 S_1(\omega) + 2\,\Phi_{11}(i\omega)\Phi_{12}(i\omega)S_{12}(\omega) + |\Phi_{12}(i\omega)|^2 S_2(\omega) \tag{3.4}$$

$$S_1(\omega) = B_1(0)S_H(\omega)\ ;\ S_2(\omega) = B_2(0)S_H(\omega)\ ;\ S_{12}(\omega) = \frac{m_2}{m_1}B_1(0)\,S_H(\omega)\ ;$$

$$B_1(0) = m_1^2\overline{F^2(t)}\ ;\ B_2(0) = m_2^2\overline{F(t)}\ ;\ S_H(\omega) = 2\alpha\,\frac{\omega^2+m^2}{\omega^4+2a\omega^2+m^4} \tag{3.5}$$

and

$$\overline{\mathcal{X}_1^2} = \overline{\mathcal{X}_{11}^2} + \overline{\mathcal{X}_{12}^2} + 2\,\overline{\mathcal{X}_1\mathcal{X}_2} = \frac{2\alpha}{m_1^2}\left[B_1(0)A_1(T_{11}-T_{12}) + \frac{B_2(0)C_{12}^2}{m_2^2}A_2(T_{21}-T_{22})\right]$$

$$+\frac{m_2B_1(0)}{\pi\,m_1}\int_{-\infty}^{\infty}\Phi_{11}(i\omega)\Phi_{12}(i\omega)S_H(\omega)\,d\omega \tag{3.6}$$

where m_1 and m_2 are masses of the system.

Tor the system with three degrees of freedom we find out the following:

$$\begin{aligned}\mathcal{X}_1^2 = \overline{(\mathcal{X}_{11}+\mathcal{X}_{12}+\mathcal{X}_{13})^2} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\Big[&|\Phi_{11}(i\omega)|^2S_1(\omega) + |\Phi_{12}(i\omega)|^2S_2(\omega)\\ &+ |\Phi_{13}(i\omega)|^2S_3(\omega) + 2\Phi_{11}(i\omega)\Phi_{12}(i\omega)S_{12}(\omega) + 2\Phi_{11}(i\omega)\Phi_{13}(i\omega)S_{13}(\omega)\\ &+ 2\,\Phi_{12}(i\omega)\Phi_{13}(i\omega)S_{23}(\omega)\Big]d\omega\end{aligned} \tag{3.7}$$

$$S_3(\omega) = m_3^2\overline{F_2(t)}\,S_H(\omega)\ ;\ S_{13}(\omega) = \frac{m_3}{m_1}B_2(0)S_H(\omega)\ ;\ S_{23}(\omega) = \frac{m_3}{m_2}B_2(0)S_H(\omega) \tag{3.8}$$

The formulae for the determination of $\Phi_{1K}(i\omega)$, $|\Phi_{1K}(i\omega)|^2$, A_{jK}, Γ_{jK} are cited in the work (8).

4. The Determination of the Design Seismic Loading of the Structure

Let us assume as a design scheme of the structure a cantilever elastic rod with masses m_j concentrated at the points 1,2... n. Then the design seismic load acting upon the structure which is considered to be the system with a single degree of freedom, may be determined according to the formula

$$P_c = \xi\, m_{red}\, a_d \tag{4.1}$$

Here ξ is the dynamical coefficient determined according to the envelope of the diagram family which is presented in Fig.5.

$m_{red} = \frac{\Sigma m_j x_j}{x_K}$ is the reduced mass of the structure, concentrated at the point k; x_j are the co-ordinates of the curve defining the first mode of free oscillations of the structure.

$a_{des} = \sqrt{\overline{F^2(t)}}$ is the acceleration of a design earthquake for a given seismic region. The design acceleration is determined on the grounds of statistical treating a series of accelerograms or seismograms recorded for a certain period of time, e.g. for 20 - 25 years. The repetition of design acceleration is evaluated according to its guaranteeing which may be taken for 1 - 2 per cent. It means that the exceeding of the design acceleration should be expected upon the average once in 50 - 100 years.

$\overline{F^2(t)}$ is the mean-square value of a design earthquake acceleration calculated according to the accelerogram for the interval of time equal to 10-15 sec.

It should be noted that the problem of seismic regionalization of the USSR territory is the basic one for determining seismic loadings of the structure. However, the solution of this problem presents some difficulties due to the absence of a sufficient number of accelerograms or seismograms of strong-motion earthquakes characterizing the seismic activity of a given region. Therefore in the present case for determining a design seismic load it is possible to use seismic coefficients $K_c = \frac{a\ des.}{g}$, adopted in standards CH-8-57 acting at the present time.

Such assumption makes design seismic loads calculated according to the probability method and those calculated according to CH-8-57 practically the same.

For systems with two and three degrees of freedom the design seismic load is determined according to the formula (4.1) for each reduced mass M_K.

Reduced masses are calculated from equations

$$\sum_{1}^{n} m_j x_{jK} = m_K x_K , \qquad (K = 1, 2, 3) \qquad (4.2)$$

here m_j - masses concentrated at points 1,2 ..... n, x_{jK} - co-ordinates of the K-mode of free oscillations of the structure. Frequencies and modes of oscillations of the structures may be determined according to the method of successive approximations.

The dynamical coefficient is

$$\xi_K = \sqrt{\overline{\xi_K^2}} = \sqrt{\frac{\overline{x_K^2}}{\overline{x_{Kcm}^2}}} \quad ; \qquad (4.3)$$

$\overline{x_K^2}$ is calculated according to the formulas (3.6) and (3.7).

For the system with two degrees of freedom

$$\overline{x_{1cm}^2} = \delta_{11}^2 B_1(0) + 2\frac{m_2}{m_1} B_1(0)\,\delta_{11}\delta_{12} + B_2(0)\,\delta_{12}^2 \ , \qquad (4.4)$$

For the system with three degrees of freedom

$$\overline{x_{1cm}^2} = \delta_{11}^2 B_1(0) + \delta_{12}^2 B_2(0) + \delta_{13}^2 B_3(0) + 2\frac{m_2}{m_1} B_1(0)\,\delta_{11}\delta_{12}$$

$$+ 2\frac{m_3}{m_1} B_1(0)\,\delta_{11}\delta_{13} + 2\frac{m_3}{m_2} B_2(0)\,\delta_{12}\delta_{13} \qquad (4.5)$$

where δ_{jk} are unit displacements of the main system.

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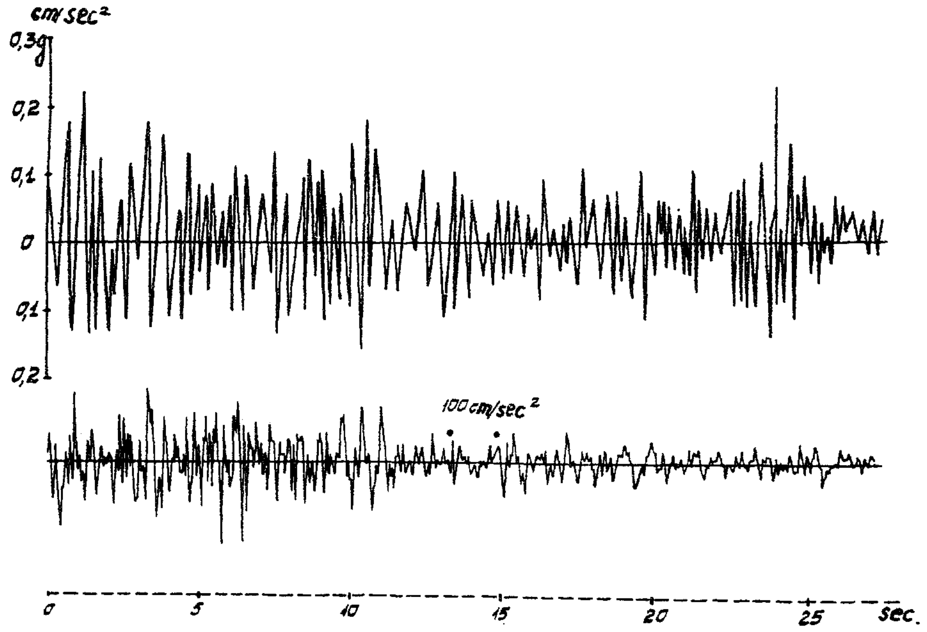

Fig. 1. Accelerograms of earthquakes.

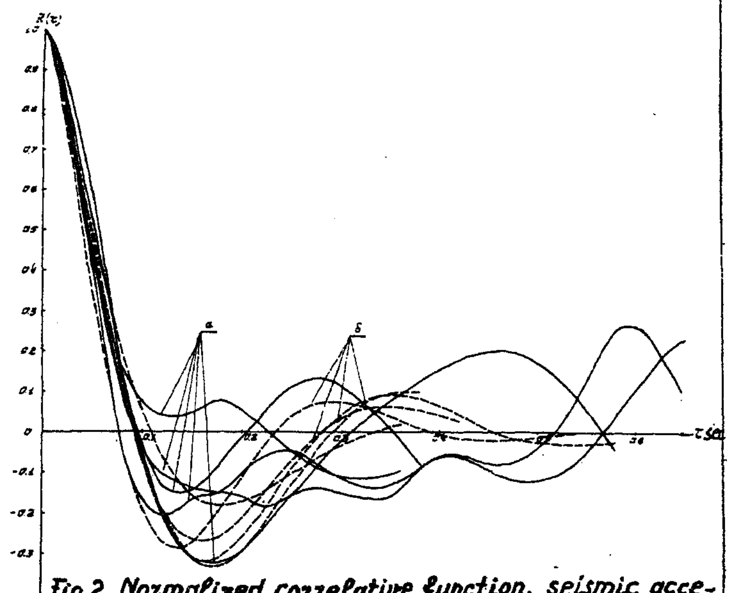

Fig. 2. Normalized correlative function, seismic acceleration a) experimental curves
б) curves of the type (1.8)

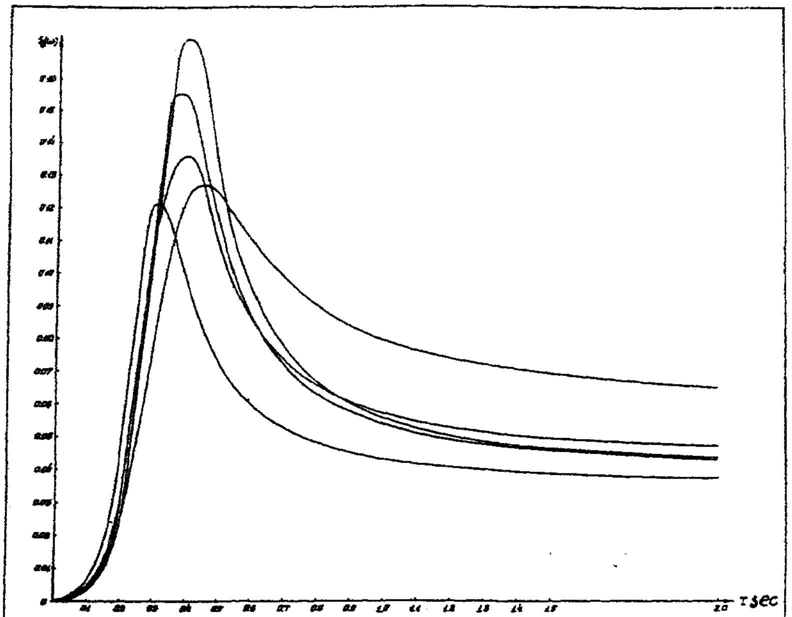

Fig 3. Normalized spectrum densities of seismic acceleration.

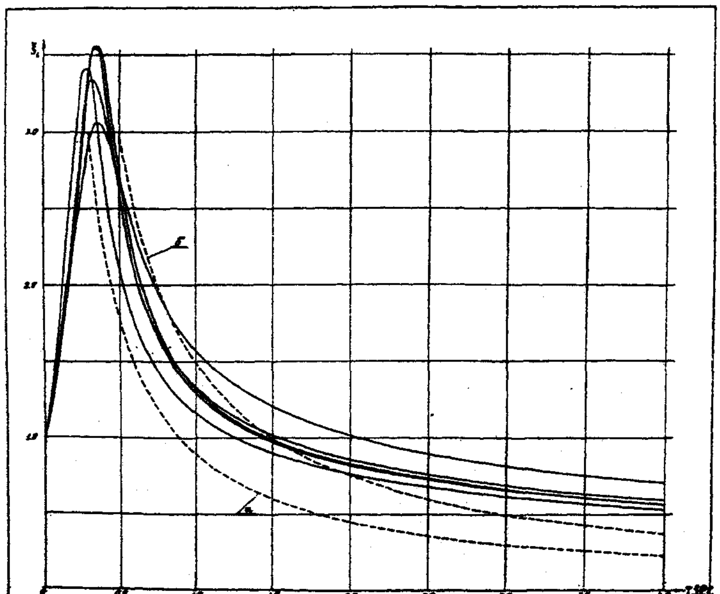

Fig.5. Diagrams of dynamical coefficients.
„a" und „b" are curves established by J.L. Korchinsky

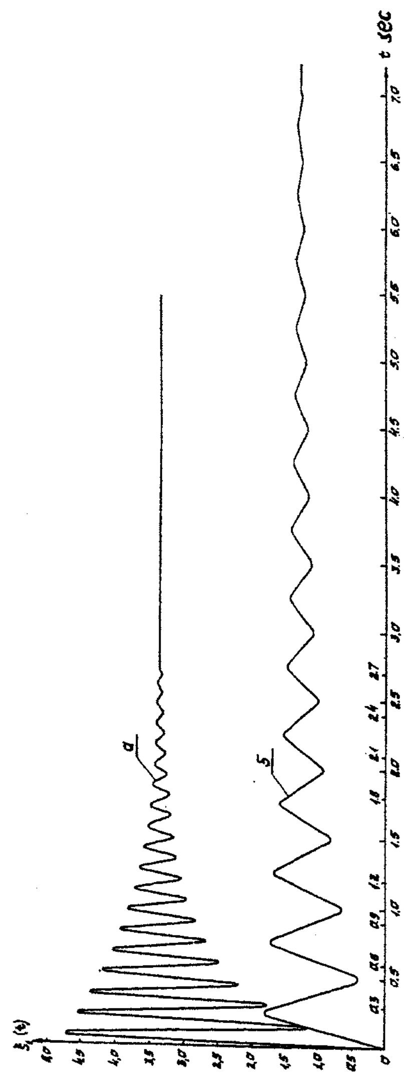

Fig.4. Diagrams of dynamical coefficients in transition regime

a. for the system with frequency $\omega = 20{,}94\ sec^{-1}$ ($T = 0{,}3\ sec$).

б. for the system with frequency $\omega = 6{,}28\ sec^{-1}$ ($T = 1\ sec$).