A THEORETICAL INVESTIGATION OF THE INTERACTION BETWEEN GROUND AND STRUCTURE DURING EARTHQUAKES

By

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Abstract

The object of the paper is to present a mathematical method of investigating the interaction between ground and structures during earthquakes.

The disturbance phenomenon of the ground motion due to the presence of the structure has been observed during actual earthquakes and taken into account theoretically on the basis of several simplifying assumptions.

In the present paper, the interaction phenomenon is mathematically reduced, starting from some data of structural dynamics and dynamic seismology, and from some currently adopted assumptions, to a system of Volterra integral equations.

The phenomenon is then discussed. Methods of approach of the equations are presented. A qualitative expression of the correction of the response spectrum is given. Stochastic aspects of the interaction phenomenon establishing the relations between the correlative functions of undisturbed and disturbed motions (taken into account like vectorial random functions) are investigated. The assumption on the instantaneous foundation compliance, on which structural analysis is often based, is discussed. Investigation of the interaction by means of stationary monochromatic forced vibrations is presented.

1) Introduction

The seismic action on structures is caused by the motion of the foundations, due to the seismic ground motion. If the dynamic displacements of the foundations are known, we could, theoretically, compute, based on the current assumptions, the seismic response of the structure.

The seismic motion of the foundations differs from that of the free ground, as recorded by an ideal seismograph, because the inertia and the rigidity of the structure and foundation lead to a disturbance in the ground motion. This disturbance is one of the phenomena that explain, at present, the influence of ground properties on structural response. The observation of earthquake effects established the evidence of the disturbance phenomenon, but its importance became obvious after the analysis of the 1952 Arvin-Tehachapi earthquake records, [6], when reductions up to 40 % of the response spectrum ordinates computed on the basis of accelerograms recorded in the basement of a rigid building were estab-

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lished, in comparison to those, computed on the basis of accelerograms recorded on the surface of the ground. Although there existed a peculiar concurrence of circumstances, leading to such an important reduction of the earthquake effects, it is clear that the disturbance of the ground motion, due to the structure, is an important phenomenon, not to be neglected.

Attempts of theoretically analysing this phenomenon pursued two different ways: either investigating structural vibrations assuming simple boundary conditions, or, on the other hand, studying the problem on a higher level, taking into account the wave propagation in the elastic semispace.

In the first procedure, we remark the investigations of the American specialists. R. Merritt and G. Housner [9] computed the dynamic response of framed structures assuming linear foundation compliance, and estimated the influence of ground stiffness on seismic loading.

In the second procedure, we remark the investigations of the Soviet and Japanese specialists. A. Nazarov studied [10] in 1939 the response of a rigid body bound to the elastic semispace, subjected to arbitrary vertical seismic motion, based on the results of Lamb's problem, reducing the phenomenon to an integrodifferential Volterra equation. Departing from I. Toriumi's study (made available for practical purposes by assuming linear stress distribution on the contact area), Y. Sato and R. Yamaguchi [12] investigated the response of a rigid cylinder bound to the elastic semispace, loaded dynamically by monochromatic, vertical or horizontal, seismic waves. They obtained diagrams of practical use for the motion amplitude as function of adimensional parameters, establishing thresholds as those of resonance, and the limits between which the seismic ground motion becomes amplified or diminished for the cylinder.

In the present paper some equations will be introduced, which permit the determination, based on currently admitted assumptions, of the interaction forces between ground and structures, taking into account the disturbance introduced into the seismic ground motion. The proposed method permits the investigation of structures, the connection to the ground of which can be reduced to an arbitrary, finite, number of degrees of freedom.

2) The equations of continuity

2.1) Assumptions

In order to investigate mathematically the ground-structure interaction problem, the following properties will be assumed:

a) The mechanical phenomena may be described by linear equations: the displacements and the strains are of negligible squares (geometrical linearity) and the mechanical properties of the materials can be described by a linear rheological law, the seismic motion being not so violent as to produce plastic strains or separation between foundation and ground (physical linearity).

b) The connection between structure and ground can be characterised by a finite number of degrees of freedom.

2.2) Elements of structural dynamics and dynamic seismology

The equations of continuity will be obtained taking into account a general manner of expressing the dynamic displacements of a solid body while satisfying the above linearity assumptions. If investigated, during the motion of the body, the dynamic displacements on the direction of a finite number, m, of degrees of freedom, noted by indices a , b ,..., can be expressed by the formula (N.B.: add from 1 to m with respect to indices a , b ,... when appearing twice in a monome),

$$(1) \quad u_a^{*}(t) = u_a^{o}(t) + \int_o^t u_{ab}(t-z)X_b(z)\, dz$$

In expression (1) following notations were used:

$X_a(t)$: dynamic forces, acting on the directions a , assumed to be equal to zero on the negative semiaxis and integrable;

$u_a^o(t)$: free displacements on the directions a (assuming $X_a(t) \equiv 0$);

$u_a^{*}(t)$: displacements on the directions a , disturbed by the action of the forces $X_a(t)$;

$u_{ab}(t)$: dynamic displacement produced on the direction a by the instantaneous pulse, equal to the unity, acting on the direction b,

$$X_b(t) = \delta(t) \text{(where } \delta(t) \text{ is Dirac's function).}$$

Sometimes, instead of expressing the disturbed displacements by (1), the following expression may be found useful:

$$(2) \quad u_a^{*}(t) = u_a^{o}(t) + \int_o^t U_{ab}(t-z)\dot{X}_b(z)\, dz$$

where $U_{ab}(t)$ is the dynamic displacement produced on the direction a by the step function force acting on the direction b , $X_b(t) = \varepsilon(t)$ (where $\varepsilon(t)$ is the step function, $\varepsilon(t) = \int_{-\infty}^{t}\delta(z)\, dz$) and where the forces $X_b(t)$ are assumed to be continuous, equal to zero on the negative semiaxis, and to have an integrable derivative.

Further on, the expression (1) will be preferred.

The expression (1) is applicable for computing the response of both structure and ground.

In order to deduce the equations of the phenomenon of dynamic contact between structure and ground, a section, as shown in fig.1, will be imagined.

After sectioning, there exists no more connection between structure and ground. If we note the degrees of freedom of the section by a , b , ..., (a,b,...=1...m), the expression (1) will be used for investigating both the free structure and the ensemble ground-foundation.

In case of the structure, the expression (1) becomes

$$(3)\quad u_a^{*s}(t) = u_a^{os}(t) + \int_o^t s_{ab}(t-z)X_b^s(z)\,dz$$

where the tensor $s_{ab}(t)$ is a linear combination, easy to compute, of the free structural vibrations, $X_a^s(t)$ representing the forces $X_a(t)$ applied to the structure, and $u_a^s(t)$ representing the displacements $u_a(t)$ of the structure.

In case of the ground, the expression (1) becomes

$$(4)\quad u_a^{*g}(t) = u_a^{og}(t) + \int_o^t g_{ab}(t-z)X_b^g(z)\,dz$$

where the tensor $g_{ab}(t)$ can be, theoretically, computed taking into account the wave propagation in the material semispace (based on Lamb's problem and using, for example, the excellent results furnished by [14]), $X_a^g(t)$ represent the forces $X_a(t)$ applied to the ground, and $u_a^g(t)$ represent the displacements $u_a(t)$ of the ground.

2.3) Deduction of the equations of continuity

In the real case, there exists a contact between structure and ground during the earthquake, and this phenomenon is expressed by the condition of geometrical continuity

$$(5)\quad u_a^{*g}(t) - u_a^{*s}(t) = 0$$

Introducing expressions (3) and (4) into (5), the equations of continuity are obtained, which may be used for investigating motions of underground origin (seismic waves, propagations of vibrations through ground, etc.), or of overground origin (wind action, blast effects, engine actions, etc.). Further on, it will be assumed that the dynamic phenomenon is of a purely seismic origin ($u_a^{os}(t) \equiv 0$). Taking into account the actio-reactio principle, $X_a^s(t)+X_a^g(t) \equiv 0$, the conditions of continuity (5) lead to (see (3), (4)).

$$(6)\quad u_a^o(t) = \int_o^t u_{ab}(t-z)X_b(z)\,dz,$$

$$\text{noting}\quad u_a^o(t) \equiv u_a^{og}(t),\ X_*(t) \equiv X_a^s(t) \equiv -X_a^g(t),$$

$$u_{ab}(t) \equiv s_{ab}(t) + g_{ab}(t)$$

The significancy of all the quantities intervening in (6) results easily from the significance of the quantities of (3) and (4).

3) Discussion of the equations of continuity

3.1) General

The relations (6) represent the equations of continuity of the problem and they constitute a linear system of Volterra integral equations with respect to the unknown interaction forces $X_a(t)$. The phenomenon thus described is a non-stationary one.

A detailed discussion of the nature of the solution of system (6) could be accomplished only based on the analytical expression of the tensorial nucleus $u_{ab}(t)$, expression which is generally very difficult to obtain. However, it can be shown that the solution of the system is unique. Therefore, it is sufficient to observe that there can be no system of dynamic forces $X_a(t)$, not to produce a relative displacement between ground and structure in the case of sectioning (mathematically speaking, Volterra equations do not admit eigenvalues).

The system (6) is a Volterra system of the 1-st kind. If in the zone of the sectioning as in Fig.1 there appeared elements of the dynamic pattern of massless-spring type, the system (6) would become a Volterra system of the 2-nd kind. In the real case, every element possesses its own mass and thus, the system (6) is adequate to the nature of the phenomenon.

Formally, the solution of system (6) will be expressed by the formula

$$(7)\quad X_a(t) = \int_o^t R_{ab}(t-z)\ddot{u}^o_b(z)\, dz$$

which will be further discussed.

The physical significance of the tensor $R_{ab}(t)$ is the following: its terms represent the system of interaction forces $X_a(t)$ that produces a relative displacement $u^s_b(t)-u^g_b(t)=t\mathcal{E}(t)$ on the direction b and maintains fixed the relative position of the system on the other directions, in the case of sectioning as in fig.1.

The disturbed, common motion of structure and ground on the directions a will be expressed by (see (4)),

$$(8)\quad u^*_a(t) = u^o_a(t) - \int_o^t g_{ab}(t-z)X_b(z)\, dz$$

The analysis of the dynamic structural response, based on the disturbed seismic motion, could be obtained by the aid of either the interaction forces $X_a(t)$, as given by (7), or the disturbed displacements $u^*_a(t)$, as given by (8).

3.2) Methods of solving the system (6)

The free dynamic displacements $u^o_a(t)$ and the tensorial nucleus $u_{ab}(t)$ once known, the system (6) can be solved. Among the methods to be employed, the most adequate appear the method of finite sums, the method of operational calculus, and the iterative method.

The method of finite sums algebrises the system (6), reducing it to a triangular succession of algebraic linear systems, which can be solved by recurrence, and offers the approximate values of $X_a(t)$ at a discrete succession of moments (assuming that the matrix of the coefficients is not a singular one at the initial moment). If the moments $t_o = 0, \dots$ $t_r = t_{r-1} + \Delta t$ are taken into account, the system (6) is approximated by the succession of systems

$$(9)\quad u_a^o(\Delta t) = u_{ab}\left(\frac{\Delta t}{2}\right)X_b\left(\frac{\Delta t}{2}\right)\Delta t$$

$$u_a^o(2\Delta t) = \left[u_{ab}\left(\frac{\Delta t}{2}\right)X_b\left(\frac{3}{2}\Delta t\right) + u_{ab}\left(\frac{3}{2}\Delta t\right)X_b\left(\frac{\Delta t}{2}\right)\right]\Delta t$$

$$\vdots$$

$$u_a^o(r\Delta t) = \sum_s^{1,r} u_{ab}\left(\left\{r-s+\tfrac{1}{2}\right\}\Delta t\right)X_b\left(\left\{s-\tfrac{1}{2}\right\}\Delta t\right)\Delta t$$

that admit the solution

$$(10)\quad X_a\left(\frac{\Delta t}{2}\right) = \frac{1}{\Delta t}\overset{-1}{u}_{ab}\left(\frac{\Delta t}{2}\right)u_b^o(\Delta t) \quad \text{where} \quad \overset{-1}{u}_{ab}\left(\frac{\Delta t}{2}\right)u_{bc}\left(\frac{\Delta t}{2}\right) = \delta_{ac}$$

$$X_a\left(\frac{3}{2}\Delta t\right) = \frac{1}{\Delta t}\overset{-1}{u}_{ab}\left(\frac{\Delta t}{2}\right)\left[u_b^o(2\Delta t) - u_{bc}\left(\frac{3}{2}\Delta t\right)X_c\left(\frac{\Delta t}{2}\right)\Delta t\right]$$

$$\vdots$$

$$X_a\left(\left\{r-\tfrac{1}{2}\right\}\Delta t\right) = \frac{1}{\Delta t}\overset{-1}{u}_{ab}\left(\frac{\Delta t}{2}\right)\left[u_b^o(r\Delta t) - \sum_s^{1,r-1} u_{bc}\left(\left\{r-s+\tfrac{1}{2}\right\}\Delta t\right)X_c\left(\left\{s-\tfrac{1}{2}\right\}\Delta t\right)\Delta t\right]$$

$$\vdots$$

The recursive solving (10) can be mechanised and is able to offer numerical results of practical use.

The method of operational calculus algebrises too the system (6) and permits more extensive discussions on this subject.

The operational correspondence original-image by the Laplace-transform will be noted by $h(t) \overset{\bullet}{=} \cdot f(p)$ [13].

The Volterra system (6) will have as operational image the algebraic system

$$(11)\quad \mathcal{U}_a^o(p) = \frac{1}{p}\mathcal{U}_{ab}(p)\,\mathcal{X}_b(p) \quad \text{where} \quad u_a^o(t) \overset{\bullet}{=} \cdot\,\mathcal{U}_a^o(p)$$

$$X_a(t) \overset{\bullet}{=} \cdot\,\mathcal{X}_a(p)$$

$$u_{ab}(t) \overset{\bullet}{=} \cdot\,\mathcal{U}_{ab}(p)$$

It is to be observed that all the originals intervening are equal to zero on the negative semiaxis.

The solution of the algebraic system (11),

$$(12)\quad \mathcal{X}_a(p) = \frac{1}{p}\mathcal{R}_{ab}(p)p^2\mathcal{U}^o_b(p) \qquad \text{where } \mathcal{R}_{ab}(p)\,\mathcal{U}_{bc}(p) = \delta_{ac}$$

$$R_{ab}(t)^\circ = \circ\mathcal{R}_{ab}(p)$$

justifies formally the adoption of relation (7) for expressing the solution.

The operational image of expression (8) will be

$$\begin{aligned}(13)\quad \mathcal{U}^*_a(p) &= \mathcal{U}^o_a(p) - \mathcal{G}_{ab}(p)\mathcal{R}_{bc}(p)\,\mathcal{U}^o_c(p)\\ &= \left[\mathcal{U}_{ab}(p) - \mathcal{G}_{ab}(p)\right]\mathcal{R}_{bc}(p)\,\mathcal{U}^o_c(p)\\ &= \mathcal{S}_{ab}(p)\,\mathcal{R}_{bc}(p)\,\mathcal{U}^o_c(p)\end{aligned}$$

$$\text{where } s_{ab}(t)^\circ = \circ\mathcal{S}_{ab}(p);\; g_{ab}(t)^\circ = \circ\mathcal{G}_{ab}(p)$$

If the series having the general term $\mathcal{T}_{ac_1}(p)\mathcal{T}_{c_1c_2}(p)\ldots\mathcal{T}_{c_nb}(p)$ converges uniformly (where $\mathcal{T}_{ab}(p)=-\mathcal{G}_{ac}(p)\mathcal{S}^{-1}_{cb}(p)$ and $\mathcal{S}^{-1}_{ab}(p)\mathcal{S}_{bc}(p)=\delta_{ac}$), the expression (13) of the disturbed motion can be written in the manner

$$(14)\quad \mathcal{U}^*_a(p) = \left[\delta_{ab} + \mathcal{T}_{ab}(p) + \mathcal{T}_{ac}(p)\,\mathcal{T}_{cb}(p) + \ldots\right]\mathcal{U}^o_b(p)$$

The values of the terms of the tensor $\mathcal{T}_{ab}(p)$ tend to zero simultaneously with the ratio of the ragidities of structure to ground and, in the limit case, the undisturbed, $u^o_a(t)$, and the disturbed, $u^*_a(t)$, motions, coincide. This fact will appear more clearly later on, when the influence of the disturbance on the response spectrum will be studied.

It becomes obvious, from (13) or (14), that, if the system of dynamic displacements is continuous and derivable at the moment $t = 0$, then the correspondence between the undisturbed and, respectively, the disturbed displacements, velocities and accelerations, will maintain itself, and so these relations are available also in order to determine the disturbed accelerograms.

The iterative method is to be used in this case not in the manner proposed by Neumann for the investigation of integral equations, but in a manner specific to our problem. In the first approximation, the structural response will be computed when subjected to the undisturbed motion, $u^o_a(t)$. The approximation of order r will be obtained by computing the structural response when subjected to the ground motion $u^{r-1}_a(t)$ produced (if sectioning as in fig.1) by the interaction forces $-X^{r-1}_a(t)$ corresponding to the approximation of order $r-1$ of the structural response. If the process converges, the response of the structure to the disturbed ground motion will be obtained by adding the successive approximations. The operational image of the method is reduced to the application of relation (14), which obtains thus an intuitive sense.

3.3) Correction of the response spectrum

The seismic disturbed motion once computed by (8), the response spectrum method becomes applicable, using the disturbed displacements $u^{*}_{a}(t)$ as input. For a more detailed discussion, it will be of use to investigate the case of a one-degree-of-freedom system.

The free seismic vertical motion of the ground, $u^{o}(t)$, given, the equation of motion of the body drawn in fig.2a will have the form and, respectively, the operational image (in case of homogeneous initial conditions),

$$(15)\quad m\ddot{u} + k(u-u^{o}) = 0 \quad \stackrel{\bullet}{=}. \quad mp^{2}\mathcal{U} + k(\mathcal{U}-\mathcal{U}^{o}) = 0$$

with the general solution given by one of the equivalent forms (when $\omega = \sqrt{\frac{k}{m}}$)

$$(16)\quad u = \omega\int_{0}^{t} \sin\omega(t-z)u^{o}(z)\,dz \quad \stackrel{\bullet}{=}. \quad \mathcal{U} = \frac{\omega^{2}}{p^{2}+\omega^{2}}\mathcal{U}^{o} = \frac{\omega}{p}\,\frac{p\omega}{p^{2}+\omega^{2}}\mathcal{U}^{o}$$

$$u - u^{o} = -\frac{1}{\omega}\int_{0}^{t} \sin\omega(t-z)\ddot{u}^{o}(z)\,dz \quad \stackrel{\bullet}{=}. \quad \mathcal{U}-\mathcal{U}^{o}$$

$$= -\frac{p^{2}}{p^{2}+\omega^{2}}\mathcal{U}^{o} = -\frac{1}{\omega p}\,\frac{p\omega}{p^{2}+\omega^{2}}\,p^{2}\mathcal{U}^{o}$$

In order to make obvious the disturbance, the pattern of fig.2b will be substituted to that of fig.2a, where the ground is compliant and the connection body-ground is represented by a support of linear dimension L. A section will be imagined like in fig.2c.

The tensor $s_{ab}(t)$ will be replaced by the function $s(t)$, given by

$$(17)\quad s(t) = \frac{\delta(t)}{K} + \frac{\mathcal{E}(t)}{m}t \quad \stackrel{\bullet}{=}. \quad \mathcal{S}(p) = \frac{p}{m}\left(\frac{1}{\omega^{2}} + \frac{1}{p^{2}}\right)$$

For the function $g(t)$, which particularises the tensor $g_{ab}(t)$, a qualitative, dimensional expression, will be adopted,

$$(18)\quad g(t) = \frac{d}{dt}\left[\frac{\mathcal{E}(t)}{LE}\,f\left(\sqrt{\frac{E}{\rho}}\,\frac{t}{L}\right)\right] \stackrel{\bullet}{=}. \quad \mathcal{G}(p) = \frac{p}{LE}\,\mathcal{F}\left(\sqrt{\frac{\rho}{E}}\,Lp\right),$$

$$\text{where}\quad \mathcal{E}(t)f(t) \stackrel{\bullet}{=}. \mathcal{F}(p)$$

the expression of $f(t)$ being not determined.

The tensor $\mathcal{T}_{ab}(p)$ of (14) will be replaced by the function

$$\mathcal{T}(p) = -\frac{\mathcal{G}(p)}{\mathcal{S}(p)} = -\frac{k}{LE}\,\frac{p^{2}}{p^{2}+\omega^{2}}\,\mathcal{F}\left(\sqrt{\frac{\rho}{E}}\,Lp\right),$$

while the operational image of the disturbed ground motion subjected to the interaction force $X(t) \stackrel{\bullet}{=}. \mathcal{X}(p) = \frac{K}{p^{2}+\omega^{2}}\,\frac{1}{1-\mathcal{T}(p)}\,p^{2}\,\mathcal{U}^{o}(p)$

will become

$$(19)\quad \mathcal{U}^{*}(p) = \left[1 - \frac{\mathcal{G}(p)}{\mathcal{G}(p)+\mathcal{F}(p)}\right]\mathcal{U}^{\circ}(p) = \frac{1}{1-\mathcal{J}(p)}\mathcal{U}^{\circ}(p)$$
$$= \left[1 + \mathcal{J}(p) + \mathcal{J}^{2}(p)\ldots\right]\mathcal{U}^{\circ}(p)$$

where the series is assumed to be uniformly convergent.

Obviously, the images of the equivalent forms of the solution (16),

$$\mathcal{U} = \frac{\omega^2}{p^2+\omega^2}\mathcal{U}^{\circ} \text{ or } \mathcal{U} - \mathcal{U}^{\circ} = -\frac{p^2}{p^2+\omega^2}\mathcal{U}^{\circ} \qquad \text{(see 13)}$$

will be replaced by

$$(20)\quad \mathcal{U}(p) = \frac{\omega^2}{p^2+\omega^2}\,\frac{1}{1-\mathcal{J}(p)}\,\mathcal{U}^{\circ}(p) \approx \frac{\omega^2}{p^2+\omega^2}\left[1+\mathcal{J}(p)\right]\mathcal{U}^{\circ}(p)$$

The expression (20) shows that the disturbance vanishes simultaneously with the ratio $\frac{k}{LE}$. The analytical expression of $\mathcal{F}(p)$ given, the effective computation of the corrected response spectrum becomes available.

The considerations above discussed remain valid in case of linear frictions.

3.4) Stochastic aspects of the interaction between ground and structure

In the former paragraphs, the ground-structure interaction phenomenon during an earthquake was dealt with from an exclusively deterministic standpoint, assuming that in the practical valuation of earthquake effects on buildings the response spectrum method will be used.

The actual tendency of applying the theory of stochastic processes in investigating the effects of earthquakes upon structures may be adopted too in the investigation of the ground-structure interaction, starting from the equations (6), [1], [4].

If admitting the assumptions of §2.1), it is practically satisfactory to know, for the vectors $u_a(t)$ and $X_a(t)$, now taken into account as vectorial random functions of a non-stationary stochastic process, the correlative functions of orders 1 (mean) and 2(covariancy), [4], [8]. These functions will be, respectively,

$$(21)\quad \overline{u_a(t)} = \int_{-\infty}^{\infty} u_a(t)W\{u_a(t);\ t\}\,du_a(t) \;\dot{=}\; \overline{\mathcal{U}_a(p)}$$

$$\overline{u_a(t_1)u_b(t_2)} = \iint_{-\infty}^{\infty} u_a(t_1)u_b(t_2)W\{u_a(t_1),u_b(t_2);\ t_1,t_2\}\,du_a(t_1)du_b(t_2) \;\dot{=}\; \overline{\mathcal{U}_a(p_1)\,\mathcal{U}_b(p_2)}$$

$$\overline{X_a(t)} = \int_{-\infty}^{\infty} X_a(t) W\{X_a(t);\ t\}\, dX_a(t) \ \overset{\circ}{=}\!\!\circ\ \overline{\mathcal{X}_a(p)}$$

$$\overline{X_a(t_1) X_b(t_2)} = \iint_{-\infty}^{\infty} X_a(t_1) X_b(t_2) W\{X_a(t_1), X_b(t_2);$$
$$t_1, t_2\}\, dX_a(t_1)\, dX_b(t_2) \ \overset{\circ}{=}\!\!\circ\ \overline{\mathcal{X}_a(p_1)\, \mathcal{X}_b(p_2)}$$

where the functions W represent probability densities of the values of the random functions previously mentioned.

From the stochastic standpoint, it is interesting to establish the relations between the correlative functions of the undisturbed displacements $u_a^o(t)$, the disturbed displacements $u_a^{*}(t)$, and the interaction forces $X_a(t)$. These relations can be established based on the equations (6), (7) and (8), or on their operational images (11)÷(14). The 2-nd alternative will be preferred, because it permits simplification of the computations.

The equations of continuity (6) or their images (11), lead, in the field of images, to the relations, [8]

$$(22)\quad \overline{\mathcal{U}_a^o(p)} = \frac{1}{p} \mathcal{U}_{ab}(p) \overline{\mathcal{X}_b(p)}$$

$$\overline{\mathcal{U}_a^o(p_1)\, \mathcal{U}_b^o(p_2)} = \frac{1}{p_1 p_2} \mathcal{U}_{ac}(p_1) \mathcal{U}_{bd}(p_2) \overline{\mathcal{X}_c(p_1)\, \mathcal{X}_d(p_2)}$$

between the correlative functions of $u_a^o(t)$ and $X_a(t)$.

The solution of the equations (22) is, obviously, if considering (12),

$$(23)\quad \overline{\mathcal{X}_a(p)} = \frac{1}{p} \mathcal{R}_{ab}(p) p^2 \overline{\mathcal{U}_b^o(p)}$$

$$\overline{\mathcal{X}_a(p_1)\, \mathcal{X}_b(p_2)} = \frac{1}{p_1 p_2} \mathcal{R}_{ac}(p_1) \mathcal{R}_{bd}(p_2) p_1^2 p_2^2 \overline{\mathcal{U}_c^o(p_1)\, \mathcal{U}_d^o(p_2)}$$

Departing from (13), it is easy to establish the relation between the correlative functions of the undisturbed and disturbed displacements:

$$(24)\quad \overline{\mathcal{U}_a^{*}(p)} = \mathcal{S}_{ab}(p) \mathcal{R}_{bc}(p) \overline{\mathcal{U}_c^o(p)}$$

$$\overline{\mathcal{U}_a^{*}(p_1)\, \mathcal{U}_b^{*}(p_2)} = \mathcal{S}_{ac}(p_1) \mathcal{S}_{bd}(p_2) \mathcal{R}_{ce}(p_1) \mathcal{R}_{df}(p_2) \overline{\mathcal{U}_e^o(p_1)\, \mathcal{U}_f^o(p_2)}$$

The relations (24) can be written in a form corresponding to (14), if the tensor $\mathcal{S}_{ab}(p)\mathcal{R}_{bc}(p)$ will be represented by series as in (14).

It follows, from the former discussions, that, if the interaction problem has once been investigated from the deterministic standpoint, it would be easy to compute the effect of the disturbance upon the correlative functions of the free displacements $u_a^o(t)$, or accelerations $\ddot{u}_a^o(t)$, by the aid of the exact relations (24), or by that of series similar to those of (14), returning then to the field of the originals.

If necessary, the relations (22), (23), (24), are directly extensible to the case of higher order correlative functions.

3.5) Position of the assumption on the instantaneous linear foundation compliance

The analysis carried out in this paper leads to the determination of the disturbed ground motion, to be used as input for the dynamical computation of the structure.

Another procedure could be used for this aim, investigating the structural response to the boundary conditions

$$(25)\quad u_a^{\mathrm{x}}(t) + \int_0^t g_{ab}(t-z)X_b(z)\,dz = u_a^{o}(t)$$

The system (25) is equivalent to the system (6), if the relations (3) are taken into account.

The elementary assumption on the instantaneous linear foundation compliance, as used in [9], substitutes to the exact boundary conditions (25) the algebraic conditions

$$(26)\quad u_a^{\mathrm{x}}(t) + G_{ab}X_b(t) = u_a^{o}(t),\quad G_{ab} = \text{const.}$$

which can be deduced from (25) if the expression $g_{ab}(t) = G_{ab}\delta(t)$ is assumed.

Instead of the boundary conditions (25), similar to the rheological law of a body with postaction, and which lead to energy dissipation into the ground, the elementary elastic conditions (26) lead to no energy dissipation.

3.6) Investigation of the disturbance by means of stationary monochromatic forced vibrations

If the free dynamic displacements $u_a^{os}(t)$ of the structure are not equal to zero, then the expression (13) of the disturbed ground motion will change (in the field of images) into one of the expressions

$$(27)\quad \mathcal{U}_a^{\mathrm{x}}(p) = \mathcal{S}_{ab}(p)\mathcal{R}_{bc}(p)\mathcal{U}_c^{og}(p) + \mathcal{G}_{ab}(p)\mathcal{R}_{bc}(p)\mathcal{U}_c^{os}(p)$$

$$= \mathcal{S}_{ab}(p)\mathcal{R}_{bc}(p)\left[\mathcal{U}_c^{og}(p) - \mathcal{T}_{cd}(p)\mathcal{U}_d^{os}(p)\right]$$

$$= \mathcal{S}_{ab}(p)\mathcal{R}_{bc}(p)\mathcal{U}_c^{og}(p) + \left[\delta_{ac} - \mathcal{S}_{ab}(p)\mathcal{R}_{bc}(p)\right]\mathcal{U}_c^{os}(p)$$

From the expressions (27), it follows that the relation between the undisturbed and the disturbed seismic motions, expressed by the tensor $\mathcal{S}_{ab}(p)\mathcal{R}_{bc}(p)$, may be investigated by means of motions of ground origin or of structural origin, taking into account one of the expressions

$$(28)\quad \mathcal{T}_{ab}(p)\mathcal{R}_{bc}(p)\mathcal{U}_c^{og}(p) = \mathcal{U}_a^{*}(p) \quad \text{where } \mathcal{U}_a^{os}(p) = 0$$

or

$$(29)\quad \mathcal{T}_{ab}(p)\mathcal{R}_{bc}(p)\mathcal{U}_c^{os}(p) = \mathcal{U}_a^{os}(p) - \mathcal{U}_a^{*}(p) \quad \text{where } \mathcal{U}_a^{og}(p) = 0$$

that means, either by comparing the disturbed ground motion to the free ground motion, or by comparing the disturbance introduced into the structural motion to the free structural motion. The second procedure may show itself to be of a more practical use.

If m linearily independent vectors $\mathcal{U}_{ar}^{os}(p)$ and the corresponding vectors $\mathcal{U}_{ar}^{*}(p)$ (r=1...m) are known, the tensor $\mathcal{T}_{ab}(p)\mathcal{R}_{bc}(p)$ may be determined by solving, with respect to it, the system of m^2 equations

$$(30)\quad \mathcal{T}_{ab}(p)\mathcal{R}_{bc}(p)\mathcal{U}_{cr}^{os}(p) = \mathcal{U}_{ar}^{os}(p) - \mathcal{U}_{ar}^{*}(p)$$

If a motion of ground origin would be used to this purpose, an algebraic system, deduced not from (29), but from (28), would be substituted to (30).

A stationary monochromatic input of the original of the relation (28),

$$(31)\quad u_a^{og}(t) = e^{G_a + i(\omega t - \varphi)}$$

leads to a stationary monochromatic output of the same relation,

$$(32)\quad u_a^{*}(t) = e^{D_a + i(\omega t - \varphi - \varphi_a)}$$

making the tensor $\mathcal{T}_{ab}(p)\mathcal{R}_{bc}(p)$ to be in this case a function of ω,

$$(33)\quad \mathcal{T}_{ab}(p)\mathcal{R}_{bc}(p) = e^{\chi_{ac}(\omega) - i\varphi_{ac}(\omega)} \quad \text{where } \omega = \text{Im}\,p$$

If the tensors $\chi_{ac}(\omega)$ and $\varphi_{ac}(\omega)$ are determined for each value of ω, the disturbed motion $u_a^{*}(t)$ can be expressed, taking into account the direct and inverse integral transforms, by the formula

$$(34)\quad u_a^{*}(t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dp \int_0^{\infty} \mathcal{T}_{ab}(p)\mathcal{R}_{bc}(p)e^{p(t-z)}u_c^{og}(z)\, dz$$

where the tensor $\mathcal{T}_{ab}(p)\mathcal{R}_{bc}(p)$ is expressed by means of (33).

The importance of studies as that of I. Toriumi, on which the paper [12] is based, becomes obvious for the case of an arbitrary ground motion.

Besides the theoretical investigation of the tensor $\mathcal{T}_{ab}(p)\mathcal{R}_{bc}(p)$ by computing the response to monochromatic stationary forces, experimental methods, starting from (30), become of use. The tensor $\mathcal{J}_{ab}(p)\mathcal{R}_{bc}(p)$ may now be determined by accomplishing an m-fold recording of the motions $u^{*}_{ar}(t)$ $(r=1...m)$ due to forces which would produce the m structural linearily independent types of vibrations $u^{os}_{ar}(t)$ and hence computing by means of (30) the tensors $\mathcal{X}_{ab}(\omega)$ and $\varphi_{ab}(\omega)$. The circular frequency ω of the periodic forces must vary, theoretically, for each of the m experiments, from 0 to ∞.

The tensor $\mathcal{J}_{ab}(p)\mathcal{R}_{bc}(p)$ once known for the case of a relatively simple structure, can be computed for the case of a more complex structure, if only the type of foundations remains unchanged. For example, if its values are known for the case of a rigid cylinder, they can be hence deduced for the case of a stack.

If, in the case of a simple structure, the expression of this tensor is

$$(35)\quad \mathcal{J}^{o}_{ab}(p)\mathcal{R}^{o}_{bc}(p) = \mathcal{A}^{o}_{ac}(p)$$

the expression of the tensor $\mathcal{G}_{ab}(p)$ will be, taking into account the relations between the various tensors above discussed,

$$(36)\quad \mathcal{G}_{ab}(p) = \left[\mathcal{A}^{o(-1)}_{ac}(p) - \delta_{ac}\right]\mathcal{J}^{o}_{cb}(p)$$

The tensor $\mathcal{G}_{ab}(p)$ must be the same for two structures having the same type of foundation, that means

$$(37)\quad \left[\mathcal{A}^{o(-1)}_{ac}(p) - \delta_{ac}\right]\mathcal{J}^{o}_{cb}(p) = \left[\mathcal{A}^{o(-1)}_{ac}(p) - \delta_{ac}\right]\mathcal{J}_{cb}(p)$$

and hence the following expression is obtained

$$(38)\quad \mathcal{J}_{ab}(p)\mathcal{R}_{bc}(p) = \mathcal{A}_{ac}(p)$$
$$= \mathcal{J}_{ab}(p)\left[\mathcal{J}_{bc}(p) - \mathcal{J}^{o}_{bc}(p) + \mathcal{A}^{o(-1)}_{cd}(p)\,\mathcal{J}^{o}_{bd}(p)\right]^{(-1)}$$

The expression (38) allows the determination of the disturbed motion for the case of a complex structure, if the tensor (35) is known for a simple structure, without solving again the whole interaction problem. Numerical results for simple structures, as those of [12] are of practical use also for the case of complex structures.

4) Conclusions

The disturbance phenomenon introduced by the presence of the structure into the seismic motion, observed in the seismographic records by several specialists and taken into account in the structural computations, or quantitatively valued, based on several simplifying assumptions, is described, in a general manner, in the bounds of the former admitted assumptions, by the equations of continuity (6), established in this paper. When the continuous contact between ground and structure can be approximated by a discrete one, then the problem can be approached too by the proposed method. Contrarily, the equations (6) would be replaced by multiple Volterra-Fredholm integral equations.

The proposed method allows one the theoretical investigation of a phenomenon not yet investigated systematically: the seismic interaction of neighbouring structures, through the mutual propagation of the disturbance produced by each of the structures.

The response spectrum method is further of use, but adopting as input the disturbed ground motion. From (20), it becomes obvious that the response spectrum method (when adopting as input the undisturbed motion) is rigorous in the limit case when the ratio of rigidities of structure to ground tends to zero, maintaining at a given value the ratio of structural rigidity and inertia.

In applying the proposed method, probabilistic deviced can be used, taking into account the free motion as a stochastic process, investigated by means of vectorial random functions.

The assumption on the instantaneous foundation compliance is valid in the practically impossible peculiar case of a massless ground (leading to an infinite propagation velocity). In a more accurate analysis, the boundary conditions (25) must be substituted to this elementary assumption.

In investigating the interaction phenomenon, a nucleus corresponding to monochromatic stationary vibrations may be used, instead of the original nucleus, corresponding to instantaneous pulses, by using in the computations the relation (34).

In the general expression of the equations (6), the analytical form of the nucleus remained unprecised. This analytical expression has not yet been generally obtained for the tensor $g_{ab}(t)$, except in some peculiar cases which do not exhaust practical problems. The theoretical approach, excessively laborious and based on too simple assumptions (ordinarily, perfectly elastic body), cannot, practically, avoid this shortcoming. It follows, logically, taking into account the actual possibilities, that the experimental determination of the nucleus $g_{ab}(t)$ must be adopted, by recording the dynamic displacements of the foundations, due to instantaneous loading. A program of systematic experimental investigation in this sense can be accomplished by a research institute possessing an adequate experimental basis.

Instead of using the equations (6) as a basis for the investigation of the interaction phenomenon, their operational image may be taken into account, avoiding in this manner the necessity of determining the tensor $g_{ab}(t)$. Thus, the problem of determining this nucleus is replaced by the problem, leading in many cases to less difficulties, of determining the tensor $\mathcal{Y}_{ab}(p)\mathcal{R}_{bc}(p)$ by means of monochromatic forced vibrations. A program of experimental investigations, accomplished this time not upon the ensemble ground-foundation, as for the tensor $g_{ab}(t)$, but upon actual structures, is now of use.

A systematization of computing methods becomes necessary.

For the finite sums method, a mechanical analyser and an investigation of the variation of the nucleus $u_{ab}(t)$ in the neighbourhood of the origin are necessary, in order to determine the admissible interval Δt of (9).

For the operational calculus, it is necessary a rapid computer of direct and inverse integral transforms for empirical functions.

For the iterative method it is necessary to investigate the convergence of the correction series of (14).

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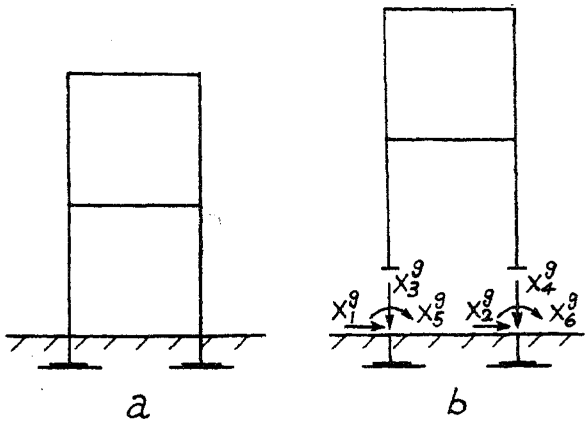

FIG. 1

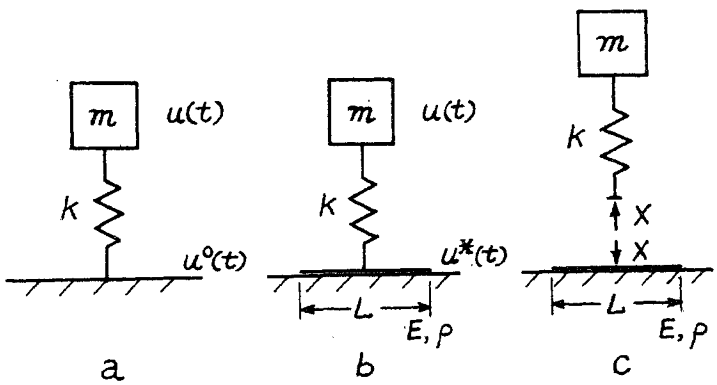

FIG. 2