THE METHOD OF DIRECT DETERMINATION OF REDUCED SPECTRA OF SEISMIC ACCELERATIONS

By

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We propose the following method of integration of the equation of the seismo-resistant theory /1,2/.

Let the system of differential equations of structure vibration in orthogonal coordinates be given when horizontal vibration of the ground corresponds to the law $y_0(t)$:

$$q''_1(t) + p_1^2 e^{i\alpha_1} q_1(t) = -\delta_1 y''_0(t) \qquad (1)$$

....

$$q''_j(t) + p_j^2 e^{i\alpha_j} q_j(t) = -\delta_j y''_0(t)$$

....

Here q_j - the generalized coordinates, responding to the fundamental functions of the structure X_j.

p_j - circular frequences;

α_j - energy diffusion measure of vibration according to Sorokin's hypothesis* and, finally

$$\delta_j = \frac{\int X_j q(x)\,dx + \sum X_j(x_m) Q_m}{\int X_j^2 q(x)\,dx + \sum X_j^2(x_m) Q_m} \qquad (2)$$

Here $q(x)$ - continuous load on the length unit;

Q_m - concentrated load, responding to the axes of the abscissa of the structure x_m.

Under the system of equation (1) it is understood that the scheme of the structure may be reduced to the beam, whereby the necessary conditions of symmetry of the beam in the plane are maintained.

The complete displacements of the structure axes in relation to the ground is written

$$y(x,t) = \sum_{j=1}^{n} q_j(t) X_j(x) \qquad (3)$$

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*There is a close inter-univalue relation between the hypothesis of Sorokin and Focht in the limits of extinguishing decrements of natural structures which are not to be considered here (2).

Let us now assume that along with the structure under consideration other seismographs are installed in similar ground conditions with indentical frequences of free vibration p_j. Let us assume further that the seismographs are provided with dampers which absorb the same amount of energy as Sorokin's hypothesis claims.

The differential equation of vibration of the seismograph pendulums, on the basis of the above mentioned conditions of frequency of their free vibration and damping, may be written down thus for the earthquake under consideration:

$$\begin{array}{c} u''(t) + p_1^2 e^{i\alpha_1} u_1(t) = -\lambda_1 y_0''(t) \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ u_j''(t) + p_j^2 e^{i\alpha_j} u_j(t) = -\lambda_j y_0''(t) \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \end{array} \qquad (4)$$

Here $u_j(t)$ - relative displacement of seismograph pendulums, caused by ground vibration according to the law $y_0(t)$;

λ_j - any constant depending on the construction of the seismographs.

Comparing differential equations (1) and (4) we come to the conclusion that between the generalized coordinates of the structures q_j, and the seismographs u_j the following relationship should take place:

$$\frac{q_j}{u_j} = \frac{q_j''}{u_j''} = \frac{\delta_j}{\lambda_j} \qquad (5)$$

This directly follows from the condition that the differential equations are linear and vary only by constant multipliers on their right hand sides.

Thus, being aware of the law of relative displacement of the seismometer pendulum $u_j(t)$, it will be possible to determine the generalized coordinate of the structures $q_j(t)$ responding to the fundamental function $u_j(t)$ by means of the formula

$$q_j(t) = u_j(t)\frac{\delta_j}{\lambda_j} \qquad (6)$$

Consequently, the complete deformed state of the structure, defined by the term (3), can be determined by means of displacing the pendulums of the series of seismographs under consideration

$$y(x,t) = \sum q_j(t) X_j(x) = \sum \frac{\delta_j}{\lambda_j} u_j(t) X_j(x) \qquad (7)$$

Thus, it will be possible to find the elements of the deformed state of the structure, if the corresponding displacements of the seismographs are known for the same earthquake. It is clear that term (7) refers to the forced as well as to the free vibrations.

The deformation of the structure once known, its strained state may be calculated at any given moment of time.

As seen from above, we have virtually realized an instrumental solution of the system of differential equation (1), to which the problem of seismo-resistant theory has been reduced.

Up to now we had proceeded from the condition that the periods of free vibration and the measurement of energy diffusion of the seismographs coincide with the same dynamic characteristics of the structure.

When a sufficient large amount of seismographs are installed with various periods of free vibration and with various dampings (1) this limitation no longer takes place because there may be an interpolation between their data. The scheme of analysis of the structural behaviour under consideration at the time of earthquakes may be useful for research purposes. From expression (7) it is possible to find such a moment of time (1) for every point of the structure at which the greatest state of tension takes place at every point under consideration. It may be taken for granted that in principal, such a course brings us to a complete solution of the equation system (1). We find the practical realization of this method of integration expendient. Of course, there is no need of using series of seismographs with various periods of free vibration and with various extinguishing decrements. It is possible to design a multi-pendulum instrument based completely on mechanical principle or to construct a multi-canal instrument based on any form of electromechanical transformations of vibration, accomplishing the integration of the equations of type (1) directly, during the process of the earthquake. The merit of this method of integration is that in this particular case the frequency distortion of neither the seismographs nor the accelerograph may trouble us at all, as in the case when accelerograms are integrated. The experimental operation of the above mentioned type of instrument has given satisfactory results (3). Further simplification of this idea will give us the possibility to introduce seismometers for wide purposes, to record severe earthquakes in order to obtain spectra of reduced seismic accelerations directly. This will help to solve the problem considered by Baiot, Housner, Korchinsky and Medvedev by a completely new method /4, 5, 6, 7/. By reduced seismic acceleration we understand such a permanent active acceleration, under the influence of which the seismometer pendulum deflects to such a maximum value as it would under the claimed earthquake. Let us consider the differential equation (4) for generalized coordinates $u_j(t)$ when the vibration of the ground is along the axes y :

$$u_j''(t) + p_j^2 e^{i\alpha_j} u_j(t) = -\lambda_j y_0''(t) \qquad (7)$$

As agreed the reduced seismic acceleration is constant, consequently its activity is statistic and therefore $u_j''(t) \equiv 0$. Denoting the maximum reduced seismic acceleration τ_j , substituting it with $y_0''(t)$ in (7) and having under consideration that $u_j'' \equiv 0$ we shall receive:

$$\tau_j = \frac{R_e\left[p_j^2 e^{i\alpha_j} u_j(t)\right]}{\lambda_j} \approx \frac{p_j^2 u_j}{\lambda_j}$$

since $\cos\alpha_j \approx 1$. Here the symbol Re shows that we have taken the really part of the term. Under u_j the maximum value of the substantial part of $u_j(t)$ is understood. The reduced seismic acceleration does not depend on the construction of the linear elastic system with one degree of freedom only if the value of p_j and α_j were invariable. The term (8) renders reduced seismic acceleration for the j seismomenter pendulum with the period of free vibrations T_j.

By determining the reduced seismic acceleration for the j structure coordinate, analogically from equation (1) we shall obtain

$$\tau_j = \frac{Re[p_j^2 e^{i\alpha_j} q_j(t)]}{\delta_j} \approx \frac{p_j^2 q_j}{\delta_j} \qquad (9)$$

The terms (8) and (9) offer the same values of reduced seismic acceleration as in virtue of relation (5).

This property of reduced seismic acceleration is convenient for practical calculations.

Having a series of pendulums with various periods of free vibration T_j, if the measurement of energy diffusion of vibration α is fixed, it will be possible to receive various values of reduced seismic acceleration τ_j. If the values of T_j on the abscissa axes and on ordinate axes τ_j are marked down, then we shall receive a spectrum of reduced seismic accelerations $\tau(T)$ at fixed α.

What is essential when giving an idea of the spectrum of reduced seismic accelerations is that in seismographs which record according to (4) the relative displacements of $u_j(t)$, the clock mechanism with the register drum may be eliminated and it is enough to record directly the maximum displacement u_j of the pendulums in relation to the ground and by their help, based on (8) to determine the ordinates τ_j of the $\tau(T)$ spectrum.

Thus, we come to the following principal of the instrument. The instrument must represent a series of linear pendulums, for example, elastic, possessing various periods of free vibration, comprising possible periods of free structural vibration, provided with dampers and marking its maximum relative deflected positions. Let us agree to call such instruments in the future maximum multi-pendulum seismometers or abbreviated, just seismometers. The substitution of seismographs with seismometers has an essential, principal significance. The seismic field is rather heterogeneous, therefore to receive statistic data on the force of the earthquake it is necessary to arrange a large amount of simple, cheap instruments which permit simplified treatment of the recorded data. These conditions are essential to avoid the maintenance of a significant number of servicemen.

Experience has shown that damping i.e. the measure of energy dissipation α in practically important limits, does not effect value τ so much. Therefore it is possible to install two or three seismometers in some points with various α, for the introduction of the correction factor and verification of the curve $\tau(T,\alpha)$ depending on the value of α.

The question of selecting the possible minimum amount of seismometer pendulum is essential. The theoretical calculations and also the analysis of spectra, obtained by Housner, show that at the decrement of logarithm extinguishment of 0,3 order, the spectra of reduced acceleration change quite smoothly. Besides, these spectra change almost according to hyperbolic law. This was the reason that we used only six pendulums for the seismometer.

In order to receive a more detail spectra two seismometers may be installed beside one another with displacements in relation to one another with periods of free vibration.

Different constructions of multi-pendulum seismometers have been constructed such as those with pendulums performing polarizing oscillation as well as with spherical pendulums. Our most widespread seismometer is AIS - 2 type with a spherical pendulum. Such instruments are installed in large quantities in Yerevan, Tbilisi, Stalinabad and other places for the recording of earthquakes.

Moreover, the AIS - 2 seismometer is widely used to investigate blast effect.

The AIS - 2 seismometer, constructed at the Institute of Building Materials and Structures in Yerevan, Armenia, U.S.S.R., represents a series of pendulums with various periods of free vibration (horizontally with T = 0,05; 0,1; 0,2; 0,8; and 1,2 sec., and vertical with T = 0,05; 0.1 and 0.2 sec.) placed on a common plate. The pendulum resembling a bar with the load at its end is led through a rubber cylinder by the other end, closely fitted into a metal ring which is tightly connected with the plate. The rubber is a spring and at the same time a damper for the pendulum. At the end of the pendulum bar which emerges from the rubber, a springy recording needle is fit on a thread. When the pendulum oscillates the needle makes a scratch on the sooty glass (with which it is in constant contact). Seismometer AIS - 2 is shown in fig. 1.

After carrying out the recording the plate-holder is taken out of the instrument, brought under the instrumental microscope and the maximum value of the relative displacement of the pendulum is measured. The examples of recordings of blasts and shocks with the AIS - 2 seismometer is shown in fig. 2.

The maximum reduced seismic acceleration is determined by means of the following simple formula :

$$\tau = g\left(\frac{T_0}{T}\right)^2 \frac{d}{a} = Kd$$

where d - measured maximum deflection of the pendulum point in centimeters;

a - distance of the end of the point from the centre of rotation of the elastic pendulum in centimeters;

T_0 - period of free vibration of the pendulum under consideration, freely suspended at the centre of rotation;

T - period of free vibration of the same pendulum, when fitted elastically in a rubber plug, i.e. in its working condition.

g = 981 cm/cek^2.

The value of coefficient $K=\left(\frac{T_0}{T}\right)^2\frac{g}{a}$ is given for every pendulum in a numerical form in the instruction accompanying the seismometer. Consequently in order to determine the reduced seismic acceleration it is enough to have only the value of maximum relative displacement of the pendulum and to multiply it by the corresponding coefficient K.

The spectral curves of reduced seismic acceleration $\tau(T)$ were first obtained in 1954, by measuring blast waves when a mass of explosions were being performed on the territory of the basin of Gumush hydroelectrical station for daily regulations/3/. These curves are seen in fig. 3.

Subsequently, such curves were obtained in a large quantity for various geological conditions, the distance from the blast location, the number of charges, form of blasts etc., when mass explosions were being performed in the regions of the Armenian SSR. The explosions measured were weak as well as very strong, bringing about responce in the pendulum with T = 0,05 sec. reduced seismic acceleration till 17 g /8.9/.

From the results of seismometric observations of mass explosions the following items have been established:

1. The AIS - 2 seismometer fully justifies itself in its performance, necessary virtual material may be received, the treatment of which does not present difficulty for the building of spectral curves.

2. In most cases the curve of reduced seismic accelerations is principally obtained decreasingly with the increase of free vibration period of the pendulum. This curve may be approximately represented as a hyperbola.

3. It is clearly seen that with the increase of the amount of charge, the curve of the reduced seismic accelerations becomes steep.

4. The change of distance from the location of the explosion shows more influence on the increase of the reduced seismic acceleration than the change of quantity of the explosive charge.

5. The value of reduced seismic acceleration decreases strongly with the increase of the distance from the place of the blast. However, it has not been possible to establish general regulations for the time being. At the present we may speak of separate blasts. For example, for T = 0,05 and 0,1 sec. a square dependence of the reduced seismic acceleration is obtained from the distance in case of some blasts, while for others we receive cubic and even higher, and sometimes on the contrary, it approaches to linear.

6. The blast effect in marls is about three times more than in cliff

rocks, for example as in dacite and andesite basalt.

7. The reduced seismic acceleration in pumice stone is many times more than in obsidian. (For the pendulum with T = 0,05 sec. almost 10 times).

8. The reduced seismic acceleration for the pendulum with T = 0,05 sec. in water-washed clay soil is about 2 times more than on embarkments: for T = 0,1 and 0,2 sec., on the countrary, on embarkments very much more than in water-washed clay soils.

9. The blast effect at the entrance of a tunnel is substantially more than in the tunnel itself.

10. The reduced seismic acceleration in a tunnel is approximately 10 times less than compared with the surface of the slope at the mark of 35 m. from the foundation of the tunnel.

11. The blast effect along the line of the prospect-hole when set linear is much less than the direction perpendicular to the prospect-hole line (approximately three times).

12. When the blast waves are directed along the ravine, the horizontal component of the reduced seismic acceleration on the precipice and on the ravine bottom are almost alike for the period of vibration in the limits of T = 0,05 - 0,4 sec.; the vertical componant for T = 0,05 sec. is less on the precipice and for T = 0,1 and 0,2 sec. is less on the ravine bottom.

13. When the blast waves pass through the layer bottom of the ravine (perpendicular to the ravine), which by character resembles the passing of earthquake wave, horizontally as well as vertically components of the reduced seismic acceleration for T = 0,05 and 0,1 sec. on the precipice are much less than on the bottom of the ravine. When the periods of free vibration are increased the value of the reduced seismic acceleration is about the same.

14. The horizontal components of the reduced seismic acceleration for T = 0,05 sec. is less on plains than on precipices. For the rest of the values of periods of free vibration of horizontal components of reduced seismic acceleration on plains seems to be somewhat more than on the precipice, while the vertical component is considerably more.

15. The presence of a ravine on the path of passing blast waves strongly decreases the horizontal components of reduced seismic acceleration (2-4 times) and has comparatively little effect on vertical components, almost without changing its value.

16. When explosion takes place in a drift (at the depth of 30 m from the surface) horizontally as well as vertically the components of the reduced seismic acceleration at the mark of the draft is 2 times less than on the surface and this shows that blast effect substantially increases at the surface.

17. In the process of explosions on territories under construction the maximum value of charges has been established by means of the AIS - 2

seismometer thus providing safety construction.

The AIS - 2 seismometers are permanently installed at five points in Yerevan, in various soil conditions and at all the four seismological stations of the Armenian SSR ("Yerevan", "Leninakan", "Goris" and "Stepanavan").

On February 14, 1957 an earthquake was recorded in Yerevan - intensity 4, with the help of three AIS - 2 instruments, which were installed in various grounds (rocky, loose ground and contemporary sedimentary deposits of medium density).

The results of recording analysis received by C.A. Piruzian show that spectra $\tau(T)$ generally has a hyperbolic character (fig. 4); for periods of free vibration in the limit of 0,2 - 0,3 sec. we observe the increase of horizontal acceleration components in loose ground in comparison with the rocky, while for the periods T = 0,4 sec. τ is equal in basalt and loamy soils and in gravel deposits of the river Gedar. For the period T = 0,05 sec. the largest τ was obtained for loose soils, two times less for the rocky and about four times less for sedimentary deposits of medium density /10/.

Five AIS - 2 seismometers were permanently installed in different parts of Tbilisi in different soil conditions.

On May 30, 1957 in the city of Tbilisi an earthquake was recorded with an intensity of 5.. The recording of this earthquake was kindly handed over to us be A.N. Safarian. The graph of reduced seismic accelerations for horizontal and vertical pendulum which were received from the above mentioned earthquake at one point is shown in fig. 5.

In 1957, June - Sept. in the city of Stalinabad in the Gissar Basin a seismometric net was installed, the profile extent being about 20 k.m. consisting of 23 points. The locations of the seismometers where chosen on such an estimation as to include various soils, hydrogelogical and morphological conditions of the Basin. A chamber of reinforced concrete was set up at every point where a AIS - 2 multipendulum seismometer was placed /11, 12/.

During the period of August 1957 to March 1958 we succeeded in getting recordings of four earthquakes the force of which were of 3, 4, 5, 3 intensity. The graph of the reduced seismic acceleration (imagined in logarithmic scale) for these earthquakes is shown in fig. 6.

The analysis of the data acquired by means of the AIS - 2 seismometer gives us the possibility to come to the following conclusions for Stalinabad conditions.

1. The curves of the reduced seismic acceleration obtained for various soil conditions at the time of earthquakes have hyperbolic character.

2. The spectral curves of reduced seismic acceleration obtained for earthquakes with an intensity force 5 show that they differ from the general

regularities of the reduced seismic acceleration at the period of vibration 0,4 sec. by the form of a small peak. Analogous phenomenon was not observed for earthquakes of weaker force.

3. For comparison, instead of the reduced seismic accelerations of separate pendulums it is possible to use the mean value according to the data given by pendulums with periods 0,2; 0,4; 0,8; 1.2 sec. in order to bring the value of the earthquake intensity to a mean.

4. With the increase of vibration period the value of reduced seismic acceleration of weak earthquakes decreases more sharply than in the case of moderate or rather strong shocks.

5. It is evident from the data that has been obtained that the value of τ depends much upon the geological - engineering conditions of the locality. This position is seen more clearly in case of moderate and rather strong earthquakes.

6. The instrumental results have shown that the higher the level of ground water the larger the value of τ .

7. The results of the data also show that τ is about two times more in loess - like deposits than in conglomerates of large thickness with a deep level of ground water.

8. τ for loess - like soils of less thickness, streching over thick conglomerates nearly coincides with τ for conglomerates of large thickness.

9. With the increase of the thickness of the loess blanket from 3,5 to 17 m. τ shows an increase of about two times.

10. The value of τ for conglomerates is 1,5 - 2 times more than for limestones.

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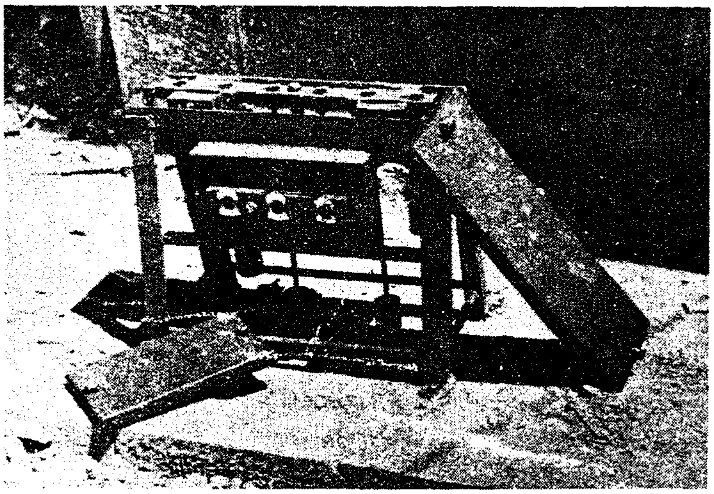

Fig. 1

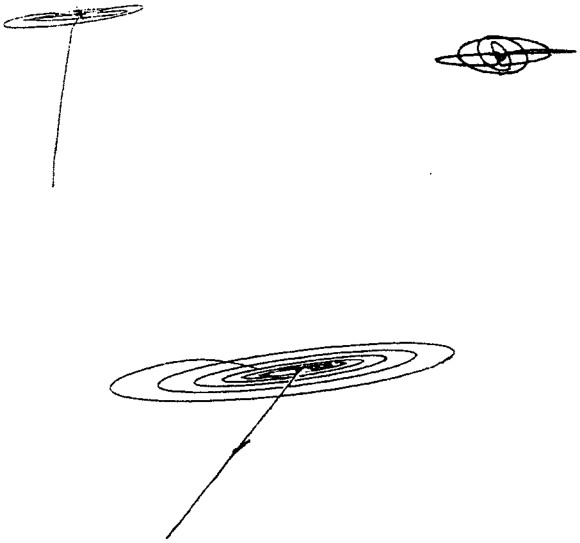

Fig. 2

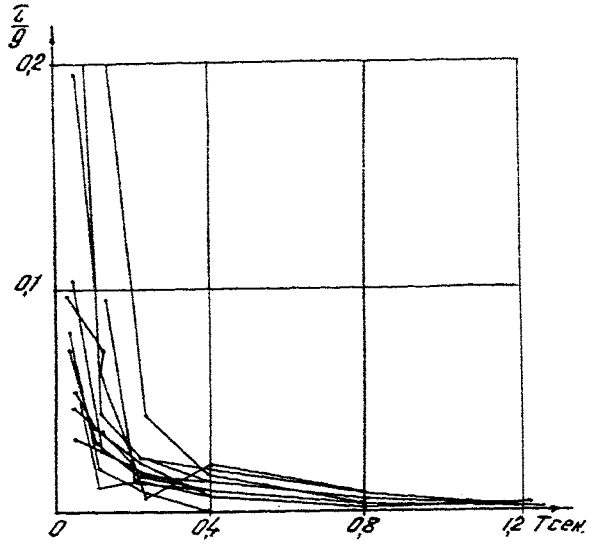

Fig. 3

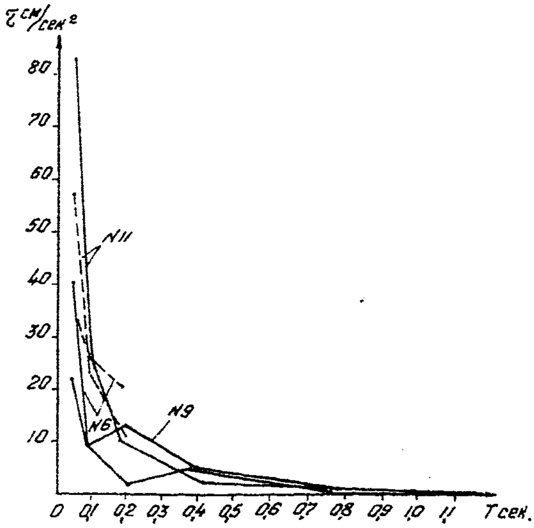

Fig. 4

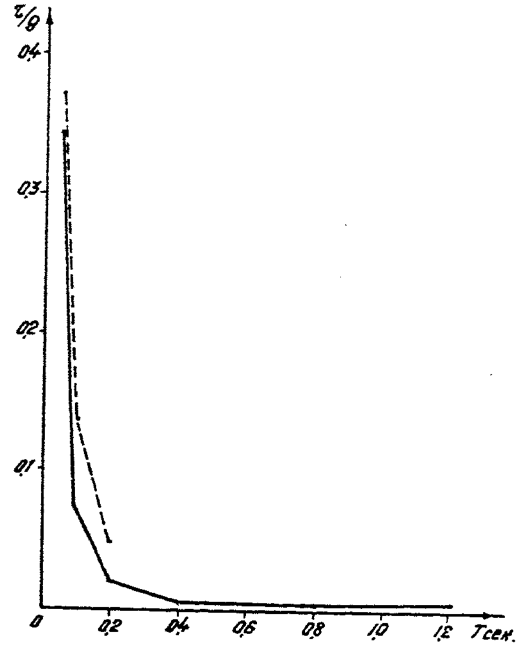

Fig. 5

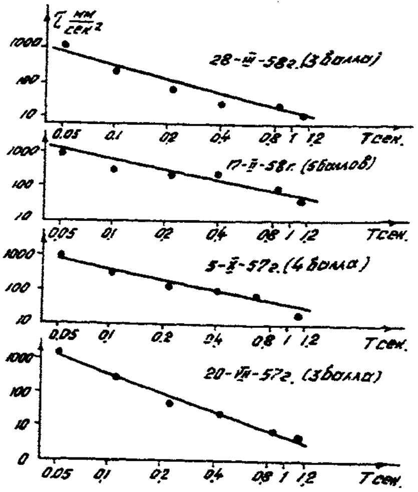

Fig. 6