A SOLUTION OF THE BASIC DIFFERENTIAL EQUATION OF SEISMIC LOADS AND HIS RELATION WITH THE EXPANSION ON SERIES OF EIGENVALUES

by

Simón Gershanik* and Georges Dedebant**

1. - Some years ago in a well known paper(1) Alford Housner and Martel have pointed out that in general, the displacements u of a building may be given by

$$u = \sum \frac{W_i}{P_i} \Phi_i \int_0^t a(\tau) e^{-n_i P_i (t-\tau)} \sin p_i (t-\tau) d\tau \qquad (1)$$

where

Φ_i is a factor characteristic of the i mode of vibration of the structure, depending on space coordinates.

P_i is 2π times the frequency of the i mode

$\frac{W_i}{P_i}$ a function of the structure's physical properties.

n_i a fraction of critical damping of the i mode.

$a(t)$ the acceleration of the ground and

t means the time.

Later one of us(2) studying the basic differential equation of seismic loads

$$\frac{\partial^4 u}{\partial z^4} + c\frac{\partial^2 u}{\partial t^2} + 2\varepsilon_2 \frac{\partial u}{\partial t} = c\,a(t) = F_2(t) \qquad (2)$$

$$\frac{\partial^2 u}{\partial z^2} - k\frac{\partial^2 u}{\partial t^2} - 2\varepsilon_1 \frac{\partial u}{\partial t} = k\,a(t) = F_1(t) \qquad (3)$$

the first corresponding to buildings mainly sensible to bending and the second to buildings mainly sensible to shear, was able to show that the series of eigenvalues expansion of the solution of this equations could be expressed by the following formulae, similar to (1)

$$u = \sum_{j=1}^{\infty} \nu_j(z) \beta_j \frac{T_j'}{2\pi} \int_0^t a(\tau) e^{-\varepsilon(t-\tau)} \sin \frac{2\pi}{T'} (t-\tau) d\tau$$

In the preceding expression c, k, and ε_2 are constants, z is the height of a point of the building above the ground, $\nu_j(z)$ is the eigenfunction of the order j of the kernel K $(z\,\xi)$ of the integral equation

* Professor of Seismology at the Observatorio Astronómico. La Plata.
** Professor of the Observatorio Astronómico, La Plata.

$$\nu_j(z) = \lambda \int_0^l K(z\xi)\,\nu(\xi)\,d\xi$$

These eigenfunctions have to satisfy the same boundary conditions as (2) and (3) and the differential equation

$$\frac{d^n \nu}{dz^n} = -\lambda \nu \qquad (n=2 \text{ or } n=4)$$

In the case of equation (2) is

$$\nu(z) = \cos h\, mz - \cos mz + \eta(\sin hmz - \text{sen}\, mz)$$

and in the case of equation (1) is

$$\nu(z) = \text{sen}\, z\sqrt{\lambda}$$

$$\sqrt{\lambda}\, l = \frac{\pi}{2}(2J-1) \qquad (J = 1, 2, \cdots\cdots \infty)$$

We have further:

$$\beta_J = \frac{\int_0^l \nu(z)\,dz}{\int_0^l \nu^2(z)\,dz}$$

l = height of building

$$T' = \frac{2\pi}{\sqrt{\gamma^2 - \varepsilon^2}}$$

$\varepsilon = \frac{\varepsilon_2}{c}$ and $\gamma^2 = \lambda/c$ in equations (2)

$\varepsilon = \frac{\varepsilon_1}{k}$ and $\gamma^2 = \lambda/k$ in equations (3)

In this paper we wish to come back to equation (3) as it seems that buildings behave more sensible to shear than to bending, and to show:

a) which is the rigurous solution of this equation and
b) how we may arrive to expression (4) by starting from such solution.

2. - Let us make the substitution

$$u = u_1 + \psi(t) \tag{5}$$

where $\psi(t)$ is a pure function of t and u_1 is the solution of the homogeneous differential equation

$$\frac{\partial^2 u_1}{\partial z^2} - k\frac{\partial^2 u_1}{\partial t^2} - 2\varepsilon_1 \frac{\partial u_1}{\partial t} = 0 \tag{6}$$

u must satisfy the following boundary conditions:

For $z = 0$ $u = 0$ (7)

for $z = 1$ $\frac{\partial u}{\partial z} = 0$ (8)

Therefore it must be:

$$u_1 = -\psi(t) \qquad \text{for} \qquad z = 0 \tag{9}$$

$$\frac{\partial u_1}{\partial z} = 0 \qquad \text{for} \qquad z = 1 \tag{10}$$

Trying to satisfy (6) with the expression

$$u_1 = g(t)\, h(z)$$

where g is a pure function of t and h is a pure function of z, we obtain as a particular solution:

$$u_1 = \left[A(\omega)e^{-i\delta z} + B(\omega)e^{i\delta z}\right]e^{-\varepsilon t + i\omega t}$$

$$\delta = k\sqrt{\omega^2 + \varepsilon^2} \tag{11}$$

δ may have any value.

From (11) it follows that a solution of (6) is also the following expression:

$$u_1 = e^{-\varepsilon t}\int_{-\infty}^{+\infty} e^{i\omega t}\left[A(\omega)e^{-i\delta z} + B(\omega)e^{i\delta z}\right]d\omega$$

From (10) we obtain

$$B = A\, e^{-2i\delta l}$$

hence

$$u_1 = e^{-\varepsilon t}\int_{-\infty}^{+\infty} A(\omega)e^{i\omega t}\left[e^{-i\delta z} + e^{i\delta(z-2l)}\right]d\omega \tag{12}$$

When $z = 0$ it must be: $u_1 = -\psi(t)$. We have therefore:

$$e^{-\varepsilon t}\int_{-\infty}^{+\infty} A(\omega)e^{i\omega t}\left[1 + e^{-2i\delta l}\right]d\omega = -\psi(t) \tag{13}$$

As $\psi(t)$ may be represented by a Fourier integral, we can write

$$e^{\varepsilon t}\psi(t) = \int_{-\infty}^{+\infty}\Gamma(\omega)\,e^{i\omega t}\,d\omega \tag{14}$$

From this and (13) we easily find that

$$A(\omega) = -\frac{\Gamma(\omega)}{1 + e^{-2i\delta l}}.$$

Taking this on account as well as the relation (5) we may provisionaly write:

$$u = \psi(t) - e^{-\varepsilon t}\int_{-\infty}^{+\infty}\frac{\Gamma(\omega)}{1 + e^{-2i\delta l}}\left[e^{-i\delta z} + e^{i\delta(z-2l)}\right]d\omega \tag{15}$$

In order to have a complete expression of the solution of (3) we have to add to (15) all the solutions of the homogeneous equation (6) allowed by the boundary conditions (7) and (8).

Let us call w one of such solutions and put

$$w = G(t)\,H(z)$$

By substituting this in (6) we get

$$H(z) = c_1 e^{i\delta z} + d_1 e^{-i\delta z} \;;\quad G(t) = a_1 e^{i\omega t} + b_1 e^{-i\omega t}$$

and from the boundary conditions

$$c_1 + d_1 = 0 \;;\quad c_1 e^{i\delta l} + d_1 e^{-i\delta l} = 0 \tag{16}$$

Whence

$$e^{i\delta l} + e^{-i\delta l} = \cos\delta l = 0 \tag{17}$$

It is not allowed now for δ to be arbitrary. As it has to satisfy (17) it only may be

$$\delta l = \frac{\pi}{2}(2j - 1) \qquad (j = 1, 2, 3, \ldots, \infty) \tag{18}$$

Calling μ_j this values of δ and α_j the corresponding values of ω, we may write, taking account of (16)

$$w_j = \sin\mu_j z\left[a_j e^{i\alpha_j t} + b_j e^{-i\alpha_j t}\right]e^{-\varepsilon t}$$

As equation (6) is satisfied by this expression it will also be by the expression

$$e^{-\varepsilon t}\int_{-\infty}^{+\infty}\Gamma(\omega)\,d\omega\sum_{j=1}^{\infty}\left(a_j e^{i\alpha_j t} + b_j e^{-i\alpha_j t}\right)\sin\mu_j z \tag{19}$$

where a_j and b_j may be considered as functions of ω

We may then add (19) to (15) and write as a general solution of (3)

$$u = \Psi(t) - e^{-\varepsilon t}\int_{-\infty}^{+\infty}\left\{\frac{e^{i\omega t}}{1+e^{-2i\delta l}}\left[e^{-i\delta z} + e^{i\delta(z-2l)}\right] + \sum_{j=1}^{j=\infty}\left(a_j e^{i\alpha_j t} + b_j e^{-i\alpha_j t}\right)\sin\mu_j z\right\}\Gamma(\omega)\,d\omega$$

On account of this, of (14) and of the relation:

$$\frac{e^{-i\delta z} + e^{i\delta(z-2l)}}{1 + e^{-2i\delta l}} = \frac{\cos\delta(l-z)}{\cos\delta l}$$

we may write

$$u = e^{-\varepsilon t}\int_{-\infty}^{+\infty}\Gamma(\omega)\left[\left(1 - \frac{\cos\delta(l-z)}{\cos\delta l}\right)e^{i\omega t} + \sum\left(a_j e^{i\alpha_j t} + b_j e^{-i\alpha_j t}\right)\sin\mu_j z\right]d\omega \tag{20}$$

a_j and b_j are arbitrary functions of ω. We can select them so that the initial conditions of u are satisfied.
For this conditions we may put:

$$u = 0 \qquad \frac{\partial u}{\partial t} = 0 \qquad \text{when } t = 0$$

and therefore by (20)

$$0 = \int_{-\infty}^{+\infty} \Gamma(\omega)\left[1 - \frac{\cos\delta(1-z)}{\cos\delta 1} + \sum (a_j + b_j)\sin\mu_j z\right] d\omega$$

$$0 = \int_{-\infty}^{+\infty} i\,\Gamma(\omega)\left[\omega\left(1 - \frac{\cos\delta(1-z)}{\cos\delta 1}\right) + \sum (a_j - b_j)\alpha_j \sin\mu_j z\right] d\omega$$

These equations are satisfied if

$$\sum_{j=1}^{\infty}(a_j + b_j)\sin\mu_j z = -\left[1 - \frac{\cos\delta(l-z)}{\cos\delta l}\right]$$

$$\sum_{j=1}^{\infty}\alpha_j(a_j - b_j)\sin\mu_j z = -\omega\left[1 - \frac{\cos\delta(l-z)}{\cos\delta l}\right]$$

Multiplying by sen $\mu_s z$, integrating between 0 and 1, and taking account that

$$\int_0^l \sin\mu_j z \sin\mu_s z\, dz = 0 \qquad \text{if} \qquad j \neq s$$

$$\int_0^l \sin^2\mu z\, dz = \frac{l}{2} \qquad\qquad \int_0^1 \sin\mu z\, dz = \frac{1}{\mu}$$

$$\frac{\int_0^l \sin\mu z\, dz}{\int_0^l \sin^2\mu z\, dz} = \beta \qquad \text{because} \qquad \mu = \sqrt{\lambda}$$

$$\int_0^l \cos\delta(l-z)\sin\mu z\, dz = \frac{1}{2}\cos\delta l\left(\frac{1}{\mu-\delta} + \frac{1}{\mu+\delta}\right) = \frac{\mu}{\mu^2-\delta^2}\cos\delta l$$

we obtain:

$$a_j + b_j = \frac{2}{l}\frac{\mu_j}{\mu_j^2 - \delta^2} - \beta_j \quad ; \quad a_j - b_j = \frac{2}{l}\frac{\omega}{\alpha_j}\frac{\mu_j}{\mu_j^2-\delta^2} - \frac{\omega}{\alpha_j}\beta_j$$

hence

$$a_j = \frac{1}{l}\left(\frac{\alpha_j + \omega}{\alpha_j}\right)\frac{\mu_j}{\mu^2 - \delta^2} - \frac{\beta}{2}\left(1 + \frac{\omega}{\alpha_j}\right)$$

$$b_j = \frac{1}{l}\left(\frac{\alpha_j - \omega}{\alpha_j}\right)\frac{\mu}{\mu^2 - \delta^2} - \frac{\beta}{2}\left(1 - \frac{\omega}{\alpha_j}\right)$$

We now remember that

$$\mu^2 = k(\alpha^2+\varepsilon^2) \quad ; \quad \delta^2 = k(\omega^2+\varepsilon^2)$$

whence

$$\mu^2-\delta^2 = k(\alpha^2-\omega^2)$$

$$a_j = \frac{\mu}{k\alpha_j l}\cdot\frac{1}{\alpha_j-\omega} - \frac{\beta_j}{2}\left(1+\frac{\omega}{\alpha_j}\right) \quad ; \quad b_j = \frac{\mu_j}{k\alpha_j l}\cdot\frac{1}{\alpha_j+\omega} - \frac{\beta_j}{2}\left(1-\frac{\omega}{\alpha_j}\right)$$

By substituting this in (20) we obtain as a rigurous solution of (3) adjusted to the boundary and initial conditions:

$$u = e^{-\varepsilon t}\int_{-\infty}^{+\infty}\Gamma(\omega)\left\{\left(1-\frac{\cos\delta(l-z)}{\cos\delta l}\right)e^{i\omega t}+\frac{1}{l}\sum_{j=1}^{\infty}\frac{\mu_j}{\alpha}\sin\mu_j z\left[\frac{e^{i\alpha t}}{\alpha-\omega}+\frac{e^{-i\alpha t}}{\alpha+\mu}\right.\right.$$

$$\left.\left.-\beta\left(\frac{e^{i\alpha t}+e^{-i\alpha t}}{2}-\frac{i\omega}{\alpha}\cdot\frac{e^{i\alpha t}-e^{-i\alpha t}}{2i}\right)\right]\right\}d\omega \qquad (21)$$

3.- We will demosntrate now that by developing (21) in series of eigenfunctions we obtain the expression (4).

First we may note for this that the function f(z)

$$f(z) = \cos\delta l - \cos\delta(l-z)$$

satisfies the differential equation

$$\frac{d^2f}{dz^2} = \delta^2\cos\delta(l-z) = -\Gamma'(z)$$

and the boundary conditions of our problem. We may write therefore:

$$f(z) = \int_0^l k(z,\zeta)\Gamma(\zeta)d\zeta \qquad (22)$$

where k has the same meaning as in §1. As a consequence, f(z) can be developed in an uniformly convergent series of eigenfunctions of K. These functions are:

$$\nu_j(z) = \sin\mu_j z$$

Whence

$$\cos\delta l - \cos\delta(l-z) = \sum_{j=1}^{\infty}c_j \operatorname{sen}\mu_j z$$

By the properties of integration detailed in §2 we obtain

$$c_j = \cos\delta l\left[\beta-\frac{1}{l}\left(\frac{1}{\mu-\delta}+\frac{1}{\mu+\delta}\right)\right] = \cos\delta l\left[\beta-\frac{\mu}{l\alpha}\left(\frac{1}{\alpha-\omega}+\frac{1}{\alpha+\omega}\right)\right]$$

Substituting this in the former equation and then dividing by $\cos\delta l$, this equation become:

$$1 - \frac{\cos\delta(l-z)}{\cos\delta l} = \sum_{j=1}^{\infty}\left(\beta_j - \frac{1}{l\alpha_j}\left(\frac{1}{\alpha_j-\omega} + \frac{1}{\alpha_j+\omega}\right)\right)\sin\mu_j z$$

and equation (21):

$$ue^{\varepsilon t} = \int_{-\infty}^{+\infty}\Gamma(\omega)\sum_{j=1}^{\infty}\Big\{e^{i\omega t}\Big[\beta_j - \frac{\mu}{\alpha_j l}\Big(\frac{1}{\alpha_j-\omega}+\frac{1}{\alpha_j+\omega}\Big)\Big] + \frac{\mu_j}{\alpha_j l}\Big(\frac{e^{i\alpha_j t}}{\alpha_j-\omega}+\frac{e^{-i\alpha_j t}}{\alpha_j+\omega}\Big)$$

$$-\beta_j\cos\alpha_j t - \frac{i\omega}{\alpha_j}\beta_j\sin\alpha_j t\Big\}\sin\mu_j z\, d\omega \qquad (23)$$

We note that the order of $\int$ and $\sum$ may be interchanged, and that due to (14)

$$\int_{-\infty}^{+\infty}\Gamma(\omega)\,d\omega = \psi(0)$$

$$\int_{-\infty}^{+\infty}\Gamma(\omega)\,\omega\, e^{i\omega t}d\omega = \frac{d}{dt}\left(e^{\varepsilon t}\psi(t)\right)$$

$$\int_{-\infty}^{+\infty}\Gamma(\omega)\,\omega\, d\omega = \varepsilon\psi(0) + \psi'(0)$$

Therefore we get from the terms containing β:

$$\int_{-\infty}^{+\infty}\Gamma(\omega)e^{i\omega t}d\omega\sum_{j=1}^{\infty}\beta_j\sin\mu_j z = \sum_{j=1}^{\infty}\beta_j\sin\mu_j z\int_{-\infty}^{+\infty}\Gamma(\omega)e^{i\omega t}d\omega = \sum_{j=1}^{\infty}\psi(t)e^{\varepsilon t}\beta_j\sin\mu_j z$$

$$\int_{-\infty}^{+\infty}\Gamma(\omega)d\omega\sum_{j=1}^{\infty}\beta_j\cos\alpha_j t\sin\mu_j z = \psi(0)\sum_{j=1}^{\infty}\beta_j\cos\alpha_j t\sin\mu_j z = \sum_{j=1}^{\infty}\psi(0)\beta_j\cos\alpha_j t\sin\mu_j z$$

$$\int_{-\infty}^{+\infty}\Gamma(\omega)d\omega\sum_{j=1}^{\infty}\frac{\beta_j}{\alpha_j}\sin\alpha_j t\sin\mu_j z = \left(\varepsilon\psi(0)+\psi'(0)\right)\sum_{j=1}^{\infty}\frac{\beta_j}{\alpha_j}\sin\alpha_j t\sin\mu_j z$$

With this, it results from (23):

$$ue^{\varepsilon t} = \sum_{j=1}^{\infty}\Big\{\beta_j e^{\varepsilon t}\psi(t) - \beta_j\psi(0)\cos\alpha_j t - \frac{\beta_j}{\alpha_j}\Big(\varepsilon\psi(0) + \psi'(0)\sin\alpha_j t\Big)\Big\}\sin\mu_j z$$

$$+\frac{1}{l}\int_{-\infty}^{+\infty}d\omega\,\Gamma(\omega)\sum_{j=1}^{\infty}\left(\frac{e^{i\alpha t}-e^{i\omega t}}{\alpha-\omega} + \frac{e^{-i\alpha t}-e^{i\omega t}}{\alpha+\omega}\right)\frac{\mu_j}{\alpha_j}\sin\mu_j z \qquad (24)$$

The integration may be carried out on the complex plane $\xi = \omega + iy$. The function to be integrated has poles only where $\Gamma(\omega)$ has. Let m_s be the values of ξ corresponding to the poles. Some of them which will be called m_r may be equal to some α_j which will be called α_r.

Taking this in account let us integrate along a path formed by a large semicircle whose radius $R \to \infty$, by the ω axis from $-\infty$ to $+\infty$ and by small circles enclosing the poles (fig. 1).

Calling J the integral, and $\chi(m)$ the residues multiplied by $2\pi i$ when $m_s \neq \alpha$ and by πi when $m_s = \alpha$, we may write:

$$J = \frac{1}{l}\sum_s \chi(m_s)\sum_{j=1}^{\infty}\left[\frac{e^{i\alpha_j t} - e^{im_s}}{\alpha - m_s} + \frac{e^{-i\alpha_j t} - e^{im_s t}}{\alpha + m_s}\right]\frac{\mu_j}{\alpha_j}\sin\mu_j z$$

$$+ \frac{1}{l}\sum_r \chi(m_r)\frac{m_r}{\alpha}\left[it\,e^{imt} + \frac{e^{-im_r t} - e^{im_r t}}{2m_r}\right]\frac{\mu_r}{\alpha_r}\sin\mu_r z$$

$$+ \frac{1}{l}\sum_s \chi(m_r)\sum_{j\neq r}^{\infty}\left[\frac{e^{i\alpha_j t} - e^{im_r t}}{\alpha_j - m_r} + \frac{e^{-i\alpha_j t} - e^{im_r t}}{\alpha_j + m_r}\right]\frac{\mu_j}{\alpha_j}\sin\mu_j z$$

Now we may note that:

$$\frac{e^{i\alpha t} - e^{imt}}{\alpha - m} + \frac{e^{-i\alpha t} - e^{imt}}{\alpha + m} = i\int_0^t\left(e^{i\alpha t - i(\alpha - m)\tau} - e^{-i\alpha t + i\tau(\alpha + m)}\right)d\tau$$
$$= -2\int_0^t e^{im\tau}\sin\alpha(t-\tau)\,d\tau$$

and this is valid for $m \neq \alpha$ as well as for $m = \alpha$. Hence we have, putting m_a for the different m:

$$J = -\frac{2}{l}\sum_a \chi(m_a)\sum_{j=1}^{\infty}\frac{\mu_j}{\alpha_j}\sin\mu_j z\int_0^t e^{im_a\tau}\sin\alpha(t-\tau)\,d\tau$$
$$= -\frac{2}{l}\sum\frac{\mu_j}{\alpha_j}\sin\mu_j z\int_0^t \sin\alpha(t-\tau)\,d\tau\sum\chi(m)e^{im\tau} \qquad (25)$$

By contour integration along the path of fig. 1 it can be shown that

$$\sum\chi(m_a)e^{im\tau} = \int_{-\infty}^{+\infty}\Gamma(\omega)e^{i\omega\tau}d\omega$$

If in addition we remember that expression (14), it becomes clear that instead of (25) we may write:

$$J = -\frac{2}{l}\sum_{j=1}^{\infty}\frac{\mu_j}{\alpha_j}\sin\mu_j z\int_0^t \psi(\tau)e^{\varepsilon\tau}\sin\alpha(t-\tau)\,d\tau \qquad (26)$$

By consecutive integration by parts we obtain:

$$\int_0^t M(\tau)\sin\alpha(t-\tau)\,d\tau = \left[\frac{M(\tau)\cos\alpha(t-\tau)}{\alpha} + \frac{1}{\alpha^2}M'(\tau)\sin\alpha(t-\tau)\right]_0^t$$

$$-\frac{1}{\alpha^2}\int_0^t M''(\tau)\sin\alpha(t-\tau)\,d\tau \qquad (27)$$

Taking account of this let us write

$$e^{\varepsilon t}\psi(t) = M(t) \qquad (28)$$

Hence

$$M''(\tau) = e^{\varepsilon\tau}(\varepsilon^2\psi + 2\varepsilon\psi' + \psi'')$$

By substituting (5) in (3) we can readily see that

$$\psi'' + 2\varepsilon\psi' = \frac{F_1(\tau)}{k}$$

As a consequence we have

$$M''(\tau) = e^{\varepsilon\tau}\frac{F_1(t)}{k} + \varepsilon^2 M(\tau)$$

From this and (27) we readily get

$$\frac{\alpha^2+\varepsilon^2}{\alpha^2}\int_0^t M(\tau)\sin\alpha(t-\tau)d\tau = -\frac{1}{\alpha^2}\int_0^t \frac{e^{\varepsilon\tau}F(\tau)}{k}\sin\alpha(t-\tau)d\tau + \frac{M(t)}{\alpha}$$

$$-\frac{M(o)\cos\alpha t}{\alpha} - \frac{M'(o)}{\alpha^2}\sin\alpha t$$

and as $\alpha^2+\beta^2=\mu^2$, if we multiply the above equation by $-\frac{2\alpha}{\mu l}$:

$$-\frac{2\mu}{\alpha l}\int_0^t M(\tau)\sin\alpha(t-\tau)d\tau = \frac{2}{\alpha\mu l k}\int_0^t e^{\varepsilon\tau}F(\tau)\sin\alpha(t-\tau)d\tau - \frac{2M(t)}{\mu l}$$

$$+\frac{2M(o)\cos\alpha t}{\mu l} + \frac{2}{\alpha\mu l}M'(o)\sin\alpha t \qquad (29)$$

By definition

$$\alpha = \sqrt{\frac{\delta^2}{k} - \varepsilon^2}$$

when $\delta = \frac{\pi}{2}(2j-1)$ or, what i.e. when $\delta^2 = \lambda$. Therefore we have

$$\frac{1}{\alpha} = \frac{T'}{2\pi}$$

By comparing the values of μl and β we moreover can see that

$$\beta = \frac{2}{\mu l}$$

With this and (28), expression (29) becomes

$$-\frac{2\mu}{\alpha t}\int_0^t \psi(\tau)e^{\varepsilon\tau}\sin\alpha(t-\tau)d\tau = \frac{\beta T'}{2\pi k}\int_0^t e^{\varepsilon\tau}F(\tau)\sin\alpha(t-\tau)\,d\tau - \beta e^{\varepsilon t}\psi(t)$$

$$+\beta\psi(0)\cos\alpha t + \frac{\beta}{\alpha}\left(\varepsilon\psi(0)+\psi'(0)\right)\sin\alpha t$$

Substituting this in (26) and the resulting expression of J in (24), it follows that:

$$u = \sum_{j=1}^{\infty}\frac{\beta T'}{2\pi k}\int_0^t d\tau\, e^{-\varepsilon(t-\tau)}F(\tau)\sin\alpha_j(t-\tau)\sin\mu_j z$$

as was expected.

Bibliography

(1) J.L.ALFORD, G.M.HOUSNER and R.R.MARTEL, Spectrum Analyses of strong motion Earthquakes, California Institute of Technology, 151.

(2) S.GERSHANIK, Mejoras en la apreciación de cargas sísmicas.

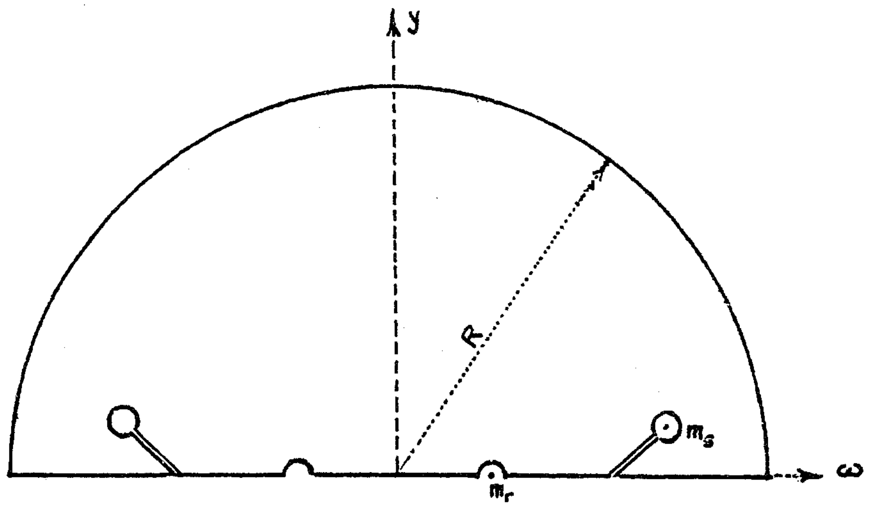

Fig. 1