THE SEISMIC STABILITY OF EARTH DAMS

by N. N. Ambraseys *

1.1 Introduction. Earth dams when subjected to seismic movements of their foundations do not behave either as absolute rigid or perfectly elastic oscillators. If earth dams were absolutely rigid, the accelerations at all points within them would be equal to those of the ground on which they are founded. Field observations and experiments however, show, at least qualitatively, an increase in response with elevation, a fact which indicates that earth dams have considerable elasticity and that they may quasiresonate with certain ground motion components. The observed increase in acceleration with elevation shows beyond doubt that the seismic coefficient should also increase with elevation, and that the crest of an earth dam may be subjected to accelerations many times larger than those of the ground.

For small seismic disturbances, earth dams respond approximately as elastic oscillators; in general however, the response of such structures to strong ground motions would be non-elastic. The analytical treatment of such a problem is extremely involved. Approximate simulation of earth dam sections by other sections possessing linear stress-strain characteristics is therefore worth attempting. The use of highly damped linear systems or yield spectra, although not rigorously consistent with the behaviour of the dam, may give a solution of the right order of magnitude and be adequate for engineering purposes.

The device of using linear oscillators is to some extent justified by the fact that, irrespectively of whether earth dams behave as elastic or elasto-plastic bodies, their initial response is elastic. The treatment of the earth dam problem in an elastic framework is a first step forward although it is by no means the complete solution to the problem.

Common practice in earth dam design in seismic regions ignores the non-rigid nature of such structures, and a constant seismic acceleration is used throughout the structure. Earthquake resistant design codes do however recognise the non-rigid response of tall buildings, although for earth dams they enforce a constant seismic coefficient, Figure 1a. The author recently made an attempt to obtain information concerning methods employed in various countries for the seismic stability analysis of earth dams. With the exception of the 1957-USSR code(17), all other codes enforce a constant seismic coefficient which is independent of the dimensions of the dam and properties of the fill material; this coefficient can be assessed from seismic probability maps.

The Russian code of 1957, however, enforces a seismic coefficient in spectal form, which depends on the seismicity of the site. The magnitude of the seismic coefficient increases linearly towards the crest. The coefficient depends also on the dimensions and properties of the fill and foundation meterials. When the amount of critical damping in the fill material, and spectral intensity at the site are known, the seismic coefficient is shown in Figure 1b.

The fact that earth dams under seismic disturbances do not behave as absolutely rigid oscillators was first discussed in a paper published by the Ministry of Construction in Tokyo(19). Papers on this problem were published by Mononobe

*Lecturer, Dept. of Soil Mechanics, Imperial College of Science, London S.W.7

(1936), Martel (1938), Heiland (1940), Hatanaka (1955), Ambraseys (1959) and Napetvaridzhe (1959). The important conclusion that can be drawn from these papers is that the highest ground accelerations are not necessarily those which will cause the greatest damage to earth dams. A study of the results shown on Table I of the paper* dealing with the seismic behaviour of earth dams, shows that the maximum destructive effects on earth dams may not occur in the immediate vicinity of a fault-break or epicentre. At a distance of say 20 to 60 miles from the fault or epicentre much more damage may occur.

From Table 1* it appears that failures of earth dams are caused by long period oscillations, and that failures can be more easily caused by large slow movements of the ground than by high frequency shaking. The high frequency shaking, predominant on epicentral regions, may however produce rock slides from steep slopes or flows and slumping of loose soils.

It is not surprising, therefore, that while earth dams situated in the epicentral region of destructive earthquakes did not fail, those at distances of 20 to 60 miles were damaged by the longer period of ground movements although the ground accelerations were much smaller.

In this paper, assuming earth dams to be elastic bodies, the seismic coefficients for different cases are discussed. The soil strength parameters appropriate for the calculation of the seismic stability of the fill and foundation material are also discussed.

The Author is grateful to his colleagues at Imperial College, in particular to Dr. D.J. Henkel, for many helpful discussions.

2.1 The seismic coefficient for an elastic symmetrical solid.

In this section, the formulation of the problem under consideration will be presented in some detail; limitation of space does not permit detailed presentation of the solution and only bare outlines of the analysis will be given; the methods used are standard and the treatment is purely formal.

Let us consider an elastic solid occupying the region defined by $L \geqslant x \geqslant 0$, $H \geqslant y \geqslant h$, with symmetric conditions for z: $b(y) \geqslant z \geqslant -b(y)$, where $b(y)$ is a continuous function of y§ , Figure 2a. Such a solid would be (i) a strip, if $b(y)$ is a constant with $H \geqslant y \geqslant \rho$, (ii) a symmetric wedge of arbitrary cross-section if $b(y) = f(y)$, and (iii) a symmetric truncated wedge of arbitrary cross-section if $b(y) = f(y)$, $H \geqslant y \geqslant h$, with $f(h)$ finite.

A particular case of (ii) or (iii) is that for the completly symmetric wedge $b(y) = a.y$, with $H \geqslant y \geqslant 0$ (Figure 2d) and that for the truncated symmetric wedge $b(y) = a.y$ with $H \geqslant y \geqslant h$, where $\underline{a}$ is a numerical constant (Figure 2e).

Suppose that the shear rigidity of the solid varies with y alone, this variation being given as a continuous function of y say $G_o(y)$, subject to the condition that it does not vanish in the finite range $[0,H]$ or $[h,H]$.

Let us consider the response in shear of the elastic solid described by $b(y)$, (Figure 2a), when its rigid boundaries at $x = o$, $x = L$, and $y = H$ are subjected

---

* Ambraseys, N.N.(1960) "On the seismic behaviour of earth dams" 2nd World Conf. Earthq. Eng., Tokyo.

§ See Appendix for explanation of symbols.

to an arbitrary unidirectional time dependent acceleration $g_o(t)$, which acts from $t = o$ uniformly at its base and over the two vertical sides and at right angles to the plane of symmetry of the solid.

If the elastic displacements due to shear, relative to the moving boundaries, at a point be $u(x,y,t)$, the oscillations of the solid in shear obey the equation

$$q(y)\nabla^2 u + \frac{dq(y)}{dy}\frac{\partial u}{\partial y} + p(y)\left[\frac{\partial^2 u}{\partial t^2} + c\frac{\partial u}{\partial t} - g_o(t)\right] = o \text{ ------(1)}$$

where

$q(y) = G(y)b(y) > o$; $\quad p(y) = -b(y).s^{-2}$; $\quad G_o(y) = G_t.G(y)$;

$G(o) = G(1) = 1$, and $\underline{s}$ is the velocity of the shear waves which correspond to G_t for a given unit mass density of the solid.

The boundary conditions for equation (1) are given by:

$u(o,y,t) = o$ ....(2); $\quad u(L,y,t) = o$ .... (3)

$u(x,H,t) = o$ ....(4); $\quad \frac{\partial u}{\partial y} = 0$ at $y = h$ .... (5)

and

$u(x,y,t) = \frac{\partial u}{\partial t} = o$ for $t = o$ .... (6)

We assume that the variable coefficients of equation (1) are restricted in the region $[h,H]$ by the following conditions: $q(y)$ and its derivatives are all continuous; $q(y) \neq o$; $p(y)$ is continuous, and the range $[h,H]$ remains for all cases finite.

The most convenient method of solving equation (1) under the conditions (2) to (6), involves the application of a multiple Fourier type transform

$$\bar{u}_s(m,y,j-c/2) = \int_0^\infty\int_0^L u(x,y,t)\sin(mx)\exp\left[-t(j-c/2)\right]dt\,dx$$

where $j > c/2$. This transformation with the boundary conditions (2), (3) and (6) reduces equation (1) to

$$L(\bar{u}_s) + j^2 p(y)\bar{u}_s + R(y,j) = o \text{ ....(7)}$$

where

$$L(\bar{u}_s) = \frac{d}{dy}\left[q(y)\frac{d\bar{u}_s}{dy}\right] - \left[m^2 q(y) + p(y)\,c^2/4\right]\bar{u}_s$$

$$R(y,j) = -\int_0^\infty\int_0^L g_o(t)p(y)\sin(mx)\exp\left[-t(j-c/2)\right]dt\,dx$$

and $m = r\pi/L$ with $\underline{r}$ odd.

It is now necessary to find solutions of equation (1) -and also find values of j for which there exists an integral of (1)-continuous and having a continuous derivative, not identically equal to zero, and satisfy the transformed boundary conditions (4) and (5). The variable parametre $p(y)$ is a negative function and j is a numerical parametre the value of which is not given beforehand.

The non-homogeneous equation (1) can be shown to correspond to a non-homogeneous integral equation with, in general, an unsymmetric nucleus, which can be easily solved in series form, provided the nucleus is first symmetrised and the existence of a solution of the Fredholm equation is justified. The complete inversion of $u_s(m,y,j-c/2)$ does not present analytical difficulties and it can easily be established that the solution of (1) under the boundary conditions (2) to (6) can be expressed by double series in normal modes. For small values of damping the seismic coefficient will be given by

$$k=\frac{4}{g\pi}\sum_{n=1,2}^{\infty}\sum_{r=1,3}^{\infty} r^{-1}\sin\left(\frac{r\pi x}{L}\right) f_n(y)\int_h^H f_n(w)p(w)\,dw\cdot S_a \text{ -----(8)}$$

where

$$S_a=W_{nr}\int_0^t g_o(t')\sin\left[W_{nr}(t-t')\right]\exp\left[-\lambda W_{nr}(t-t')\right]dt'$$

and $f_n(y)$ are the fundamental functions, forming an orthonormal system, and satisfying the homogeneous equation associated with (7) in the range $[h,H]$ and also satisfy the appropriate boundary conditions. The w_{nr} are the undamped frequencies of the oscillating solid.

Many physical and engineering problems depend for solution on the homogeneous equation of (7). Such solutions, however, can be obtained only for particular forms of the variable coefficients involved. It can be established that if the homogeneous equation of (7) possesses less than three regular singularities, a fact which will depend on the form of $G(y)$ and $b(y)$, its solution can be obtained in terms of elementary functions. With three regular singular points the solution, while no longer expressible in terms of elementary functions, can be obtained in terms of hypergeometric functions. When two of the regular singularities coalesce an irregular singularity is obtained and the resulting equation has confluent hypergeometric functions as solutions.

Hence, although formally a wide variety of problems can be solved, for engineering purposes the evaluation of results from transcendental functions becomes extremely involved, and in some instances when the zeros of confluent hypergeometric functions with respect to one of its parameters are required, solutions become impracticable.

In the following sections we shall derive the seismic coefficient for a number of structural forms.

2.2 Shear response of overburdens. Let us consider the case of an overburden of constant rigidity occupying the region $L \geqslant x \geqslant o$, $H \geqslant y \geqslant o$, $-\infty < z < +\infty$, (Figure 2b). It can easily be shown that the fundamental functions in this case are given by

$$f_n(y) = \sqrt{2}\cos\left[\frac{2n-1}{2}\pi y'/D\right](-1)^{n+1}, \quad n = 1, 2, 3, \text{------}$$

The undamped frequencies of the overburden will be given by

$$W_{nr} = \left(\frac{S}{D}\right)\left\{\left[\frac{2n-1}{2}\pi\right]^2 + \left(\frac{r\pi}{\mu}\right)^2\right\}^{1/2} \qquad \text{.............(9)}$$

and the seismic coefficient is

$$k = \frac{4}{g\pi}\sum_{n=1,2}^{\infty}\sum_{r=1,3}^{\infty} 4(-1)^{n+1}\cos\left(\frac{2n-1}{2}\pi y'/D\right)\sin\left(\frac{r\pi x}{L}\right)\frac{Sa}{(2n-1)r\pi} \text{ ---(10)}$$

In the case when the shear rigidity of the overburden diminishes upwards with a constant gradient, there will be a lower finite limit to the rigidity at the surface, G_t. By inspecting the fundamental equation it will be noticed that the characteristic values and functions involve confluent hypergeometric functions as well as the zeros of such functions with respect to their first parameter. A detailed discussion of this problem here is not possible; it is of interest, however to note that if μ tends to infinity (one-dimensional problem) then the hypergeometric functions reduce to Bessel functions. In this case the frequencies of free oscillation are given by

$$W_r = \left(\frac{S}{D}\right)\frac{1-k'^2}{2K}\cdot a'_n \qquad \text{.................... (11)}$$

and the seismic coefficient is given by

$$k = \frac{\pi}{2g}\sum_{n=1,2\cdots}^{\infty}\frac{J_0(a'_n Y')Y_0(a'_n) - J_0(a'_n)Y_0(a'_n Y')}{J_0^2(a'_n)/J_1^2(k'a'_n) - 1}\cdot Sa \qquad \text{........ (12)}$$

<u>2.3 The embankment problem. Complete wedge.</u> **In this case the** solid occupies the region $H \geqslant y \geqslant o$, $L \geqslant x \geqslant o$, and $-ay \leqslant z \leqslant +ay$, (Figure 2d). It may be assumed that the shear rigidity of the solid is independent of y. The orthonormal fundamental functions $f_n(y)$ can easily be shown to be given by

$$f_n(y) = \sqrt{2}\, J_0(\hat{a}_n y'/D)\,/\,J_1(\hat{a}_n)\,D$$

The frequencies of free oscillation of the wedge are

$$W_{nr} = \left(\frac{S}{D}\right)\left[\hat{a}_n^2 + \left(\frac{r\pi}{\mu}\right)^2\right]^{1/2} \qquad \text{............ (13)}$$

and the seismic coefficient is given by

$$k = \frac{4}{g\pi} \sum_{n=1,2}^{\infty} \sum_{n=1,3}^{\infty} 2r^{-1} \sin\left(\frac{r\pi x}{L}\right) \frac{J_0(\hat{a}_n y'/D)}{\hat{a}_n J_1(\hat{a}_n)} Sa \text{ .... (14)}$$

which is identical with the expression derived by Hatanaka (1955).

2.4 Truncated wedge. The solid in this case occupies the region $H \geqslant y \geqslant h$, $L \geqslant x \geqslant o$, and $-ay \leqslant z \leqslant +ay$, (Figure 2e). The frequencies of free oscillation of the solid are now given by

$$W_{nr} = \left(\frac{S}{D}\right)(1-k'')\left[a_n''^2 + \left\{\frac{\pi r(1-k'')}{\mu}\right\}^2\right]^{1/2} \text{ .... (15)}$$

while the seismic coefficient can be expressed in the form

$$k = \frac{4}{g} \sum_{n=1,2}^{\infty} \sum_{r=1,3}^{\infty} r^{-1} \sin\left(\frac{r\pi x}{L}\right) \frac{J_0(a_n'' Y'')\, Y_0(a_n'') - J_0(a_n'')\, Y_0(a_n'' Y'')}{J_0^2(a_n'')/J_1^2(k'' a_n'') - 1} Sa \text{ ---- (16)}$$

2.5 Discussion. All symbols used in the preceeding sections are explained in the Appendix. A complete treatment of sections 2.2 to 2.4, though following a different analytic approach, is given in references (11) and (12). Numerical values for the parameters a_n', $\hat{a}_n$, and a_n'' as functions of k', k'' and n are given in Tables I and II.

For the one-dimensional problem it is of interest to note that for $k''=o$ the seismic coefficient for a truncated wedge, (equation 16), reduces to that for a complete wedge (equation 14), and for $k''=1$ it reduces to the case of an overburden of constant rigidity (equation 10). In Figure 3 the variation of the seismic coefficient pertaining to the first mode, at the crest or surface of the oscillating medium, and for the one-dimensional case, is depicted as a function of the parameters k' and k''. Point A on Figure 3 corresponds to the case of an overburden with constant rigidity; point B to that for a complete wedge. The curve between points A and B corresponds to the truncated wedge problem, while the curve between A and C corresponds to the case of an overburden the rigidity of which increases linearly with depth. For higher modes of oscillation plots similar to those in Figure 3 can be obtained.

Figure 4 shows the variation with depth of the seismic coefficient for the one-dimensional case when the coefficient of truncation k'' varies from one to zero, that is when the solid degenerates from a layer of finite thickness to a complete wedge.

Figure 5 gives a plot of the first six dimensionless periods of oscillation $\bar{T}$ which are characterised by a value of the truncation coefficient between zero and and one. For the two-dimensional case a nomographic representation for the periods of free oscillation $\bar{T}_n$ are given in reference (13).

3.1 <u>The seismic coefficient for a symmetric wedge and underlying elastic layer of finite thickness.</u> The geometric configuration of the dam foundation system to be considered in this section is shown in Figure 6a. For the sake of simplicity we shall consider here the one-dimensional problem ($\mu \to \infty$) with $k'' = o$ and $G(y)$ = constant. It can be shown that the equations describing the motion of the dam-foundation system in shear deformation are:

$$\frac{\partial^2 u_1}{\partial y^2} + \frac{1}{y}\frac{\partial u_1}{\partial y} - S_1^{-2}\left[\ddot{u}_1 + C\cdot\dot{u}_1 - g_o(t)\right] = o \quad \text{for the dam}$$

and

$$\frac{\partial^2 u_2}{\partial y^2} - S_2^{-2}\left[\ddot{u}_2 + C\cdot\dot{u}_2 - g_o(t)\right] = o \quad \text{for the foundation}$$

The prescribed boundary conditions are given by

$$u_1(h_1,t) = u_2(h_1,t); \quad u_2(H,t) = o; \quad \frac{\partial u_1}{\partial y} = o \quad \text{at} \quad y = o;$$

$$G_1\cdot\frac{\partial u_1}{\partial y} = G_2\frac{\partial u_2}{\partial y}, \quad \text{at} \quad y = h_1; \quad u_1(y,o) = u_2(y,o) = o$$

and

$$\dot{u}_1(y,t) = \dot{u}_2(y,t) = o, \quad \text{at} \quad t = o$$

The solution of these equations for u_1 and u_2 is rather involved, and we shall only give expressions for the frequencies of free oscillation of the system and the seismic coefficients without going into analytical details. The frequencies of free oscillation of the system are

$$W_n = \left(\frac{S_1}{D_1}\right)\bar{a}_n \quad \dots\dots (17)$$

and the seismic coefficients for the dam and its foundation are given respectively by

$$k_1 = 2\sum_{n=1,2}^{\infty}\frac{J_o(\bar{a}_n y'/D_1)}{\bar{a}_n\cdot P_o(q,m,n)} S_a \quad \text{------}(18)$$

$$k_2 = 2\sum_{n=1,2}^{\infty}\frac{M_o(y')}{\bar{a}_n\cdot P_o(q,m,n)} S_a \quad \text{------}(19)$$

where

$$P_o(q,m,n) = q\left[m\cdot\cos(q\bar{a}_n)J_1(\bar{a}_n) + \sin(q\bar{a}_n)J_o(\bar{a}_n)\right]$$
$$-\left[m\cdot\sin(q\bar{a}_n)\left\{J_1(\bar{a}_n)/\bar{a}_n - J_o(\bar{a}_n)\right\} - \cos(q\bar{a}_n)J_1(\bar{a}_n)\right]$$

and
$$M_o(Y) = \cos\left(q\bar{a}_n \frac{Y' - D_1}{D_2}\right) J_o(\bar{a}_n) - m \cdot \sin\left(q\bar{a}_n \frac{Y' - D_1}{D_2}\right) J_1(\bar{a}_n)$$

Again the symbols used in this section are explained in the Appendix. Numerical values of the parameter $\bar{a}_n$ are given in Table III, while the functions $P_o(q,m,n)$ and $\bar{a}_n P_o(q,m,n)$ are plotted in Figure 6b.

4.1 Application of the seismic coefficient in design problems.

In this section the practical use of the seismic coefficient derived in the earlier sections of the paper is discussed.

When standard acceleration spectra for the dam site are available, the seismic coefficient may be expressed either as the root sum square of the first m modes of oscillation:

$$k = \left\{ \sum_{n=1}^{n=m} k_n^2 \right\}^{1/2} \text{------------------} (20)$$

or as the maximum absolute value of the first m modes:

$$k = |k_n|_{max} \text{-------------------} (21)$$

where for any particular problem m need not be greater than four. For earth dams the appropriate expression for k appears in practice to be that given by (21), and this coincides with the expression employed in the USSR[18].

Hence, in computing the design k, the values of k_n to be used may be obtained from either of the expressions derived in the preceeding sections, by replacing S_a with the appropriate acceleration spectrum. It should be noted that the value of critical damping to be used for the higher modes of oscillation must be greater than that used for the fundamental.

When acceleration spectra for the dam site are not available, it may be assumed that the dam, under damped conditions, will resonate in its first mode for one full cycle with a ground acceleration component of amplitude equal to the maximum seismic acceleration α, given for the region in seismic probability maps. The chance of more than one cycle of ground motion occurring (a) consecutively, (b) with period equal to the fundamental of the structure, (c) in a direction at right angles to the axis of the dam, and (d) with an acceleration equal to the maximum for the region, is remote.

However, when this method (Resonance design method) is used, careful examination of the prevailing site conditions, geology and structural details must be observed.

When the resonance design method is used, S_a in the expression for the seismic coefficient is replaced by $\alpha \cdot \bar{S}(\lambda)$ and only the first term of the series is used. In this case $\bar{S}(\lambda)$ is the magnification factor for one cycle of synchronous damped oscillation of the structure in its first mode, and values for $\bar{S}(\lambda)$ are given in Table IV. For undamped conditions, a somewhat similar expression was derived by Hatanaka (1955) where the magnification was taken as 3.14.

TABLE IV

λ	0	5%	10%	15%	20%	25%	30%
$S(\lambda)$	3.14 ...	2.75	2.34	2.12	1.80	1.63	1.40

After the distribution of the seismic coefficient for a particular problem has been established, this is introduced in the stability analysis of the structure.

The stability analysis of an earth dam should be examined by the method based on the principle of limit design. The shear stresses in the structure and in its foundation are principally due to gravity and inertia body forces, and the state of limiting equilibrium is considered to be brought about by a reduction in the shear strength of the soil. The factor of safety may then be defined as the ratio of the actual strength of the soil to that required to maintain equilibrium. The strength required to maintain equilibrium is calculated from the two dimensional state of equilibrium along a potential failure surface, which in the presence of horizontal inertia forces appears to diverge considerably from the circular arc usually employed. For practical purposes, however, the failure surface may be considered to be a circular arc, or, a combination of arcs and planes. The horizontal inertia forces, which will depend on the magnitude of the seismic coefficient and corresponding weights of the slices, may easily be introduced in Bishop's method of analysis [9].

In designing on the principle of limiting equilibrium, although local overstressing within the body of the dam will occur, the average strength of the soil along a potential failure surface should not be less than that required to maintain equilibrium. If this condition is violated, a portion of the fill will slide along a potential failure surface. Movement may be a few inches or many yards. In designing, therefore, on the basis of limiting equilibrium there is no point in questioning how long it takes for a slide to develop or that before there is time for it to do so the direction of the forces which produced loss of equilibrium will be reversed. Once a potential failure surface is developed and a portion of the fill slides on it, no matter how small the displacement, on the basis of limiting equilibrium, a failure has occurred.

For cohesionless material of low degree of saturation, with an angle of friction ϕ', on a slope inclined at β to the horizontal, the factor of safety F against local sliding under the influence of a horizontal acceleration k.g can be shown to be

$$F = \tan\phi' \frac{1 - k\tan\beta}{k + \tan\beta} \quad \text{------------------(22)}$$

When the resonance design method is used, the factor of safety against sliding over the whole surface of the slope is given by

$$F = \tan\phi' \frac{1 - 0.98\bar{S}(\lambda)\alpha\tan\beta}{0.98\bar{S}(\lambda)\alpha + \tan\beta} \quad \text{---------(23)}$$

Bjihovskii (1956) derived a similar expression based on the distribution of k shown in Figure 1b, which, however, may overestimate F by as much as 30%.

5.1 Strength parameters. The soil's strength parameters to be used for the stability analysis of an earth dam under seismic conditions must be examined very carefully. When an earth dam and its foundation are shaken by a strong earthquake, the available shear strength to resist the seismically imposed shear stresses is not the full strength of the soil, since for the static stability of the structure a certain amount of the soil's strength has been mobilized by static forces. The strength mobilized for static equilibrium is often 60% or more of the total strength. The strength available to resist the seismic stresses will not in general be the remaining strength on a static basis. The reason for this is that the application of the seismic forces will lead to a change in the pore water pressures in the fill or foundation material. The time of loading by the seismic forces is so short that no drainage or dissipation of pore water pressure can occur and the failure conditions in these circumstances will involve a pore pressure different from that implicit in the assumptions made when computing the static stability. Hence, the strength available to resist the seismic stresses will be that which the soil will have if, after being stressed to the point which represents the stress conditions for static equilibrium, it is then sheared under undrained conditions.

One important characteristic of the resistance of soils to seismic stresses is that, when saturated they will fail under undrained conditions. In these circumstances the saturated soil appears to behave as though the angle of shearing resistance were zero and the available strength is independent of the magnitude of the stress changes. Hence, saturated soils and soft weakly cemented sandstones under seismic conditions apparently behave as cohesive materials and the value of their apparent cohesion c_u will depend on their stress history.(8)

Without going into details, the appropriate testing procedure to be followed in determining shear strength parameters in soils may be summarised as follows.

5.2 Immediately after construction.

(i) In the case of fills of comparatively low permeability there may be no appreciable decrease in pore water pressure in the compacted or foundation material by the end of construction. The shear strength parameters c' and ϕ' of the soil must, therefore, be determined from undrained tests with pore pressure measurements, and the appropriate values of pore pressure for the particular factor of safety must be used in the stability analysis.

(ii) If the fill material is comparatively permeable so that by the end of construction no appreciable pore water pressures will exist in the fill, the shear strength parameters c' and ϕ' must be determined from undrained tests carried out on anisotropically consolidated samples. A range of consolidation principal stress ratios for the consolidation process must be used so that the appropriate effective stress parameters may be obtained for any point in the fill.(10)(11)

(iii) For normally consolidated saturated foundation materials of low permeability the appropriate tests on specimens remoulded at the natural water content will show c_{ur} and ϕ=o when sheared under undrained conditions.

(iv) For permeable saturated foundation materials, for which the pore pressure initially set up by the structural load may be considered to have dissipated by the end of construction, the appropriate strength parameters will be c_u and ϕ = o. These values can be obtained from saturated samples consolidated allowing no lateral yield, and sheared under undrained conditions. Since the consolidation pressure decreases along the base of the dam approximately in proportion to the height of the fill above it, the undrained strength of the foundation material can be expressed in terms of the ratio $(c_u/p)_a$, where p is the consolidation pressure, c_u the undrained strength or apparent cohesion and a a suffix denoting anisotropic consolidation. Bishop (1955) has shown that for sands consolidated under K_o conditions and which have not been unloaded, the ratio of the undrained strength to the maximum consolidation pressure is only 1/3 of that for isotropic consolidation. For materials which have been precompressed this difference is likely to be smaller.

5.3 Long term stability.

(i) For the fill material below the phreatic line, the appropriate tests will be undrained tests on samples initially consolidated under the anisotropic stress condition which obtains in the field and then saturated before shearing. The strength parameters will be $(c_u/p)_a$ and $\phi = o$.

(ii) For the fill above the phreatic line, the strength parameters will be c' and ϕ', obtained from consolidated undrained tests on compacted samples, with pore water pressure measurements.

(iii) For the foundation material, assuming full saturation, the strength parameters will be $(c_u/p)_a$ and $\phi = o$ for non-sensitive soils and c_{ur}, $\phi = o$ for sensitive deposits.

The testing procedures mentioned in this section are described in reference (10).

6.1 General recommendations for earthquake resistant design.

1) Monolithic or articulated concrete diaphragms endanger the stability of an earth dam and these as well as concrete revetments upon which the impermeability of the structure may depend, should be avoided.

2) Thin layers or lenses of weak superficial material should be removed from the surface of the foundation.

3) Placing of the spillway on the main structure, or crossing of the dam by outlet or draw-off pipes should be avoided.

4) Cut-off walls should preferably be of puddle clay of generous thickness, and where compacted clay core walls are used they should be of generous proportions and should preferably slope upstream. Filters of generous dimensions are absolutely necessary.

5) Substantial free-board should be allowed for, but even so, rip-rap protection of the downstream slopes is essential.

6) Weak abutments should be avoided, and when weak formations are grouted care should be taken that grouting pressures are not excessive.

7) In the reservoir basin potentially steep banks which may become unstable under seismic shocks should be flattened to a seismically stable slope before impounding in order to avoid tsunami-type water waves.

8) Outlet pipes should be provided with valves both on the reservoir and outlet sides and the mechanism for the control gates and valves should be simple and d designed for rapid operation in an emergency.

9) It is advisable to examine the stability of the upstream slope of major earth dams under rapid draw down conditions coinciding with an after-shock. It is not unlikely that tilting of, or slides in the reservoir basin may set up waves after the major shock, which may produce almost instantaneous draw down. In this case the top part of the dam may be subjected to a 30 to 50 feet drew down

plus the action of inertia forces.

10) Foundation materials such as saturated silts, loose or medium saturated sands and sensitive clay deposits should be avoided. Materials of very different elastic properties should be also avoided.

11) The post-seismic stability, both of the fill and foundation should be examined. High stresses applied to and removed from soil masses will result in setting up residual pore water pressures and their dissipation with time may lead either to further consolidation or swelling of the fill and foundation material (1)(7).

12) Straddling of an active fault by major earth dam should be avoided.

7.1 Bibliography

1.- Henkel, D.J. (1952) "The stability of overconsolidated clays" Professional Interview, The Institution of Civil Eng. (Unpublished)

2.- Ambraseys, N. (1959) "The seismic stability of earth dams" Ph.D. Thesis, University of London, January 1959

3.- Ambraseys, N. (1959) "A note on the response of an elastic overburden of varying rigidity to an arbitrary ground motion" Bull. Seism. Soc.Amer.,Vol.50,p.45

4.- Ambraseys, N. (1960) "On the shear response of a two-dimensional truncated wedge subjected to an arbitrary disturbance" Bull.Seism.Soc.Amer., Vol.49,p.221

5.- Ambraseys, N. (1960) "Über die freie Transversalschwingung eines zweidimensionalen Damms von trapezförmigen Querschnitt" Verein Deutscher Ing. Zeitsch., Vol.102, Juli H.

6.- Bjihovskii, V. (1956) "Earthquake resistant structures" Chentral'noy Stroitel'noy Biblioteki, Moscow (R)

7.- Bishop, A. Henkel D.(1953) "Pore pressure changes during shear in two undisturbed clays" Proc.3rd Conf.Soil Mech.&Found. Eng.,Vol.1,p.94, Zürich

8.- Bishop, A., Eldin G.(1953) "The effect of the stress history on the relation between ø and porocity in sands" Proc.3rd Conf.Soil.Mech.& Found.Eng.,Vol.1, p.105, Zurich

9.- Bishop, A. (1955) "The use of the slip circle in the stability analysis of slopes" Geotechnique, Vol.5,p.7,London

10.- Bishop A., Henkel D. (1957) "The measurement of soil properties in the triaxial test" E.Arnold Ltd., London

11.- Bishop A. (1960) "The measurement of pore pressure in the triaxial cell" Proc.Pore Pressure & Suction in Soils Confer. Paper J, p.59, London

12.- Hataraka, M. (1955) "Fundamental considerations on the earthquake resistant properties of earth dam" Bull.Disaster Prevention Research Inst., Nr.11, Kyoto.

13.- Heiland, C.(1940) "Geophysical investigation concerning the seismic resistance of earth dams" Trans.Amer.Inst.Min.& Metal.Eng., Vol.138,p.350

14.- Mononobe N. et al (1936) "Seismic stability of earth dams" Proc. 2nd Congr.Large Dams, Q.7, D22,p.435, Washington

15.- Martel, R.(1938) "Effect of earthquakes on earth dams" The Military Engineer, Vol.30,Nr.173,p.359

16.- Napetvaridzhe Ch.(1959) "Seismic design of hydrotechnical structures" Gostroyzdat, Moscow (R)

17.- ---------- (1957 "Standard specifications for structures in seismic regions; hydrotechnical projects" S.N.-8-1957 Gosudarstvennoe Izdatel'stvo Literaturi po Stroytel'stvu i Arhitekture, Moscow (R)

18.- ---------- (1958) "Methods of calculation for buildings and structures allowing for earthquake loads" Sbornik Statei, Chentral'niy Nauchno-Isledobatelskii Institut Stroytel'nih Konstruksii, p.20, Moscow (R)

19.- ---------- (1934) "Investigation of earth dams and of their deformation due to earthquakes" Report Nr.28, Ministry of Construction, p.67, Tokyo (J)

8.1 Appendix

L = Length of dam L

D = Height of dam or thickness of overburden L

b = Width at crest level L

B = Width at foundation level L

G = Modulus of rigidity .... $M\,L^{-1}\,T^{-2}$

g = Acceleration due to gravity .... $L\,T^{-2}$

k = Seismic coefficient 1

s = Velocity of shear waves .... $L\,T^{-1}$

c = Damping coefficient .... T^{-1}

λ = Fraction of critical damping (%) 1

w = Undamped frequency .... T^{-1}

T = Undamped period .... T

u = Displacement at a point due to shear, relative to moving boundaries 1

$g_o(t)$ = Seismic ground acceleration .... $L\,T^{-2}$

x,y,z = Rectangular coordinates L

y' = Coordinate measured downwards from crest or surface of overburden L

H	= Coordinate of rigid base of dam or foundation measured from origin of x,y,z axes	L
h	= Coordinate of crest of dam or surface of overb..............	L
μ	= Ratio of length to heigh of oscillating body	1
k'	= Rigidity ratio, pertinent to overburden problem $k' = (G_t/G_b)^{1/2}$	
k"	= Coefficient of truncation, pertinent to embankment problem	
q	= Stiffness coefficient; $q = m\ (D_2/D_1)$	
m	= Stiffness ratio; $m = (s_1/s_2)$	
a'_n	= n-th root of $J_o(a'_n)Y_1(k'a'_n) - J_1(k'a'_n)Y_o(a'_n) = 0$	
a''_n	= n-th root of eq. similar to that for a'_n, but with k"	
a_n	= n-th root of $J_o(a_n) = 0$	
$\bar{a}_n$	= n-th root of $m.\tan(q\bar{a}_n) = J_o(\bar{a}_n)/J_1(\bar{a}_n)$	
1,2	= Indices denoting properties in the dam and foundation respct.	
b,t	= Indices denoting properties at the top and base of dam/overb.	
n,r	= Indices denoting transverse and longitudinal properties	

$$Y' = a'_n \left\{ \frac{(1-k'^2)}{D} y + k'^2 \right\}^{1/2}$$

$$Y'' = a''_n \left\{ \frac{(1-k'')}{D} y + k'' \right\}$$

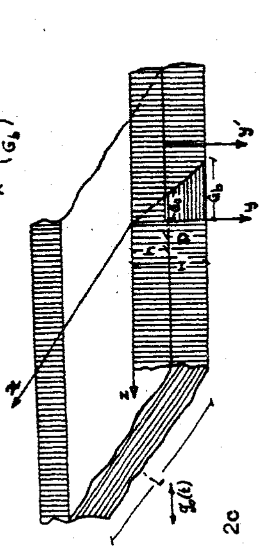

Fig. 2c

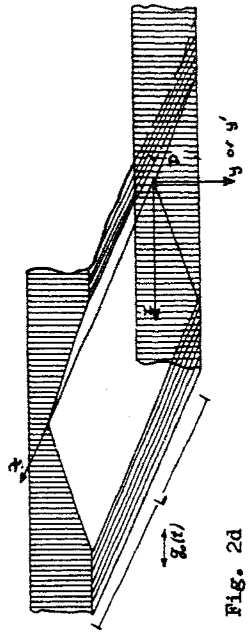

Fig. 2d

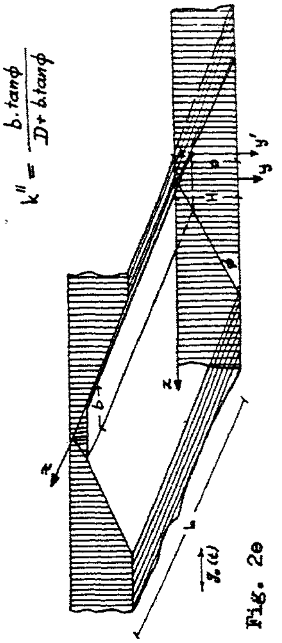

Fig. 2e

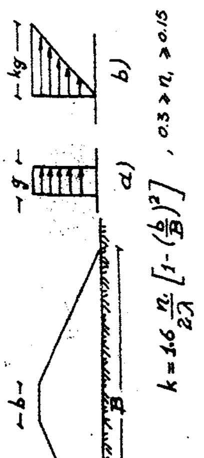

Fig. 1

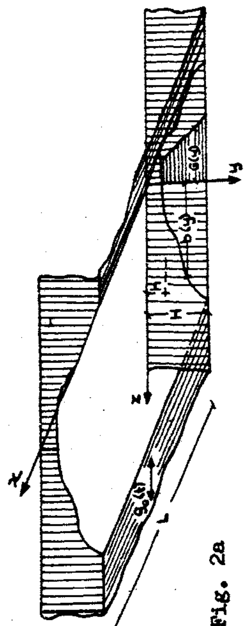

Fig. 2a

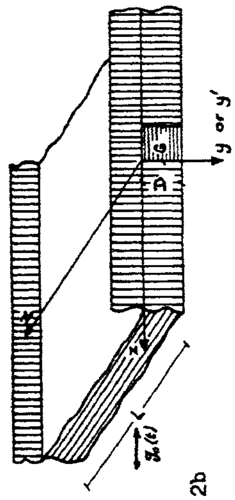

Fig. 2b

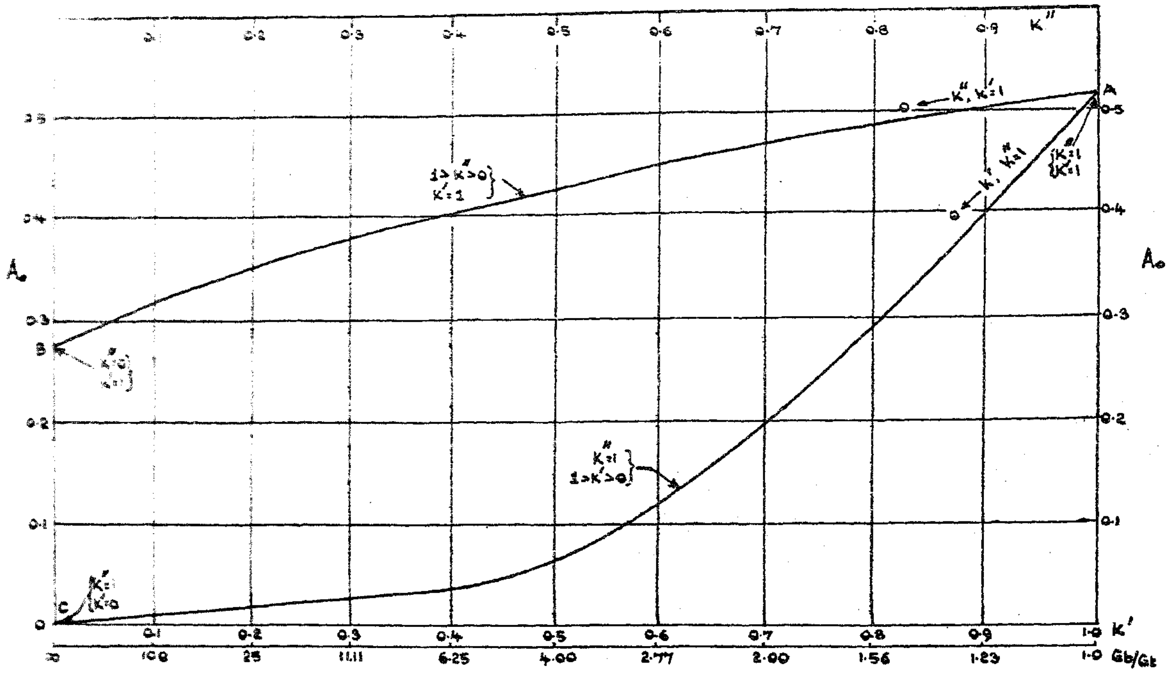

Fig. 3 $\qquad k_1 = A_0 \frac{\omega_{01}^2}{g} . S_{a1} \left(\frac{D}{S}\right)^2$

$$k_n = A_n \frac{\omega_{0n}^2}{g} S_{an} \left(\frac{D}{S}\right)^2$$

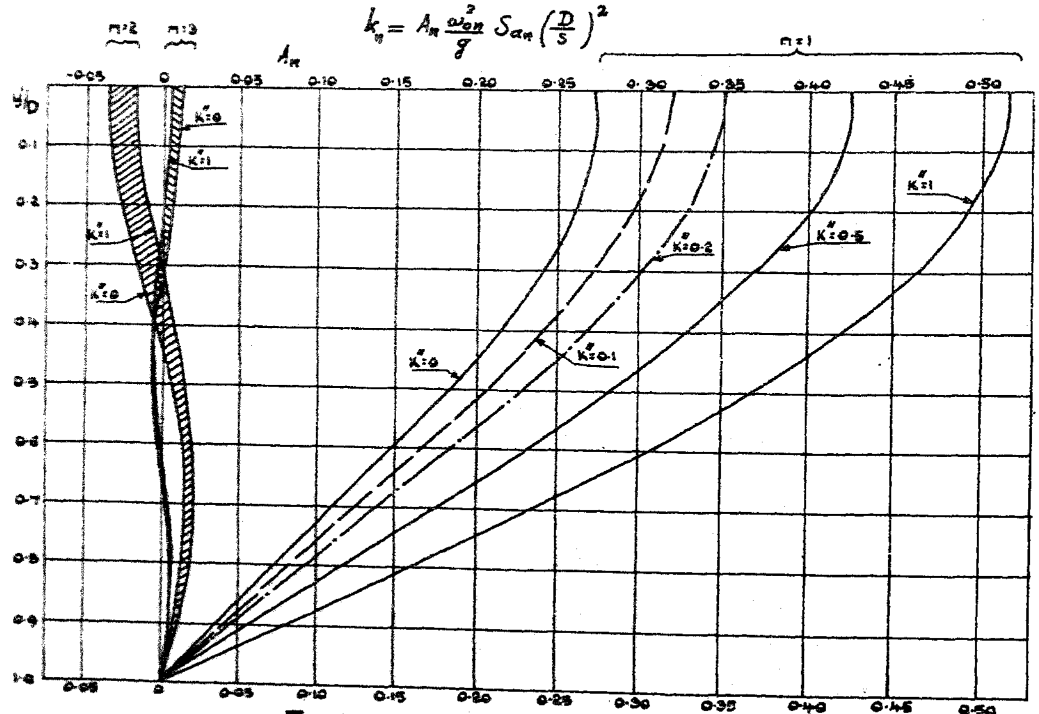

Fig. 4 $\quad A_n \quad (n = 1, 2, 3)$

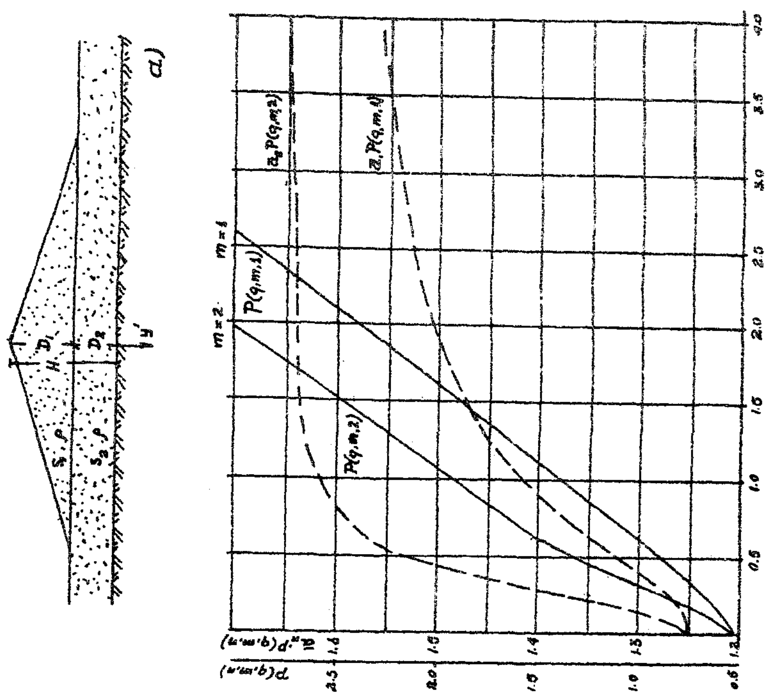
a)
b)
$m=1$
$m=2$
$P(q,m,1)$
$P(q,m,2)$
$\bar{a}_2 P(q,m,2)$
$\bar{a}_1 P(q,m,1)$
q

Fig. 6

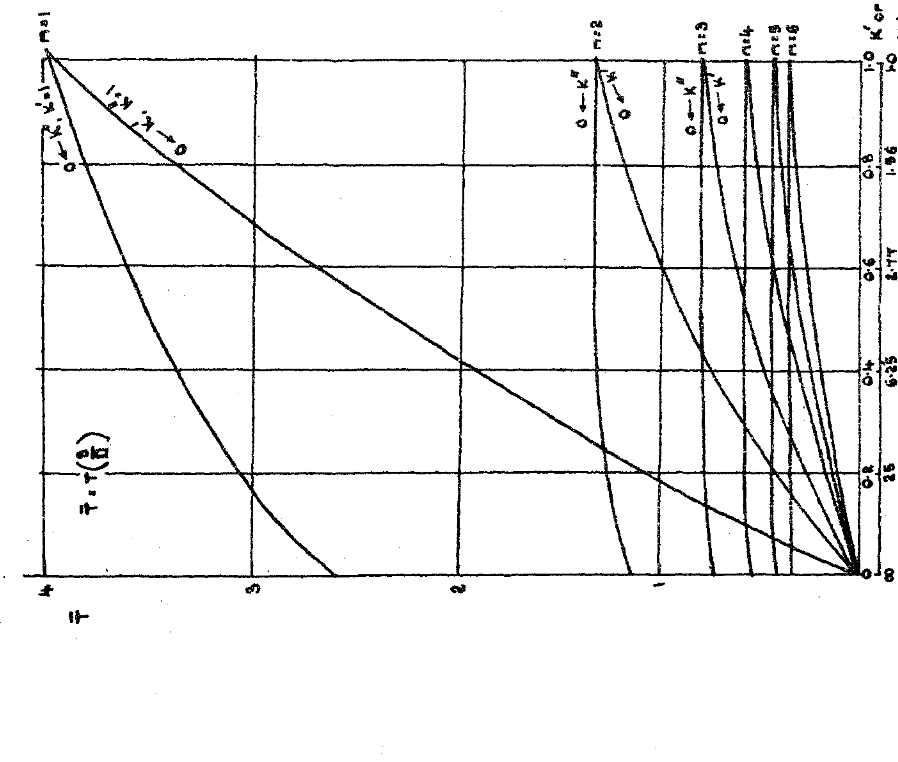
$\bar{T} = T\left(\frac{b}{H}\right)$
$\bar{T}$
$m=1$
$m=2$
$m=3$
$m=4$
$m=5$
$m=6$
K' or K''
Gb/Gt

Fig. 5

T A B L E I

k'	G_b/G_t	a'_1	a'_2	a'_3	a'_4	
0.9534	1.100	34.080	101.370	168.833	236.321	
0.9434	1.123	28.079	83.363	138.820	194.303	
0.9259	1.166	21.538	63.730	106.097	148.489	
0.9091	1.210	17.614	51.950	86.462	121.000	
0.8696	1.325	12.386	36.246	60.285	84.350	
0.8333	1.440	9.775	28.396	47.197	66.026	
0.7692	1.690	7.172	20.549	34.111	47.703	
0.6666	2.250	5.104	14.281	23.649	33.049	
0.4000	6.250	3.107	8.066	13.222	18.421	
0.2500	16.000	2.668	6.580	10.666	14.803	
0	∞	2.405	5.520	8.654	11.791	... $\hat{a}_n$

T A B L E II

k''	a''_1	a''_2	a''_3	a''_4	a''_5	a''_6	
0	2.40	5.52	8.65	11.79	14.93	18.07	... $\hat{a}_n$
0.100	2.45	5.72	9.30	12.60	15.98	19.41	
0.158	2.51	5.97	9.66	13.26	16.90	20.66	
0.200	2.57	6.23	10.05	13.92	17.81	21.71	
0.250	2.67	6.58	10.67	14.80	18.96	23.13	
0.300	2.79	6.99	11.39	15.83	20.29	24.76	
0.500	3.59	9.60	15.82	22.07	28.34	34.61	
0.800	8.21	23.69	39.34	55.03	70.73	86.43	

TABLE III

q	$\bar{a}_1$	$\bar{a}_2$	$\bar{a}_1$
0	2.404	5.520	2.404
0.2	1.961	-	-
0.4	1.631	-	-
0.6	1.375	3.440	1.090
0.8	1.185	3.035	-
1.0	1.030	2.763	-
1.2	0.915	2.515	0.752
1.4	0.831	2.313	-
2.0	0.625	1.823	0.544
3.0	0.448	1.325	-
4.0	0.349	1.040	-
	m = 1		m = 2

TABLE IIIa

	m = 1		m = 2	
q	$P_o(q,1,1)$	$\bar{a}_1 P_o(q,1,1)$	$P_o(q,2,1)$	$\bar{a}_1 P_o(q,2,1)$
0	0.519	1.250	0.519	1.250
0.2	0.641	1.260	-	-
0.4	0.795	1.300	-	-
0.6	0.980	1.346	1.440	1.570
0.8	1.170	1.385	-	-
1.0	1.380	1.420	-	-
1.2	1.580	1.444	2.170	1.630
1.4	1.777	1.475	-	-
2.0	2.410	1.510	3.030	1.600
3.0	3.421	1.532	-	-
4.0	-	1.560	-	-