The C. I. T. Mark II Response Spectrum Analyzer for Earthquake Engineering Studies

by

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Introduction. Electric Analog determinations of earthquake response spectrum curves at the California Institute of Technology have been described in a series of papers (1-4)***. This work has culminated in the development of the C. I. T. Mark II Response Spectrum Analyzer, which has been in service for the past year. Since this instrument is now in use both at the Earthquake Engineering Research Laboratory of the California Institute of Technology and the United States Coast and Geodetic Survey for routine calculations of earthquake spectra as well as for research studies of earthquake and blast phenomena, a brief description of the device may be of interest. Complete details of the circuit arrangements are available in a special report, so that the present paper will give mainly a summary of the basic principles (5).

Design Principles. In Fig. 1 are shown the basic elements of a typical structural system, along with its electrical analog, the equations of motion, and the definition of the Maximum Relative Velocity Response Spectrum. S_V can be measured directly in the electrical analog circuit by displaying the voltage across the resistor on the screen of a cathode-ray oscilloscope and by noting the maximum value of this voltage for various period and damping settings. It is only required that the shape of the voltage input E(t) to the analog circuit be the same as the shape of the ground acceleration-time record $\ddot{y}(t)$, in order that a direct experimental determination of the system response be possible.

The special features of the analog circuit design which make the device particularly useful for spectrum analysis applications are: (1) The circuits are arranged so that the natural period can be adjusted throughout its range by decade switches while the damping remains constant, (2) Various prescribed values of damping, including zero damping, can be obtained without changing other circuit characteristics. A simple test is available to check the actual damping present at any time. (3) Response measurements can be made in terms of relative displacement, relative velocity, or of relative or absolute acceleration by a simple switching operation. (4) The time scale of the electric circuit is speeded up compared with the structural prototype so that a large number of spectrum points can be calculated in a relatively short time.

The means by which these various features are realized are as follows: (1) The natural period of the analog circuit is $2\pi\sqrt{LC}$, and the damping resistance for critical damping is $2\sqrt{L/C}$. By changing L and C simultaneously in such

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*** Numbers in parentheses refer to Bibliography at the end of the paper.

a way that the quantity $\sqrt{L/C}$ remains constant, the period can be changed without altering the damping. This is accomplished by mechanically coupling the switches which change L and C, as indicated in Fig. 1. (2) Zero damping is attained by introducing an amplifier which can be employed as a negative resistance to cancel the small resistance associated with the inductance. This same amplifier makes it possible to improve the accuracy of the measurement of the voltage drop across the resistance, by the introduction of the measuring resistor R_0 of Fig. 1. To check for zero damping, a test switch position makes it possible to apply an "initial displacement" periodically to the circuit. The resulting oscillations can be visually observed on the screen of a cathode-ray-oscilloscope, and the amplifier gain can be adjusted until a constant amplitude of vibration results. Once the damping has been adjusted to zero, known fractions of critical damping can be switched in to determine the damped spectrum curves. (3) As indicated in Fig. 1 the output signal of the analog circuit will be proportional to the relative displacement, the relative velocity, or the relative or absolute acceleration, depending upon the point in the circuit at which the measurements are made. A front panel selector switch makes it possible to pick any of these response parameters for various applications. (4) If the time scale of the analog system is speeded up by a factor N, so that $N = T_p/T_a$, the basic equations of motion of Fig. 1 can be written in the form:

$$\ddot{z} + \frac{4\pi}{T_p}\left(\frac{c}{c_c}\right)\dot{z} + \frac{4\pi^2}{T_p^2} z = A_o f(t) \tag{1}$$

$$\ddot{Q} + \frac{4\pi}{T_p}\left(\frac{c}{c_c}\right)\dot{Q} + \frac{4\pi^2}{T_p^2} Q = \frac{E_o}{LN^2} f(t) \tag{2}$$

In these equations, both mechanical and electrical systems have been expressed in the same real time t, so that corresponding terms can be directly identified. For example, the expression for the relative velocity response can be obtained as:

$$\frac{\frac{4\pi}{T_p}\left(\frac{c}{c_c}\right)\dot{z}}{\frac{4\pi}{T_p}\left(\frac{c}{c_c}\right)\dot{Q}} = \frac{A_o f(t)}{\frac{E_o}{LN^2} f(t)}$$

$$\dot{z} = \dot{Q}\,\frac{A_o L N^2}{E_o} \tag{3}$$

The voltage E_R across the measuring resistor is $E_R = R_o N \dot{Q}$, and $T_p = 2\pi N\sqrt{LC}$. Thus equation (3) becomes:

$$\dot{z} = \frac{T_p}{2\pi}\left(\frac{E_R}{E_o}\right)\frac{\sqrt{L/C}}{R_o} A_o \tag{4}$$

For the particular analog elements used, $\sqrt{L/C} = 200\pi$ and $R_0 = 1000$ ohms. The relative velocity response thus finally becomes

$$\dot{z} = 0.1\, T_p \left(\frac{E_R}{E_0}\right) A_0 \tag{5}$$

where A_0 is the magnitude of the ground acceleration peak associated with input voltage E_0. Note that voltage ratios only are involved in the scope measurements, and hence no absolute calibration of the cathode ray oscilloscope is required.

Input Function Generation. An electric signal which varies with time in the same way as the earthquake ground acceleration is produced by a variable-width film trace which is moved between a light source and a photocell. A schematic diagram of the Function Generator together with the Spectrum Analyzer itself is given in Fig. 2. The variable width trace is photographically produced on a one-inch band on the periphery of a thin celluloid negative film disk of approximately 12 in. diameter. This film disk is rotated at a speed of 10 revolutions per second about a vertical axis. The earthquake is thus repeated periodically 10 times per second and the input electric signal and the circuit response signals can be presented as stationary patterns on the screen of the cathode ray oscilloscope. In this way, a visual measurement of the maximum response magnitudes can be made without the necessity of photographing the cathode ray oscilloscope output.

The film disk record for the function generator is interchangeable with those used in previous models of the electric analog computer, and has also the advantage that photographic copies can be made for the use of other laboratories with similar function generators. The film disk records are made on a special plotting table. The earthquake ground acceleration-time record drawn to a suitable scale is wrapped around a drum which can be slowly rotated as the curve is manually traced. The tracing stylus is coupled to a shutter system which exposes the film (3).

The same rotating disk which carries the film record also carries a small metallic plug which energizes a variable reluctance element once per revolution. An electric timing pulse is thus produced which is used to synchronize the various elements of the analyzer. Also controlled by this timing signal is a circuit which introduces a large amount of damping into the analog circuit at the end of each cycle of computation. This damping brings the system to rest ready for the next cycle of force application. The timing of the zero damping test signal is also controlled from this same pulse.

General Layout and Design. The diagram of Fig. 2 shows the main interconnections and controls in the complete system. The two amplifiers shown there are standard negative gain packaged units of stable design. They are combined in this way to produce the necessary positive gain zero damping amplifier.

The mechanical arrangement of the new system may be seen in Fig. 3. The cabinet at the left is the function generator, with controls for damping duration, damping phase, and output signal amplitude. The middle cabinet houses the analyzer itself, the upper two controls being the decade switches for the 100 period settings. The lower controls are the zero damping test signal switch, the zero damping adjusting control, the analog damping switch, and the response parameter selector switch. An interior view of the analyzer is given in Fig. 4 to show the style of construction.

Testing and Accuracy. In Fig. 5 are shown typical input and response records as photographed from the screen of the cathode ray oscilloscope. Fig. 5(a) is a half-sine test signal which was made by a regular film disk on which an accurately plotted half-sine function had been photographically reproduced. Fig. 5(b) shows a typical earthquake ground acceleration input signal as reproduced by the function generator and Fig. 5(c) is a typical analog response to the earthquake excitation of Fig. 5(b) for a specific period and damping.

To show a comparison of results obtained on the new model Response Spectrum Analyzer with past calculations certain function generator film disk records from past investigations were re-run. In Fig. 6, the solid line is the 0. 20 damping S_v relative velocity spectrum for the El Centro Earthquake of May 18, 1940 N. S. component as determined during the original damped spectrum studies made on the general purpose electric analog computer of the Analysis Laboratory of the California Institute of Technology. This is the earthquake shown above in Fig. 5(b). The circled points were obtained by using the original film disk and the first model analyzer. The points marked with triangles were obtained using the original film disk on the new Mark II Spectrum Analyzer and Function Generator System. The variations between the three different calculations can easily be accounted for by reasonable errors in reading the cathode ray oscilloscope amplitudes.

In Fig. 7 the solid curve is the undamped response spectrum as determined analytically for a half-sine pulse. The points are experimentally obtained on the new Mark II Spectrum Analyzer using the half-sine input of Fig. 5(a). Note that this comparison gives an evaluation of the overall accuracy of the whole process under relatively severe conditions. Such small deviations as are indicated in Fig. 6 could easily be accounted for by reasonable errors in reading the cathode ray oscilloscope amplitudes. Accuracies of this order are believed to be entirely adequate for the engineering applications contemplated.

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Nomenclature:

$y(t) =$ absolute ground displacement, a function of time

$\ddot{y}(t) = \frac{d^2y}{dt^2} =$ absolute ground acceleration $= A_o f(t)$

$x =$ absolute displacement of structural mass

$z = (x - y) =$ relative displacement between ground and structural mass

$T_p =$ undamped natural period of vibration of structure

$T_a =$ natural period of electric analog circuit

$\left(\frac{c}{c_c}\right) =$ fraction of critical damping of structure

$L =$ inductance in analog circuit

$C =$ capacitance in analog circuit

$Q =$ electric charge in analog circuit

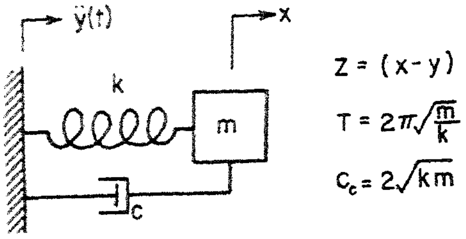
$E(t) =$ voltage applied to analog circuit $= E_o f(t) \Rightarrow A_o f(t)$

$S_v =$ maximum Relative Velocity Response Spectrum defined in Fig. 1

$R_o =$ measuring resistor in analog circuit

$N =$ time factor $= T_p / T_a$

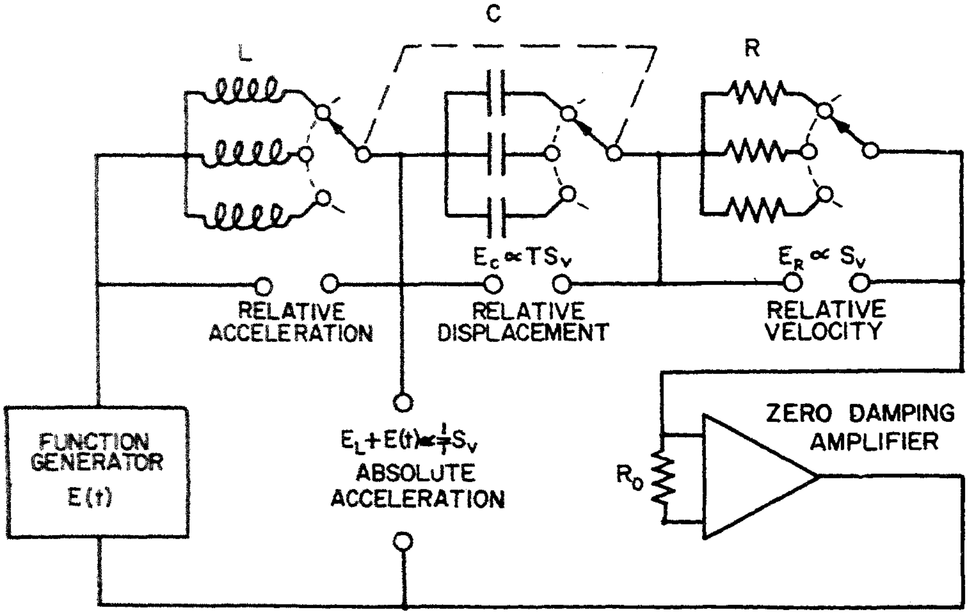
$E_R =$ voltage across measuring resistor



STRUCTURAL SYSTEM

$$\ddot{z} + \frac{4\pi}{T}\left(\frac{c}{c_c}\right)\dot{z} + \left(\frac{2\pi}{T}\right)^2 z = -\ddot{y}(t)$$

$$S_v = (\dot{x}-\dot{y})_{max} = \left[\int_0^t \ddot{y}(\tau)\, e^{-\frac{2\pi}{T}\left(\frac{c}{c_c}\right)(t-\tau)} \sin\frac{2\pi}{T}(t-\tau)\,d\tau\right]_{max}$$



ELECTRIC ANALOG.

Fig. 1. Basic Mechanical and Electrical Circuit Elements for the Response Spectrum Analyzer.

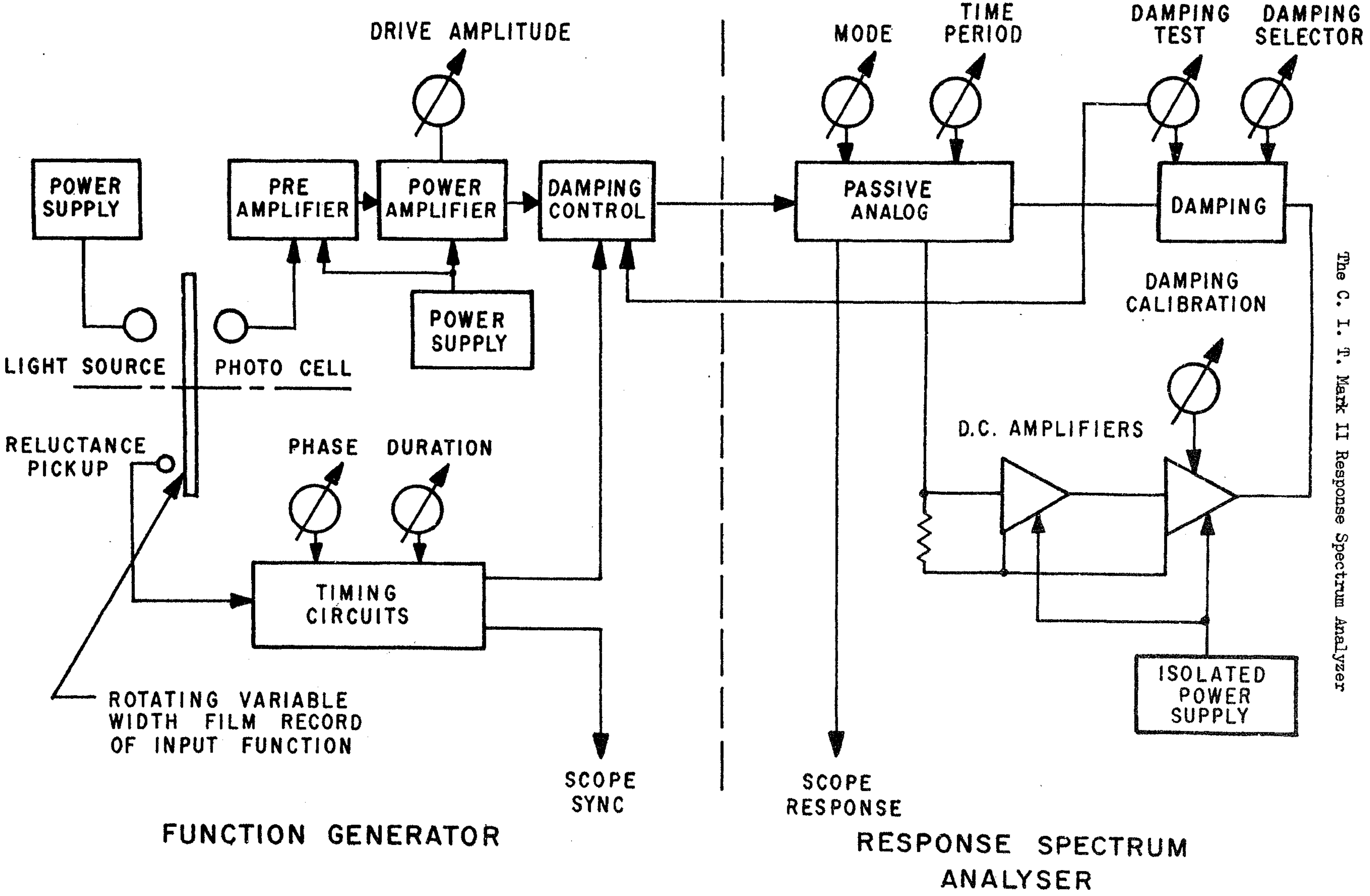


Fig. 2. General Circuit Layout for Response Spectrum Analyzer and Function Generator.

Fig. 3. General View of Function Generator-Spectrum Analyzer System in Use, Showing Mechanical Arrangement.

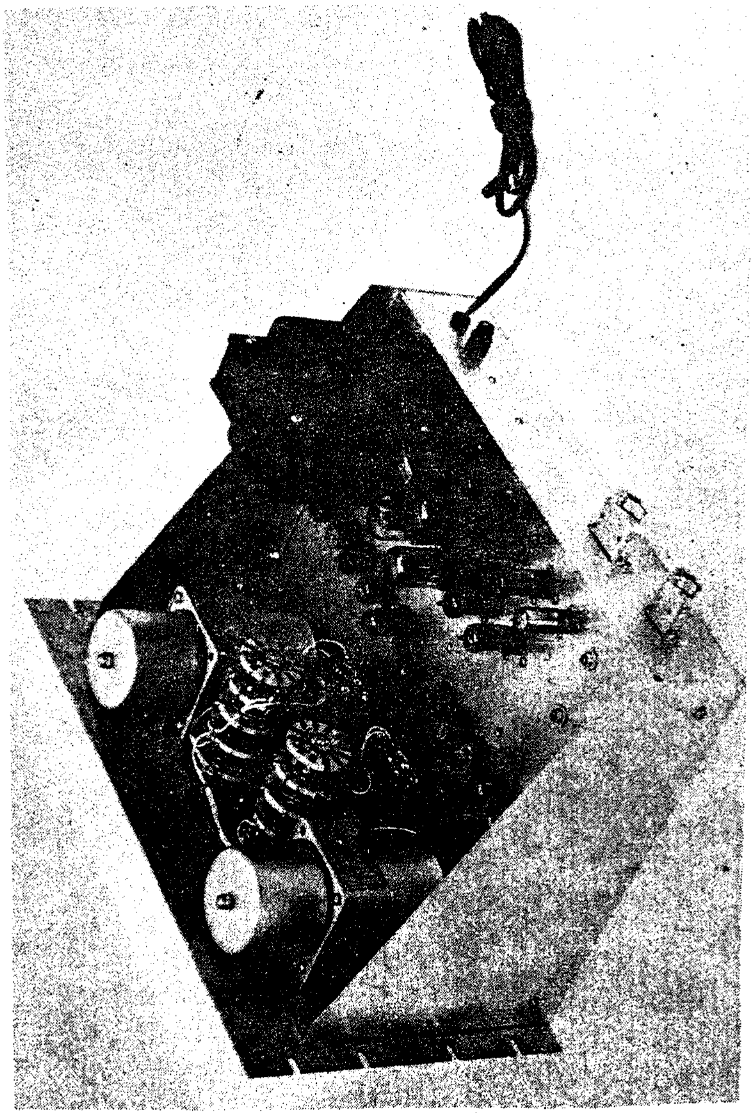


Fig. 4. Interior View of Spectrum Analyzer Showing Construction Details.

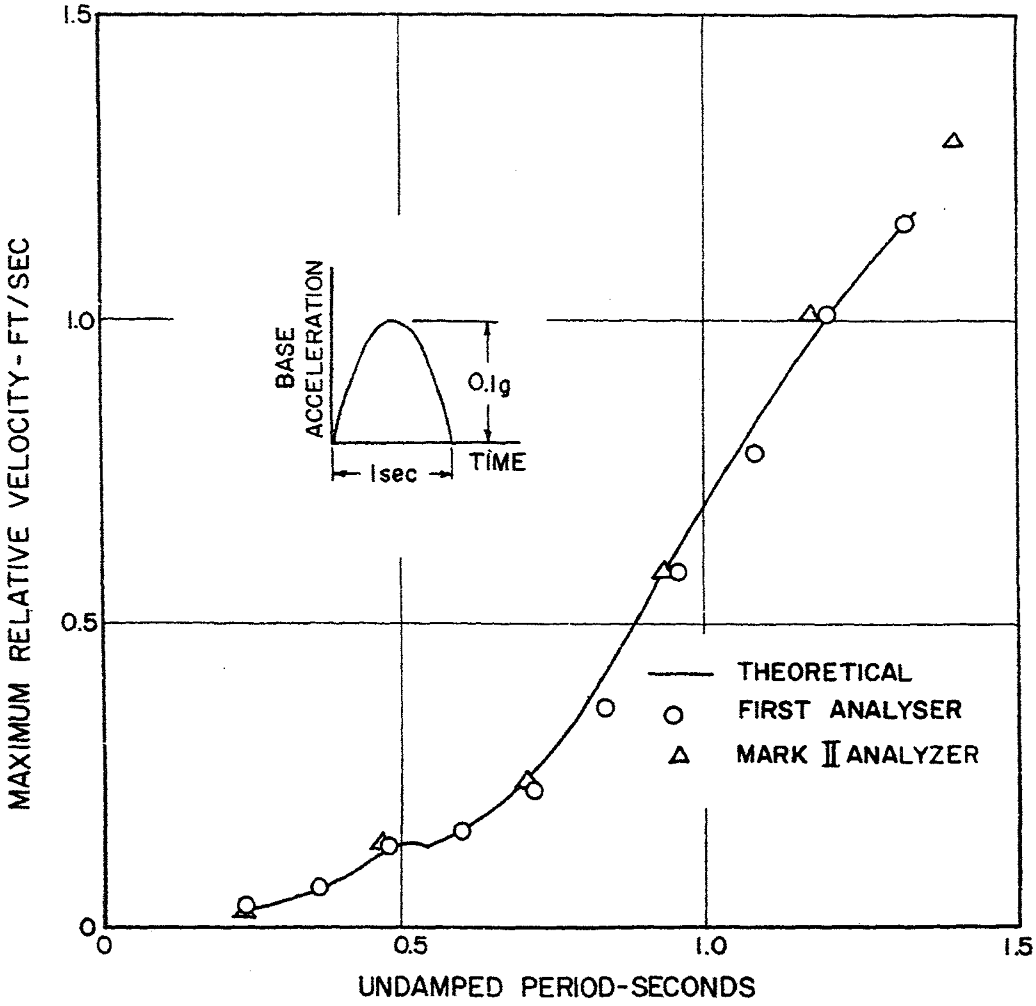


Fig. 7. Evaluation of Overall Accuracy of Spectrum Analyzer with a Half-Sine Pulse Input.