NONLINEAR TRANSIENT VIBRATION OF STRUCTURES

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SYNOPSIS

In this paper the writer discusses two important problems regarding to the nonlinear transient vibration of structures against earthquake shocks. The former studied in part I is the problem of analog computer analysis, and the latter considered in part II is one of the theoretical analyses of the torsional vibration responses of framed structures.

PART I. ANALOG COMPUTER ANALYSIS

Preface: For the design of earthquake-resistant structures, behavior of the structures under the action of earthquakes would have to be clarified as exactly as possible, and hence a series of analyses on this problem have been carried out in recent years. When a structure is subjected to large dynamic loads, the stresses exceed the yield point and the behavior of the structure will be of nonlinear type. Then, it is very difficult to analyze the problem if the structure itself is not simple. An electronic, low-speed analog computer, recently built at the Faculty of Engineering, Kyoto University, was equipped with a backlash element and enabled us to obtain accurate transient responses of a vibrating system with an idealized bi-linear, hysteretic restoring force.

Application of Analog Computer: Computations by means of the analog computer are made with assemblage of a set of elements such as amplifiers, potentiometers, servo-multipliers, etc. The computer simulates a collection of basic building blocks interconnected so that they are governed by the same set of fundamental equations as those describing the system to be analyzed. In the electronic, differential analyzer, the components are capable of summation, integration, multiplication, and generation of arbitrary functions. Thus it is capable of all the operations necessary to solve ordinary nonlinear differential equations.

Then, the nonlinear vibration of building structures is studied in the behaviors of idealized multiple-mass structures for which case the fundamental differential equations of motion for the system subjected to an earthquake shock can be written as follows.

$$M_i \ddot{X}_i + G_i(u_i\,;t) - G_{i+1}(u_{i+1}\,;t) = -M_i \ddot{X}_G \qquad (1)$$

where $u_i = X_i - X_{i-1}$. (See Nomenclature) Introducing the quantities $p_i = \sqrt{k_i/M_i}$, $G_i(u_i) = k_i g_i(u_i)$,

Eq. (1) can be written as

$$\ddot{X}_i + p_i^2 g_i(u_i) - p_i^2 g_{i+1}(u_{i+1})\, k_{i+1}/k_i = -\ddot{X}_G \ . \qquad (2)$$

In the case of i = 5, for example, the block diagram of the analog

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computer corresponding to Eq. (2) is shown in Fig. 1.

When the nonlinear, hysteretic restoring force curves are assumed as in Fig. 3, restoring force functions, g_i, shown in Eq. (2) or Fig. 1 are represented by the circuits of the backlash elements, but in the case of single mass structure with the restoring force curve assumed to Fig. 4, the block diagram of the analog computer is not so complicated as in Fig. 2.

Although it is desirable that the actual strong ground motion is adopted as for the program source of input, we have no reliable records of strong motion in Japan as yet. Therefore, the full-cycle, sine-, cosine- and rectangular-acceleration pulses are introduced by way of input functions, X_G/p^2, corresponding to earthquake shocks. For example, Fig. 6 shows the block diagram of the function generator of full-cycle cosine pulse.

Results of Analysis and Discussions: The analysis has been made for the seven values of φ; 0°(ideal plastic), 13°, 23°, 30°, 36°, 41° and 45°(linear). Ground motions were assumed to have a combination of various values of α and T/T_o. Parameter α related to the value of α' has been chosen from among the five values, 0.2, 0.4, 0.6, 0.8 and 1.0; ratio T/T_o ranged from 0.6 to 2.0 at intervals of 0.2.

The ground acceleration as well as the responses of the system, i.e., the displacement and velocity, were automatically recorded by the computer as functions of time. A total of 840 sets of response curves were analyzed for single-mass systems. And, in arrangement of these responses, our attention has been paid to the quantities X_P, X_D, X_F and t as shown in Fig. 5, which is an example of the displacement response curve obtained.

(A) Spectra X_R and X_D relating to T/T_o

The transient maximum amplitude of the nonlinear system, X_R, and the maximum displacement measured from the initial equilibrium position, X_P, are plotted in Figs. 7~12 for the ratio T/T_o with parameters φ and α. In Figs. 7~9, spectra drawn by broken lines and chained lines show the responses corresponding to the ground motions which have the same velocity or the same displacement but different durations. Observation of the spectra will indicate what follows.

1) When the absolute value of ground acceleration, α, is small, i.e., $\alpha = 0.2$, vibration of the system is elastic regardless of the shape of ground motions and the nonlinear characteristic of the restoring force φ.

2) For the range of φ from 30° to 45°, each of the spectrum curves shows a distinct peak, which lies in the region of small T/T_o. The location of the peak moves toward a larger value of T/T_o as parameter φ decreases. When φ approaches zero and α is large, the spectrum curves have no peaks.

3) Comparison of spectra for different values of φ indicates that the "backbone curve" of the spectra stands steeply if φ is large enough, while the slope of backbone curves decreases with decreasing of φ.

4) The spectra of X_R plotted for the ground motions with the same ground velocity or displacement show distinct peaks around $T/T_o = 0.7$ with small variation in their locations.

From the above observation, it can be mentioned that the maximum displacement of a vibrating system with a certain value of φ cannot be determined even when α may be constant. This implies that the response of the system has nothing to do with α alone but it is also closely related to T/T_o, and that a reasonable criterion of specifying the maximum distortion of the system during an earthquake would be the velocity or displacement of the ground motion rather than the ground acceleration.

(B) Spectra of X_R/X_G

In order to study the results in detail from another point of view, in Figs. 13~15 plotted were the spectra of the ratio of X_R to the corresponding amplitude of ground motion X_G. These diagrams will clarify the following features.

1) With a few exceptions, the maximum amplitude X_R/X_G decreases uniformly as T/T_o increases.

2) If the ground acceleration is small, say, $\alpha = 0.2$, we can see that the vibration of the systems remains elastic, and that the shape of the pulse is of little consequence for any value of φ.

3) The greatest maximum amplitude X_R/X_G occurs for small α, and with increase in α the peak shifts its location toward a smaller value of T/T_o. When α is further large, no peaks can be seen at all.

4) In most cases, the slope of the spectrum curves decreases as the restoring force of the systems enters into the plastic range. Consequently, in the region of small values of T/T_o, X_R/X_G becomes smaller for a larger value of α, whereas in the region of larger T/T_o this goes in the other way. Therefore, the spectrum curves as a whole intersect each other.

(C) Relationship between X_F and T/T_o

After the action of a ground motion is over, the position of equilibrium of a vibrating system differs from the initial location because of plastic deformation occurred in the system during vibration. We present the residual displacement of the system from the initial position of equilibrium, X_F, which is shown in Fig. 16. Features of the same kinds of this diagram may be summarized as follows:

1) For the larger value of φ than 30^o, it is noticed that X_F is approximately equal to zero for any values of α and T/T_o.

2) When φ is zero. X_F increases with T/T_o, but regularity in the shapes of spectra cannot be found.

3) When φ is small, however, we see that for smaller values of T/T_o than 1.0 dispersion of spectra is not remarkable regardless of α, and that for the larger values of T/T_o than 1.0 the dispersion of spectra is

now notable regardless of α.

(D) Relationship between t/T_o and α (with T/T_o and φ as parameters)

The values of the ratio of the period of vibration of the system at the maximum amplitude, t, to the elastic natural period, T_o, have been plotted against α and shown in Fig. 17 in which the case of $T/T_o = 1.4$ is represented. From the same kinds of this diagram we can obtain the same conclusions for two kinds of ground motions.

1) It may be said that when the value of α becomes large, the period of vibration of the system is considerably elongated. Moreover, the elongation of the period may be associated with T/T_o in such a way that larger values of t/T_o is effected by the increase in T/T_o.

2) When α is small, the value of t/T_o is smaller than unity for a small value of T/T_o. This means that in this case the maximum amplitude of the system occurs during the forced vibration era.

3) Dispersion of the spectrum curves, depends upon the value of T/T_o, namely, remarkable dispersion is observed when T/T_o is relatively large.

4) When T/T_o is larger than 1.2, the largest value of t/T_o is resulted in the system with an ideally-plastic restoring force characteristic. Also, when T/T_o is smaller than 1.6, the value of t/T_o for the linear system is constant and approximately equal to unity.

Conclusions: In this analysis, it has been shown that the electronic, low-speed analog computer is extremely useful and time-saving in obtaining responses of the vibration systems with bi-linear, hysteretic restoring force characteristics. The accuracy of the responses obtained by the analog computer has been found to check favorably with those computed by the Phase-plane-delta method under the same conditions. It has therefore been confirmed that we can employ this computer to the asseismic analysis of structures.

PART II. NONLINEAR TORSIONAL VIBRATION OF FRAMED STRUCTURES

Preface: In general, displacements of both structures and the ground will not necessarily be in the same direction because there are unavoidable asymmetricities in mass- and rigidity-distributions of the structures in many practical cases. Then, the structural behaviors contain inevitably so-called torsional vibrations. Therefore, it may be said that the results of analyses on the structural responses in the direction of the principal axis of structures are likely to confine the complicated phenomena to one possibility which arises as in a special case.

On these torsional vibration problems of the structures several theoretical analyses have been reported already so far as the linear vibration systems are concerned. However, these results did not seem to point out the occurence of the phenomena of unstable torsional vibrations, as will be discussed later. From this point of view, this paper treats the analysis on a simplified structure of specific type by Meissner's graphical construction.

Equations of the Unstationary Torsional Vibrations of Multi-story Framed Structures: Let us consider a rectangular N-story framed structure of L by L' spans, planted in a rigid foundation. Since dead loads of the structure seem to concentrate usually on each floor level, we may idealize the dead load on each floor level as a mass. Even in the nonlinear behaviors of the structure, shearing deformation of the floors in their plane are so small that these deformation will be neglected.

Now we adopt X, Y rectangular coordinates and nomenclature following the body of this paper.

Denoting restoring force functions, which do not contain time t explicitly and relate to the relative displacement between the stories, by $F_s^{(r)}$, $G_s^{(r')}$, we can obtain the general equations of the nonlinear torsional vibration as follows:

$$\left.\begin{aligned}
M_s\ddot{y}_s &+ \sum_{r=1}^{m}\Big[F_s^{(r)}\{(y_s+\xi_s^{(r)}\theta_s)-(y_{s-1}+\xi_{s-1}^{(r)}\theta_{s-1})\} \\
&\qquad - F_{s+1}^{(r)}\{(y_{s+1}+\xi_{s+1}^{(r)}\theta_{s+1})-(y_s+\xi_s^{(r)}\theta_s)\}\Big] = -M_s\ddot{y}_0 \\
M_s\ddot{x}_s &+ \sum_{r'=1}^{m'}\Big[G_s^{(r')}\{(x_s+\eta_s^{(r')}\theta_s)-(x_{s-1}+\eta_{s-1}^{(r')}\theta_{s-1})\} \\
&\qquad - G_{s+1}^{(r')}\{(x_{s+1}+\eta_{s+1}^{(r')}\theta_{s+1})-(x_s+\eta_s^{(r')}\theta_s)\}\Big] = -M_s\ddot{x}_0 \\
I_s\ddot{\theta}_s &+ \sum_{r=1}^{m}\Big[\xi_s^{(r)}F_s^{(r)}\{(y_s+\xi_s^{(r)}\theta_s)-(y_{s-1}+\xi_{s-1}^{(r)}\theta_{s-1})\} \\
&\qquad - \xi_{s+1}^{(r)}F_{s+1}^{(r)}\{(y_{s+1}+\xi_{s+1}^{(r)}\theta_{s+1})-(y_s+\xi_s^{(r)}\theta_s)\}\Big] \\
&+ \sum_{r'=1}^{m'}\Big[\eta_s^{(r')}G_s^{(r')}\{(x_s+\eta_s^{(r')}\theta_s)-(x_{s-1}+\eta_{s-1}^{(r')}\theta_{s-1})\} \\
&\qquad - \eta_{s+1}^{(r')}G_{s+1}^{(r')}\{(x_{s+1}+\eta_{s+1}^{(r')}\theta_{s+1})-(x_s+\eta_s^{(r')}\theta_s)\}\Big] = 0
\end{aligned}\right\}\quad (3)$$

In the linear problems, the terms

$$F_s^{(r)}\{(y_s+\xi_s^{(r)}\theta_s)-(y_{s-1}+\xi_{s-1}^{(r)}\theta_{s-1})\} - F_{s+1}^{(r)}\{(y_{s+1}+\xi_{s+1}^{(r)}\theta_{s+1})-(y_s+\xi_s^{(r)}\theta_s)\}$$

and $G_s^{(r')}\{(x_s+\eta_s^{(r')}\theta_s)-(x_{s-1}+\eta_{s-1}^{(r')}\theta_{s-1})\} - G_{s+1}^{(r')}\{(x_{s+1}+\eta_{s+1}^{(r')}\theta_{s+1})-(x_s+\eta_s^{(r')}\theta_s)\}$

in Eq. (3) are products of force and displacement, but these terms in the nonlinear analysis appear as nonlinear functions of displacements.

Eq. (3) then appears to be simultaneous nonlinear equations, so it is always very troublesome to get solutions of the equations in transient states even when the distribution of mass and rigidity is symmetric in either the X- or Y-direction of the structure.

Although the exact solutions of Eq. (3) can not be given analytically, such a step-by-step procedure as Meissner's graphical construction or the phase-plane-delta method introduced by Prof. L. S. Jacobsen, or numerical computations by finite difference method are available to obtain approximate solutions corresponding to several initial conditions. Thus we are able to analyze this problem by seeking many a solution for each initial condition presumed and to study the nonlinear torsional vibration phenomena by comparing these solutions to each other.

Solutions of Transient Vibrations of One-story, One-spaned Structures: Let us consider first a rectangular structure of one-story and one-span on a rigid foundation as shown in Fig. 18. Then the equations of motion of the structure are written by giving N = 1; r = 1, 2; r' = 1, 2, as follows,

$$M\ddot{y} + F^{(1)}(y+\xi^{(1)}\theta) + F^{(2)}(y+\xi^{(2)}\theta) = -M\ddot{y}_0 \tag{4a}$$

$$M\ddot{x} + G^{(1)}(x+\eta^{(1)}\theta) + G^{(2)}(x+\eta^{(2)}\theta) = -M\ddot{x}_0 \tag{4b}$$

$$I\ddot{\theta} + \xi^{(1)}F^{(1)}(y+\xi^{(1)}\theta) + \xi^{(2)}F^{(2)}(y+\xi^{(2)}\theta) + \eta^{(1)}G^{(1)}(x+\eta^{(1)}\theta) + \eta^{(2)}G^{(2)}(x+\eta^{(2)}\theta) = 0 \tag{4c}$$

From Eqs. (4a), (4b) and (4c), we have

$$M\ddot{z}_1 + (1+e\xi_1^2)F_1(z_1) + (1+e\xi_1\xi_2)F_2(z_2) + e\xi_1\eta_1 G_1\left(x+\eta_1\frac{z_1-z_2}{\xi_1-\xi_2}\right) + e\xi_1\eta_2 G_2\left(x+\eta_2\frac{z_1-z_2}{\xi_1-\xi_2}\right) = -M\ddot{y}_0 \tag{5a}$$

$$M\ddot{z}_2 + (1+e\xi_1\xi_2)F_1(z_1) + (1+e\xi_2^2)F_2(z_2) + e\xi_2\eta_1 G_1\left(x+\eta_1\frac{z_1-z_2}{\xi_1-\xi_2}\right) + e\xi_2\eta_2 G_2\left(x+\eta_2\frac{z_1-z_2}{\xi_1-\xi_2}\right) = -M\ddot{x}_0 \tag{5b}$$

$$M\ddot{x} + G_1\left(x+\eta_1\frac{z_1-z_2}{\xi_1-\xi_2}\right) + G_2\left(x+\eta_2\frac{z_1-z_2}{\xi_1-\xi_2}\right) = -M\ddot{x}_0 \tag{5c}$$

where

$$y + \xi^{(1)}\theta = z_1, \quad y + \xi^{(2)}\theta = z_2 \quad \text{and} \quad e = \frac{M}{I}.$$

The natural periods of small-amplitude vibration of the structure are obtained from the dimensions shown in Fig. 18. They are

$T_x = 2\pi/\sqrt{5.00k/M}$ .. in the X-direction,

${}_1T_{y\theta} = 2\pi/\sqrt{4.55k/M}$ .. the fundamental mode of simultaneous vibration of Y, and

${}_2T_{y\theta} = 2\pi/\sqrt{25.47k/M}$ .. the first higher mode of simultaneous vibration of Y.

Here we let T_r and T_s be periods of the structure, both when it consists of either rigid of soft frame alone,

$T_r = 2\pi/\sqrt{20.00k/M}$ .. of rigid frames,

$T_s = 2\pi/\sqrt{2.50k/M}$ .. of soft frames.

Assuming two kinds of elasto-plastic behaviors of the structure as given in Figs. (19A) and (19B), we obtain the solutions of Eq. (5) by Meissner's graphical construction, and show them in Figs. 20 and 21. The ground motions are considered now to act in the Y-direction only, i.e.,

$\ddot{x}_o = 0$, and to be of the same mean velocity. Periods of the ground motions are adopted by T_r, T_s and ${}_1T_{y\theta}$ as mentioned above (which correspond to indices in Fig. 20 and 21).

Between the structural behaviors subjected to such ground motions for one period duration, any striking difference is not recognized. But we are able to say that the structural behavior subjected to a ground motion of ${}_2T_{y\theta}$ period, though it is not shown here, appears so little that we can not exhibit its response curve together with others in the same scale. The authors have observed, in latter case, that the structural behavior oscilating in ${}_2T_{y\theta}$ period under action of ground motion shifts itself into the vibration in ${}_1T_{y\theta}$ period after the action of ground motion disappears.

Stable and Unstable Problems of Structural Behaviors due to Torsional Displacements: Since the restoring force characteristics of structures usually seem to be of soft spring type, we write them as

$$G(\zeta) = k\zeta - \beta\zeta^3 + \gamma\zeta^5 + \cdots \quad (k > 0,\ \beta > 0). \tag{6}$$

We now assume that the displacement x is so small in the vicinity of the origin that powers of x above the third may be ignored in comparison with lower powers and thus replace Eq. (5c) by the following equation

$$M\ddot{x} + (2k - 6\beta\eta^2\theta^2)x - 2\beta x^3 = -M\ddot{x}_o \quad . \tag{7}$$

We can observe easily that one solution of our problem is given by $x = 0$. The angular displacement θ arises due to the ground displacement in the Y-direction alone even though $\ddot{x}_o = 0$ and $x = 0$, so that the question is whether this solution is stable or not when the angular displacement increases.

It is perhaps of interest to interpolate at this point the treatment of the stability problem on a purely static basis, as in the theory of elasticity. By dropping the acceleration term we obtain from Eq. (7) the following equilibrium condition

$$a_1 x + a_3 x^3 = 0 \tag{8}$$

with a_1 and a_3 given by

$$\left.\begin{aligned} a_1 &= 2k - 6\beta\eta^2\theta^2 \\ a_3 &= -2\beta \ . \end{aligned}\right\} \tag{9}$$

In the theory of elasticity, displacements x are usually restricted so small that nonlinear terms in x can be ignored: in this case the equilibrium condition takes the form $a_1 x = 0$, with a_1 defined by Eq. (9). Consequently $x = 0$ unless $a_1 = 0$, and the latter equation in turn furnishes from Eq. (9) the following "critical" value for θ:

$$\theta_{crit.} = \pm\sqrt{\frac{k}{3\beta\eta^2}} \tag{10}$$

at which the structure would begin to collapse, since the equilibrium becomes indifferent for this value of θ in the sense that x is now arbitrary.

We turn now to the dynamical treatment of the problem which yields the same value for $\theta_{crit.}$. We replace $\ddot{x}$ in Eq. (7) by $v(dv/dx)$ and obtain the first order equation

$$\frac{dv}{dx} = -\frac{1}{M}\frac{a_1 x + a_3 x^3}{v} \tag{11}$$

with a_1 and a_3 as defined in Eqs. (8) and (9). Consideration about the character of Eq. (11) at the origin is analogous to examining the stability of

$$\frac{dv}{dx} = -\frac{1}{M}\frac{a_1 x}{v} \tag{12}$$

on the phase plane by Poincaré-Liapounoff's theory. Therefore, if a_1 is positive, the equilibrium point (0,0) of the frame is a "center", while it is a "saddle point" if a_1 is negative.

If $a_1 > 0$, a slight disturbance from the equilibrium position results in a small oscillation, but if $a_1 < 0$, the motion departs widely from the equilibrium position even upon the slightest disturbance. From Eqs. (9) and (11) we can conclude, therefore, as follows:

i) if $\theta < \theta_{crit.}$, vibration of the center of mass in the X-direction is stable, but

ii) if $\theta > \theta_{crit.}$, the vibration is unstable.

In this case, the coordinates of singular points are expressed by

$$\begin{aligned} x &= \pm\sqrt{(-a_1)/a_3} = \pm\sqrt{(k-3\beta\eta^2\theta^2)/\beta} \\ &= \pm\sqrt{3\eta^2(\theta^2_{crit.}-\theta^2)} \end{aligned} \tag{13}$$

from Eqs. (9) and (10).

When $a_1 > 0$, i.e. $\theta < \theta_{crit.}$, there are two positions of equilibrium besides $x = 0$. The fact that these positions correspond to the saddle point is clarified by the following considerations: Now, let us consider one of the singular points in question $(+\sqrt{(-a_1)/a_3}, 0)$. By a transfer of the coordinates by $x - \sqrt{(-a_1)/a_3} = \sigma$, Eq. (11) becomes

$$\frac{dv}{d\sigma} = \frac{1}{M}\cdot\frac{2a_1\sigma - 3a_3\sqrt{(-a_1)/a_3}\,\sigma^2 - a_3\sigma^3}{v} \tag{14}$$

Application of Poincaré-Liapounoff's theory to Eq. (14), as well as Eq. (11), equalizes the singular points of Eq. (14) with the ones of

$$\frac{dv}{d\sigma} = \frac{1}{M} \cdot \frac{2a_1\sigma}{v} \tag{15}$$

in character. Since $2a_1 > 0$, the two equilibrium points, $x = \pm\sqrt{(-a_1)/a_3}$, are saddle points. As the rotating angle of the structure θ gradually increases from zero, the absolute value of x decreases. When θ reaches $\theta_{crit.}$, x tends to zero. In this case of $\theta > \theta_{crit.}$, these three singular points concentrate into one point $x = 0$, and consequently, it is known that the origin $x = 0$ is now an unstable singular point. The trajectories of these two cases are indicated in Figs. 22 and 23. Fig. 23 shows the movement of the positions of the two unstable singular points due to the increase in θ when $\theta < \theta_{crit.}$.

Since the frames can not have any stable state so that it behaves very unstablly in regard to the amplitudes of torsional vibrations if θ becomes greater than $\theta_{crit.}$, it is desirable to make $\theta_{crit.}$ larger. If characteristics of restoring force of a frame in the x-direction is of hard spring type, we have to reconsider the case where $\beta < 0$ in Eq. (6). In this case, $\theta_{crit.}$ does not exist by Eq. (10), and there are no unstable phenomena observed.

However, unless twisted wires are prestressed, the effect of wires is not recognized very much in the case of small amplitudes. Therefore, Eq. (5) can be changed into

$$G(\zeta) = k\zeta - (\beta - \alpha\theta^2)\zeta^3 \tag{16}$$

in which $\quad k > 0, \quad \alpha > 0, \quad \beta > 0$.

The reason why we have the coefficient of ζ^3 containing θ^2 is the fact that the unstable phenomena are directly related to the increase of θ. Also, θ^2 is convenient to carry out the following analysis. We obtain

$$\left.\begin{aligned} a_1 &= 2\{k - 3(\beta - \alpha\theta^2)\eta^2\theta^2\} \\ a_1 &= -2(\beta - \alpha\theta^2) \\ \theta_{crit.} &= \pm\{\beta(1 - \sqrt{1 - 4\alpha k/3\beta^2\eta^2})/2\alpha\}^{1/2} \\ &\qquad (\ 0 < 4\alpha k/3\beta^2\eta^2 \leqq 1\ ) \end{aligned}\right\} \tag{17}$$

corresponding to Eqs. (9) and (10). In this case, though the origin is stable for $\theta < \theta_{crit.}$ but unstable for $\theta > \theta_{crit.}$, as well as mentioned before, there are no other singular points except the origin, so far as $a_3 < 0$ in $\theta > \theta_{crit.}$. Since the increase in θ leads to $a_3 > 0$, two new equilibrium points given by Eq. (13) arise and these positions are provided for

$$x = \pm\sqrt{(-a_1)/a_3} = \pm\sqrt{-\{k - 3(\beta - \alpha\theta^2)\eta^2\theta^2\}/(\alpha\theta^2 - \beta)}\ . \tag{18}$$

Having the coordinates transferred into these positions we know that Eq.

(15) is available. However, since $2a_1 < 0$ in this case, these new singular points $\sigma = 0$, $v = 0$ are centers. Thus the frame, which was unstable for $\theta_{crit.} > \theta$, moves to two other stable states. It is noticed hereby that the effect of strengthening by twisted wires is remarkable, even if it is slight.

Conclusions: Since most structures are asymmetric in the mass and rigidity distributions, and since ground motions are rarely acted only in one direction, torsional vibration of the structures in the event of earthquakes will be unavoidable. Therefore, it is indispensable to investigate unstable phenomena which may lead the structures to collapse.

For the practical design of buildings, it will be the best idea to endow structures restoring force characteristics of hard spring type, and at the same time, we shall need to make the rigidity of structures distributed as uniformly as possible in regard to the ultimate state rather than the elastic range of stiffness members.

In this sense, it will be more advantageous that the torsional behaviors of the structure should be taken into account by assuming a seismic wave resonating with the natural periods of rigid frames T_r as a measure of destructive earthquakes. This fact is shown in Figs. 20 and 21. Resonance with the natural periods of rigid frames concentrates a seismic force on the rigid frames, and if yielding of the rigid frames precedes hereby, it lets their rigidity approach those of soft frames, and at last this makes amplitudes of torsional vibrations small, being influenced also by dissipation of energies due to yielding. This may be the same thing as the effect of "plastic equilibrium of stress" or "self-help".

Further, we have to pay much attention to the fact that ununiformity of loading masses may bring a worse effect in the plastic region of stiffness members in the structures.

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NOMENCLATURE

Part I

M : Mass

X : Relative displacement of the mass with respect to the ground

G : Nonlinear restoring force function

X_G : Distinct ground displacement

p : Circular frequency of vibrating system at its initial state

k : Spring constant at initial state of vibrating system

i : As a subscript, refers the symbol to its appropriate floor of building

φ : Slope of plastic range of restoring force characteristics of vibrating system

α' : Maximum value of ground acceleration

α : Parameter proportional to the value of α'

T : Duration of ground motion

T_o : Elastic natural period of vibration of system

X_R : Maximum total amplitude of system

X_D : Maximum displacement of system

X_F : Residual displacement

t : Period of vibration corresponding to maximum total amplitude

Part II

N : Number of the stories

L, L' : Number of the spans in the X, Y-directions

X, Y : Axes of rectangular coordinates

t : Time

n, s, r, r' : Positive integers

x_s, y_s, θ_s : Relative displacements in the X, Y-directions and rotating angle around the center of mass of the s-th floor, respectively

M_s, I_s : Mass concentrated at the s-th floor level and the moment of inertia about its center of mass

$F_s^{(r)}$, $G_s^{(r')}$: Restoring force functions both in the Y, X-directions of the s-th floor level of the r, r'-frame

$\zeta_s^{(r)}$, $\eta_s^{(r')}$: Distances from the center of the s-th floor to the r-th and r'-th frame

x_0, y_0 : Ground displacement

T_x, ${}_1T_{y\theta}$, ${}_2T_{y\theta}$: Natural periods of vibrations in small amplitude of the structure in the X-direction, of the fundamental mode of simultaneous vibration of the Y-direction, and of the first higher mode of simultaneous vibration of the Y-direction, respectively

T_r, T_s : Natural periods of rigid and soft frames

k : Linear spring constant

$G(\zeta)$: Nonlinear restoring force function

β, γ : Coefficients of the ζ^3, ζ^5 terms

M : Mass

$\theta_{crit.}$: Critical value for θ

v : dx/dt

α : Positive constant

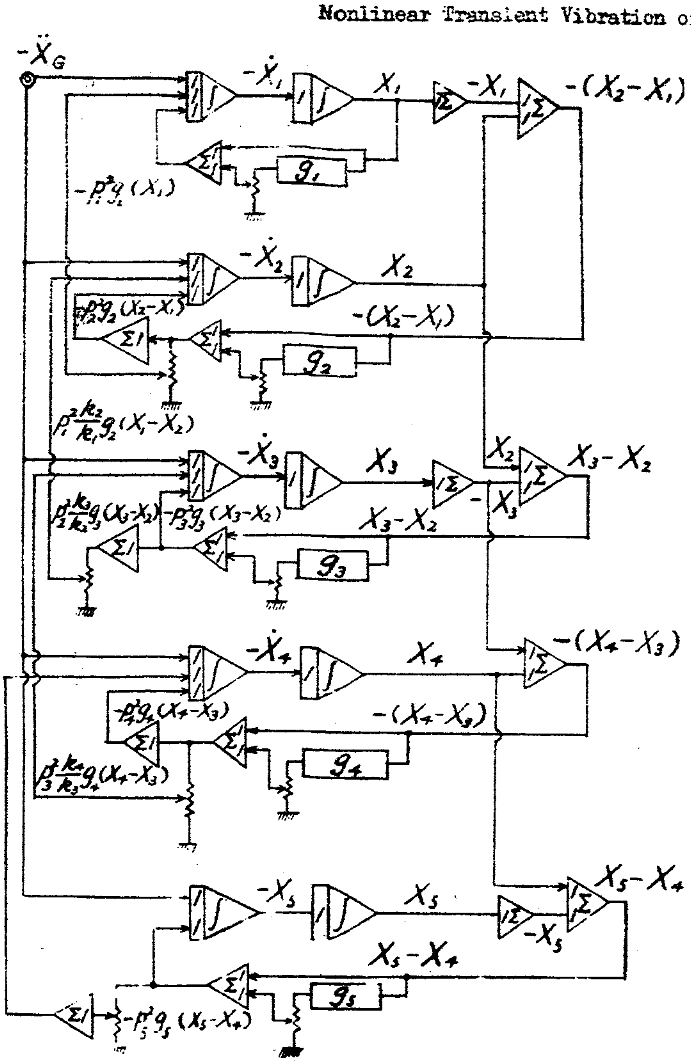


Fig. 1

Block diagram of the analog computer in the case of idealized multiple-mass structures subjected to an earthquake shock

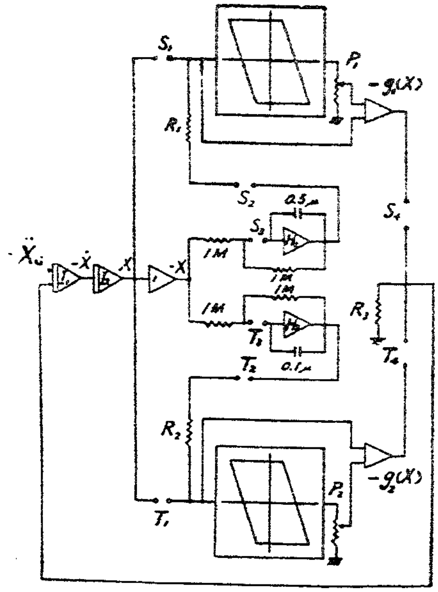


Fig. 2

Block diagram of the analog computer in the case of single mass structures with the restoring force curve assumed to Fig. 4

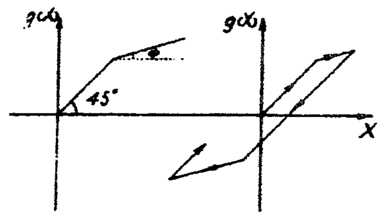


Fig. 3

Nonlinear, hysteretic restoring force curves

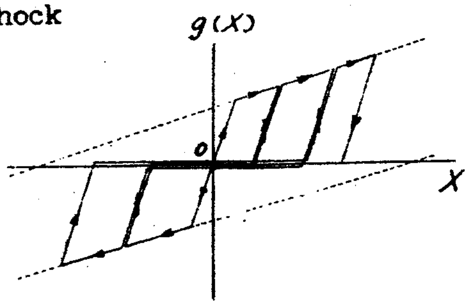


Fig. 4

Restoring force curve with dead zones in the case of wooden or steel structures

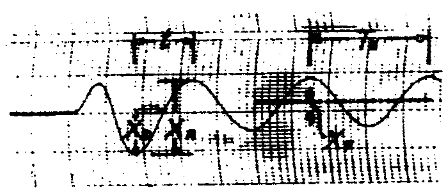


Fig. 5

Example of the displacement response curve obtained by means of the analog computer together with the quantities

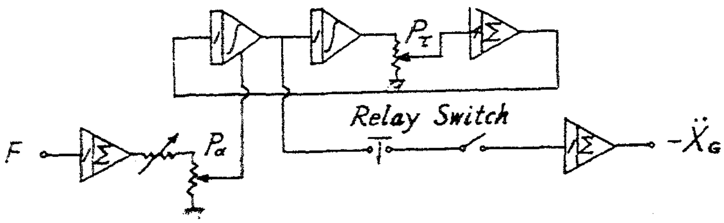


Fig. 6

Block diagram of the function generator
of full-cycle cosine

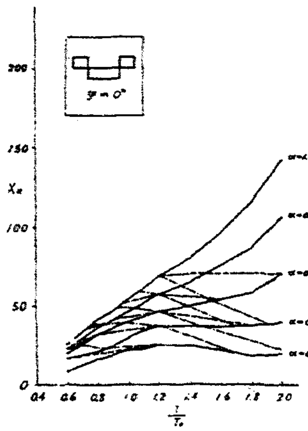


Fig. 7

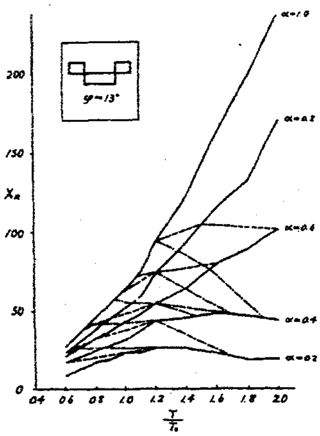


Fig. 8

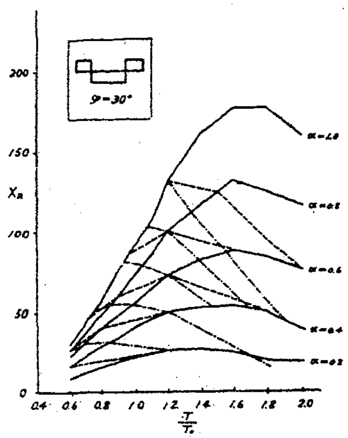


Fig. 9

Spectra where the transient maximum amplitude of the nonlinear system X_R are plotted for the ratio T/T_o with parameters φ and α

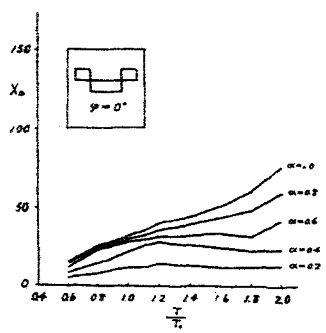


Fig. 10

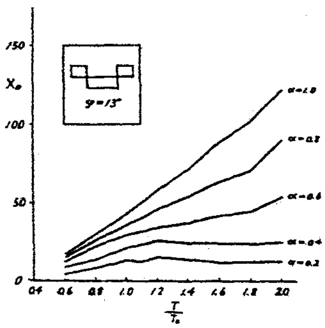


Fig. 11

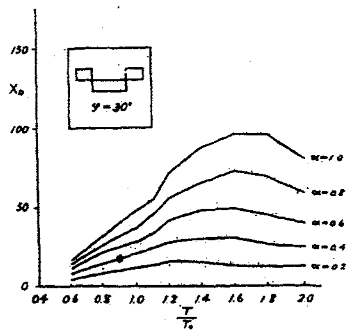


Fig. 12

Spectra where the maximum displacement measured from the initial equilibrium position X_D are plotted for the ratio T/T_o with parameters φ and α

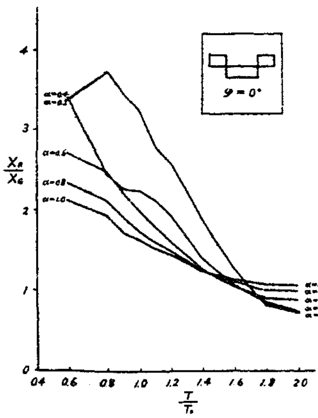


Fig. 13

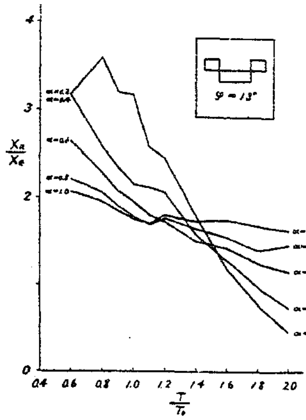


Fig. 14

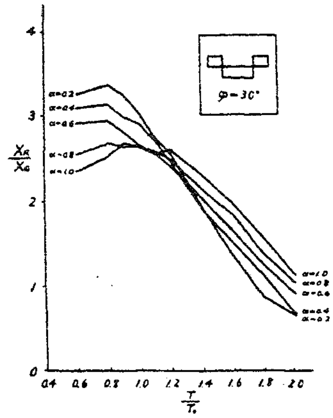


Fig. 15

Spectra of the ratio of X_R to the corresponding amplitude of ground motion X_G for the ratio T/T_o

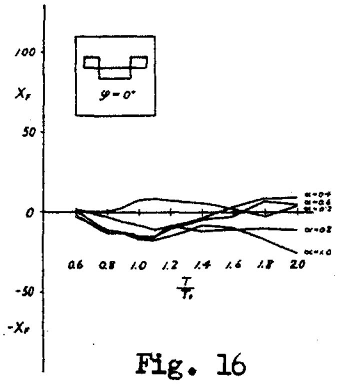


Fig. 16

Spectra of the residual displacement of the system from the initial position of equilibrium X_F for the ratio T/T_o

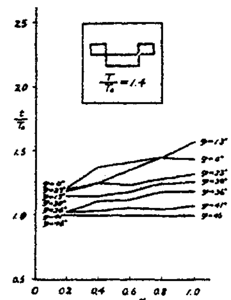


Fig. 17

Diagram where the ratio of the period of vibration of the system at the maximum amplitude, t, to the elastic natural period, T_o, are plotted against α

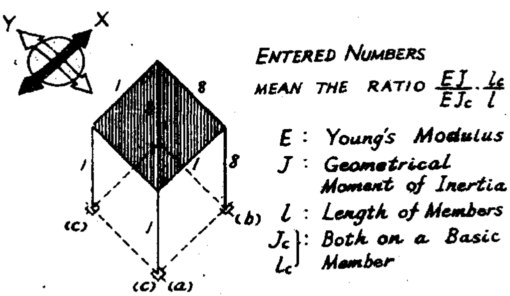


Fig. 18

Rectangular structure of one-story and one-span on a rigid foundation, for which the solutions are obtained by Meissner's graphical construction

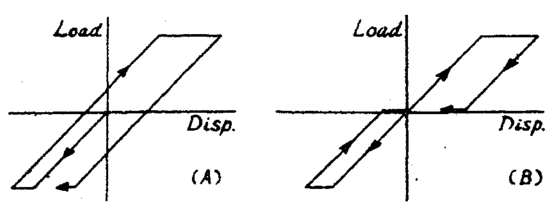
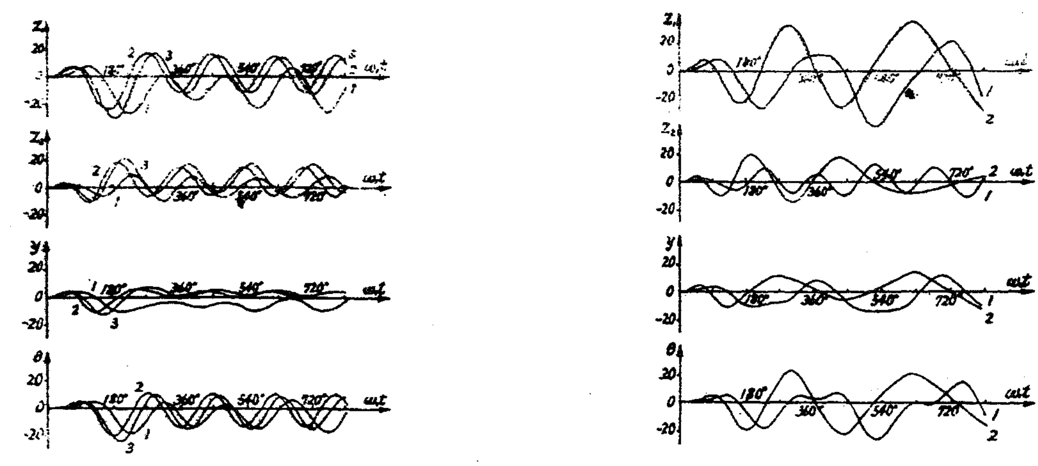


Fig. 19

Two kinds of elastic-plastic restoring force curves



Solutions of Eq. (4) obtained by Meissner's graphical construction for the restoring force curves shown in Figs. 19 (A) and (B)

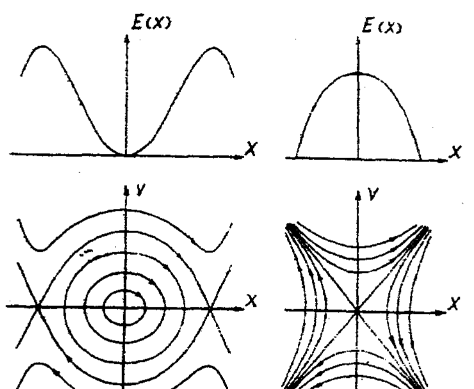


Fig. 22

Phase trauectories of the two cases, $\theta < \theta_{crit.}$ and $\theta > \theta_{crit.}$

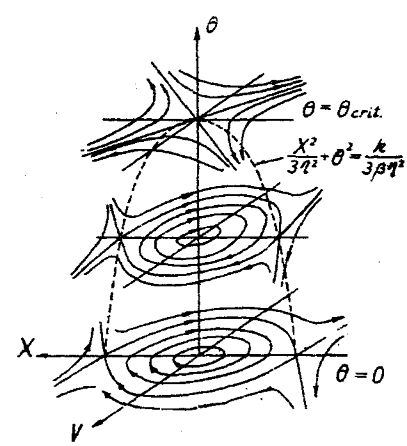


Fig. 23

Phase trajectories showing movement of the positions of the two unstable singular points due to the increase in θ when $\theta < \theta_{crit.}$