DYNAMIC PROPERTIES OF BUILDING DECIDED BY MEASUREMENT OF VIBRATION DURING EARTHQUAKE

by
Hiroyoshi Kobayashi*

INTRODUCTION

Observation of the vibration state of a building during earthquake is influenced by both the dynamic properties of the building and the ground condition supporting it. So it is not adequate to discuss the dynamic behavior of other buildings on the basis of the observed data, even if they are observed on the base floor in that building. It is difficult to eliminate from the data the effect of the building's own property and the condition of the surrounding soil.

On the other hand, as the following expression (1), we can get the seismic force which produce the deflection of the structure statically. And we call " K_r" seismic coefficient. *1

$$\sum_s \omega_s^2 y_{rs}/g = K_r \qquad \ldots\ldots\ldots\ldots\ldots(1)$$

Although it will be possible to calculate the seismic coefficient by the expression (1) theoretically, it might hardly be possible by an actual observation alone. Then the shearing force of the storey, calculated by expression (1), is equal to the shearing force as shown in expression (2).

$$Q_r = k_r(y_r - y_{r-1}) \qquad \ldots\ldots\ldots\ldots\ldots(2)$$

If we have the deflection of each storey of that structure, then we can find its shearing forces by multiplying each deflection by its rigidity, and finally it will be possible to calculate that seismic coefficient. But it must be very troublesome to observe the relative displacements between some storeys of a building. *2 Furthermore it might be difficult to find their rigidity. For such reason, even if we can get the data by observation, they will not be adequate for determination of the distribution of seismic coefficient.

Considering these problems the author tried to establish a consistent procedure, that is composed of the following three processes. The measurement of the vibration of building due to earthquake, the determination of dynamic properties of building by feeding the record measured mechanically into an electric analogue circuit, and the analysis of the seismic force acting on the

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* Assistant Professor, Tokyo Institute of Technology

structure by means of the analogue circuit. Taking this method we can save more instruments and labours. So we can get the required data without a vibration test.

1. DECIDING THE DYNAMIC PROPERTIES OF BUILDING BY THE MEASUREMENT DURING EARTHQUAKE

Now let us describe the author's method which decides the dynamic properties. Setting two accelerographs in a building, one on the top of the building and the other in the basement. The instrument takes the electro-magnetic method, and the phenomena are recorded on the magnetic tape. The tape recorder is a special one, and its heads which are made for stereo-recording purpose are assembled carefully to record simultaneously without any difference of the phase between the two elements of the measurement. Recorded tape is played back at "n" times speed as compared with recording speed. The record at the basement floor is sent to the analogue circuit as the true acceleration, from which the effect of the pendulum has been eliminated. Figure 2 shows the analogue circuit which is analogous to the mechanical model of the structure shown in Fig.1. The analogy between the mechanical model and the electrical circuit is illustrated in appendix. Now the exciting power of circuit means the acceleration of the bed rock, but we are not able to measure that motion directly. As the exciting power of the analogue circuit, we used the acceleration $\ddot{Y}$ of the ground, but it must not be a true value because $\ddot{Y}+\ddot{y}_0$ is measured on the foundation, not on the ground. So we used a feed-back circuit which deducts $\ddot{y}_0$ from the exciting power. In the analogue circuit, the voltage between two points corresponding to the acceleration of the top of the building, is picked up and sent to the X-axis of the oscilloscope through a filter, which transform the current to a form of mechanically measured acceleration using a pendulum. On the other hand, the record at the top of building is sent to the Y-axis of the oscilloscope.

If the constants of the analogue circuit agree with the assumed constants of the structure, the same phenomena will appear on the both axes, the figure on the oscilloscope must be a straight line inclined 45° to the axis. If not, irregular forms will appear on the oscilloscope. The irregular form may be set straight line by adjusting the circuit constants to proper value, which then correspond to the structures constants such as rigidity and viscosity. By applying the expression (6), we are able to decide the rigidity and damping of the structure and supporting soil.

From the analogue circuit we can get the shearing force and deflections of column and wall, velocity and acceleration of each floor of the building, by knowing the voltage between the specific points.

Exciting the basement with the stationary sine wave, we can get the same behaviour under the stationary vibrational state as in the case of a vibration test by shaking table or by vibration generator.

The operation is carried out from the measurement to the calculation, without any checking or tracing by man, except only watching the figure on the oscilloscope. And it will be possible to find out the result from the measurement of vibration during the earthquake in the comparatively short time, compared to other methods.

2. DATA OF THE MEASURED BUILDINGS

The author adapted this method to the following three buildings:

1) Reserch Institute of Building Material, called "BLDG-NO.1", in Tokyo Institute of Technology, at Oookayama, Meguroku, Tokyo. This building is three storeyed reinforced concrete construction, plans and skeleton are shown in Fig.4, and data in Table 1.

2) An office building, called "BLDG-NO.2", at Ginza, Chuooku, Tokyo. This one is steel reinforced concrete construction, 9 storeys above ground and two basements, plans and skeleton are shown in Fig.9 and data in Table 2.

3) An office and apartment house building, called "BLDG-NO.3", at Nampeidai, Shibuyaku, Tokyo. This one is steel reinforced concrete construction, 10 storeys above ground and one basement, plans and skeleton are shown in Fig.14 and data in Table 3.

These buildings have simple, box-like, plan and skeleton, and the foundation are set upon the soil directly without using piles.

3. DYNAMIC PROPERTIES OF BUILDING

By this method, we could find the several problems about the dynamic properties of the buildings described above.

1) Rigidity and damping of each storey

Rigidity and damping, decided by this method, are shown in Table 1, 2, and 3. These results include the influence of the aseismic wall, the frame, the soil condition, foundation construction and others. But these are not discussed in this paper.

2) Resonance curve

Analogue circuit with the decided constants is excited by sine wave, and gives the acceleration and the deflections of each storey at various frequencies of excited wave. The acceleration resonance curves are plotted in Figs.5, 10, and 15. The deflection of each storey and inclination of foundation are plotted in Figs.6, 11, and 16. Resonance period about these curves are shown in Tables 1, 2, and 3. On the other hand the response spectra of the basement motion are shown in Figs.7, 12, and 17. These data indicate that the resonance periods agree with the peak of the response spectrum, but the response spectrum has other peaks not corresponding with the peaks of resonance curve, these peaks might be reasoned by other effects.

3) Normal mode

The shape of the resonance curve, nearing the first resonance point, is very likely to the forming of the first normal mode, so we could decide the mode by the resonance curve. That is shown in Figs.8, 13, and 18.

4) Acceleration during earthquake

About a few cases of earthquakes the calculated acceleration of each storey, the maximum values of them are shown in Figs.8, 13, and 18, and Tables 1, 2, and 3, are shown on the oscilloscope. The figures and tables show the ratio of the maximum acceleration at each storey and on the basement.

5) Shearing force and seismic force distribution

About the same examples we show the shearing forces and seismic forces of each storey. These are also maximum values during the earthquakes, excited by unit acceleration of basement, as shown in the same figures and tables. Seismic factors defined as a quotient of the shearing force acting on some storey devided by the total value of the mass above that storey. Supposing the maximum value of the shearing force of each storey, acting on each storey simultaneously, we can get the seismic coefficient which is conventional one. These are different from the accelaration distribution.

6) Deflections of building during earthquake

The maximum deflections of each storey during the earthquake are shown in the same figures and tables.

7) Overturning moment and inclination of base

The overturning moment will be produced by the vibration of building during earthquake, by this moment occurres the inclination of base. This moment and the inclination of base are shown in the tables.

CONCLUSION

This paper has indicated a method to get the dynamic properties and seismic forces of the buildings by measuring vibrations during earthquake. The following are special points:

1) It is possible to get good data by least measurement.
2) The method is easy and speedy without any tracing and calculation by man.
3) The method can produce many constants of building which we could not find before.

ACKNOWLEDGEMENTS

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APPENDIX

ANALOGY BETWEEN DYNAMIC PROPERTIES OF STRUCTURE AND ELECTRICAL CIRCUIT

Now consider the multi-mass system, shown in Fig.1, to illustrate the vibration of the structure. The frame is assumed three-mass system, and the masses are connected by spring and damper. The foundation is assumed two-mass system, one of them shows for the sway of foundation and the other is for the rotation of it, they are also connected by spring and damper. Equilibrium equation is as follows:

$$\left\{\begin{aligned}
& m_3\ddot{y}_3 + c_3(\dot{y}_3-\dot{y}_2) + k_3(y_3-y_2) + m_3H_3\ddot{\theta} + m_3\ddot{y}_0 + m_3\ddot{Y} = 0 \\
& m_2\ddot{y}_2 + c_2(\dot{y}_2-\dot{y}_1) - c_3(\dot{y}_3-\dot{y}_2) + k_2(y_2-y_1) - k_3(y_3-y_2) \\
& \qquad + m_2H_2\ddot{\theta} + m_2\ddot{y}_0 + m_2\ddot{Y} = 0 \\
& m_1\ddot{y}_1 + c_1\dot{y}_1 - c_2(\dot{y}_2-\dot{y}_1) + k_1y_1 - k_2(y_2-y_1) + m_1H\ddot{\theta} + m_1\ddot{y}_0 + m_1\ddot{Y} = 0 \\
& m_0\ddot{y}_0 + c_0\dot{y}_0 - c_1\dot{y}_1 + k_0y_0 - k_1y_1 + m_0\ddot{Y} = 0 \\
& I\ddot{\theta} + c_\theta\dot{\theta} + k_\theta\theta + \sum m_iH_i\ddot{Y} + \sum m_iH_i^2\ddot{\theta} + \sum m_iH_i\ddot{y}_0 + \sum m_iH_i\ddot{y}_i = 0
\end{aligned}\right.$$

$$\cdots\cdots\cdots\cdots(3)$$

putting $y_i + y_0 = z_i$, $y_0 = z_0$

$$\left\{\begin{aligned}
& m_3\ddot{z}_3 + c_3(\dot{z}_3-\dot{z}_2) + k_3(z_3-z_2) + m_3H_3\ddot{\theta} = -m_3\ddot{Y} \\
& m_2\ddot{z}_2 + c_2(\dot{z}_2-\dot{z}_1) - c_3(\dot{z}_3-\dot{z}_2) + k_2(z_2-z_1) - k_3(z_3-z_2) \\
& \qquad\qquad + m_2H_2\ddot{\theta} = -m_2\ddot{Y} \\
& m_1\ddot{z}_1 + c_1(\dot{z}_1-\dot{z}_0) - c_2(\dot{z}_2-\dot{z}_1) + k_1(z_1-z_0) - k_2(z_2-z_1) \\
& \qquad\qquad + m_1H_1\ddot{\theta} = -m_1\ddot{Y} \\
& m_0\ddot{z}_0 + c_0\dot{z}_0 - c_1(\dot{z}_1-\dot{z}_0) + k_0z_0 - k_1(z_1-z_0) = -m_0\ddot{Y} \\
& (I + \sum m_iH_i^2)\ddot{\theta} + c_\theta\dot{\theta} + k_\theta\theta + \sum m_iH_i\ddot{z}_i = -\sum m_iH_i\ddot{Y}
\end{aligned}\right.$$

$$\cdots\cdots\cdots\cdots(4)$$

On the other hand the equilibrium equation of circuit potential, shown in Fig.2, is as follows:

$$
\left\{
\begin{aligned}
&\left[L_3+L_{33}\frac{\lambda_3}{\lambda_3+1}\right]\frac{di_3}{dt}+\sqrt{\frac{L_{13}}{L_{23}}}E+\sqrt{L_{63}L_{33}}\frac{\sqrt{\lambda_3}}{\lambda_3+1}\frac{di_\theta}{dt}+R_3(i_3-i_2)\\
&\qquad+\frac{1}{C_3}\int(i_3-i_2)dt=0\\
&\left[L_2+L_{32}\frac{\lambda_2}{\lambda_2+1}\right]\frac{di_2}{dt}+\sqrt{\frac{L_{12}}{L_{22}}}E+\sqrt{L_{62}L_{32}}\frac{\sqrt{\lambda_2}}{\lambda_2+1}\frac{di_\theta}{dt}+R_2(i_2-i_1)\\
&\qquad-R_3(i_3-i_2)+\frac{1}{C_2}\int(i_2-i_1)dt-\frac{1}{C_3}\int(i_3-i_2)dt=0\\
&\left[L_1+L_{31}\frac{\lambda_1}{\lambda_1+1}\right]\frac{di_1}{dt}+\sqrt{\frac{L_{11}}{L_{21}}}E+\sqrt{L_{61}L_{31}}\frac{\sqrt{\lambda_1}}{\lambda_1+1}\frac{di_\theta}{dt}+R_1(i_1-i_0)\\
&\qquad-R_2(i_2-i_1)+\frac{1}{C_1}\int(i_1-i_0)dt-\frac{1}{C_2}\int(i_2-i_1)dt=0\\
&L_0\frac{di_0}{dt}+\sqrt{\frac{L_{10}}{L_{20}}}E+R_0i_0-R_1(i_1-i_0)+\frac{1}{C_0}\int i_0dt-\frac{1}{C_1}\int(i_1-i_0)dt=0\\
&\left[L_{61}\frac{1}{\lambda_1+1}+L_{62}\frac{1}{\lambda_2+1}+L_{63}\frac{1}{\lambda_3+1}+L_\theta\right]\frac{di_\theta}{dt}+\sqrt{\frac{L_{1\theta}}{L_{2\theta}}}E+\sqrt{L_{61}L_{31}}\frac{\sqrt{\lambda_1}}{\lambda_1+1}\frac{di_1}{dt}\\
&\qquad+\sqrt{L_{62}L_{32}}\frac{\sqrt{\lambda_2}}{\lambda_2+1}\frac{di_2}{dt}+\sqrt{L_{63}L_{33}}\frac{\sqrt{\lambda_3}}{\lambda_3+1}\frac{di_3}{dt}+R_\theta i_\theta+\frac{1}{C_\theta}\int i_\theta dt\\
&\qquad=0\\
&\lambda_1=\frac{L_{51}}{L_{41}},\qquad\lambda_2=\frac{L_{52}}{L_{42}},\qquad\lambda_3=\frac{L_{53}}{L_{43}}.
\end{aligned}
\right.
\qquad\cdots\cdots\cdots\cdots(5)
$$

Then the analogy between the properties of structure and electrical circuit is substantiated, as shown in the following expressions. The number "n" is the ratio of the running velocity of exciting current of circuit and the acceleration of earthquake motion. And "α" is a arbitrary constant.

$$
\begin{aligned}
&L_{23}=L_{13}(m_3/m)^2\alpha^2, && L_{22}=L_{12}(m_2/m)^2\alpha^2, && L_{21}=L_{11}(m_1/m)^2\alpha^2,\\
&L_{20}=L_{10}(m_0/m)^2\alpha^2, && L_{2\theta}=L_{1\theta}\sum(m_iH_i/mH)^2\alpha^2, &&\\
&L_\theta=I/mH^2, && L_\theta=m_0/m &&\\
&L_1=0, && L_2=0, && L_3=0,\\
&L_{31}=(\lambda_1+1)/\lambda_1\cdot\frac{m_1}{m}, && L_{32}=(\lambda_2+1)/\lambda_2\frac{m_2}{m}, && L_{33}=(\lambda_3+1)/\lambda_3\frac{m_3}{m},\\
&L_{61}=(\lambda_1+1)\,m_1H_1^2/mH^2, && L_{62}=(\lambda_2+1)m_2H_2^2/mH^2, && L_{63}=(\lambda_3+1)m_3H_3^2/mH^2\\
&R_\theta=2\frac{h_\theta}{\omega_\theta}\cdot\frac{1}{C_\theta}\cdot\frac{1}{n}, && \frac{1}{C_\theta}=\omega_\theta^2\frac{I}{mH^2}n^2, &&\\
&R_i=2\frac{h_i}{\omega_i}\cdot\frac{1}{C_t}\cdot\frac{1}{n} && \frac{1}{C_i}=\omega_i^2\frac{m_i}{m}n^2, &&
\end{aligned}
\qquad\cdots\cdots\cdots\cdots(6)
$$

$$E = mn^2\ddot{Y}/\alpha$$

$$\int i_r dt = mZ_r \qquad \int i_\theta dt = mH\theta$$

$$i_r = mn\frac{dZ_r}{dt} \qquad i_\theta = mnH\frac{d\theta}{dt}$$

$$\frac{di_r}{dt} = mn^2\frac{d^2Z_r}{dt^2} \qquad \frac{di_\theta}{dt} = mn^2H\frac{d^2\theta}{dt^2} \quad \ldots\ldots\ldots\ldots\ldots(7)$$

BIBLIOGRAPHY

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NOMENCLATURE

s : suffix, vibratio mode

r : suffix, storey's or mass's number

ω_s : s-th natural frequency

y_{rs} : deflection between r-th floor and basement of the s-th mode

g : acceleration of gravity

K_r : seismic coefficient acting on the r-th floor

Q_r : shearing force of the r-th storey

k_r : rigidity of the r-th storey

m_r : mass of the r-th floor, mass of the r-th concentrated mass

c_r : viscous damping of the r-th storey

H_r : height of the r-th floor

θ : inclination of base

I : moment of inertia of the basement

m : mass of the virtual concetrated mass

H : height of the virtual concentrated mass

h_r : damping ratio of the r-th storey , c/c_{cri}

p : natural frequency of the r-th storey alone

Y ; displacement of bed rock

Y_r : displacement of the r-the floor

α : arbitrary constant

n : speed ratio between the exciting current of circuit and the acceleration of earthquake motion

E : in put voltage

i_r : electric current

C_r : capacitance

R_r : resistance

L_r : reactance

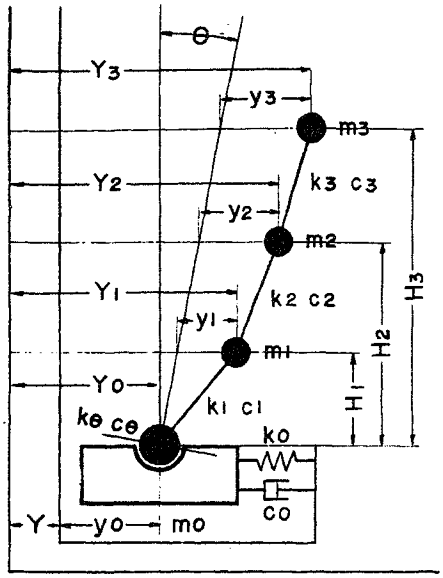

Fig.1 Mechanical model of structure.

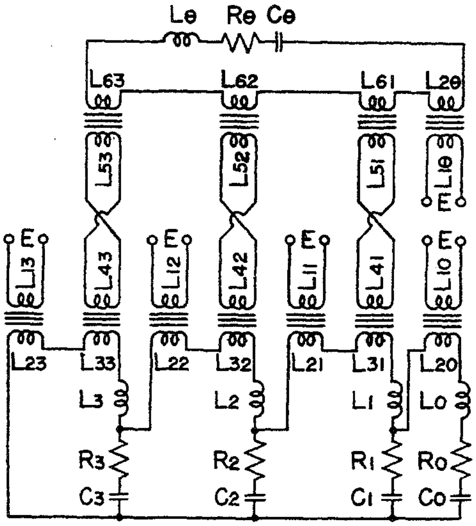

Fig.2 Analogue circuit

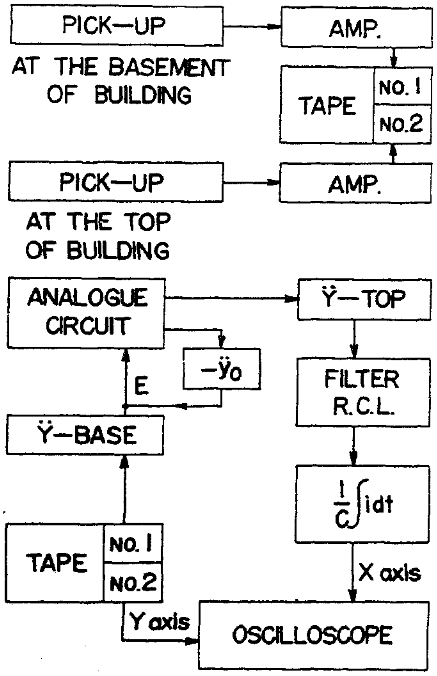

Fig.3

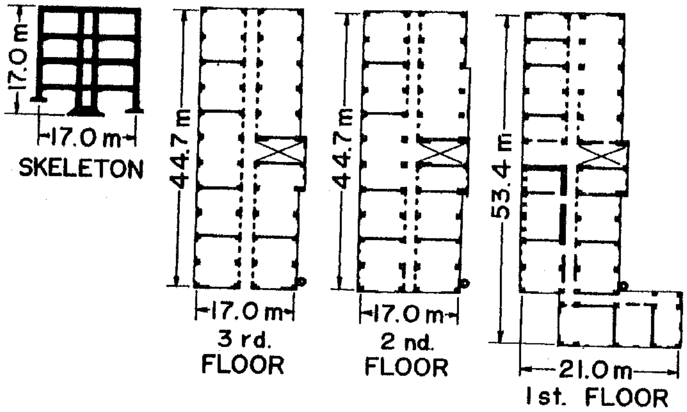

Fig.4 Plans and skeleton of BLDG-NO.1

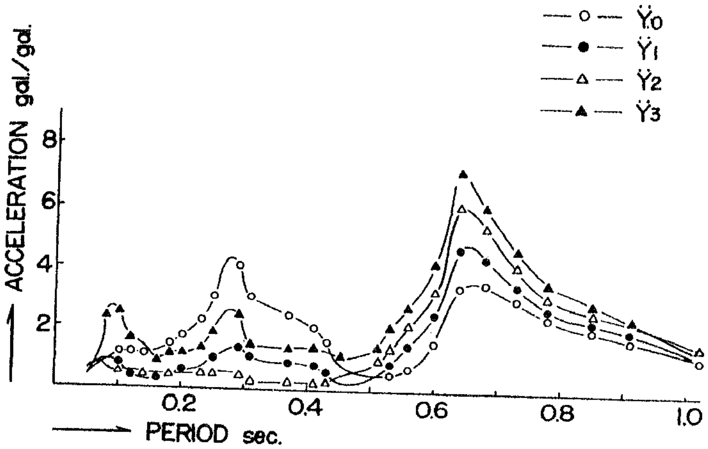

Fig.5 Acceleration resonance curve of BLDG-NO.1 NS component

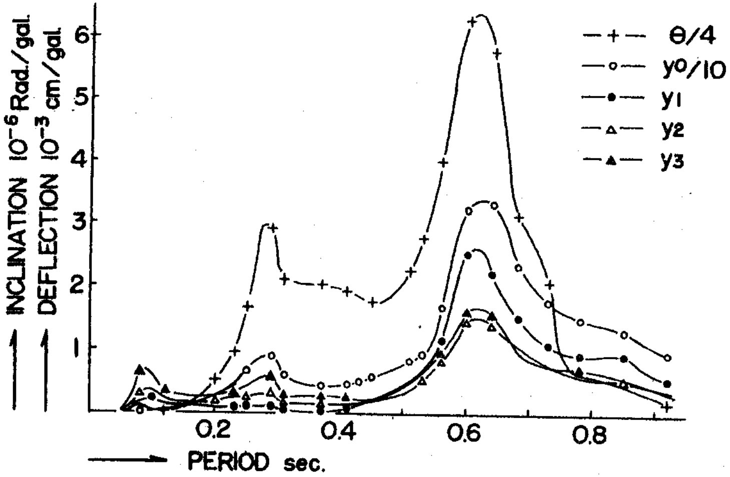

Fig.6 Deflection resonance curve of BLDG-NO.1 NS component

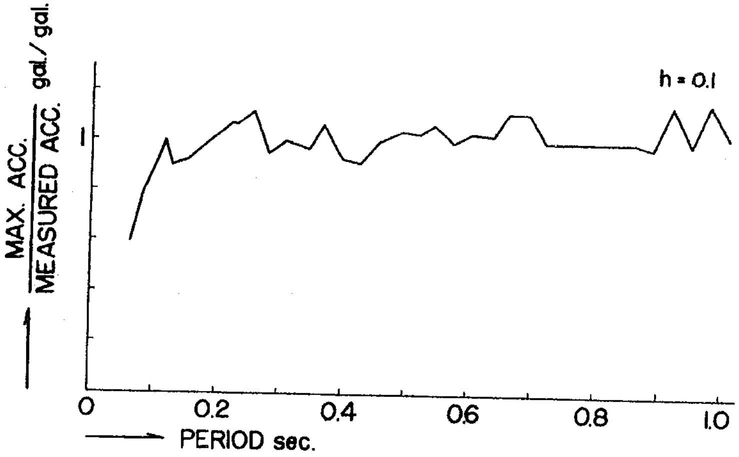

Fig.7 Acceleration response spectrum of the basement motion of BLDG-NO.1 NS component

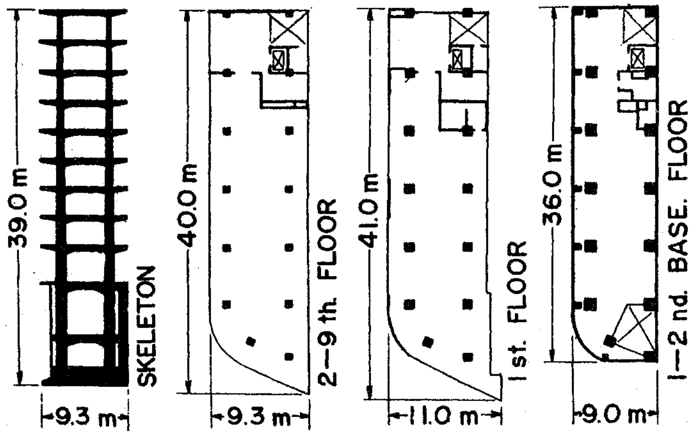

Fig.9 Plans and skeleton of BLDG-NO.2

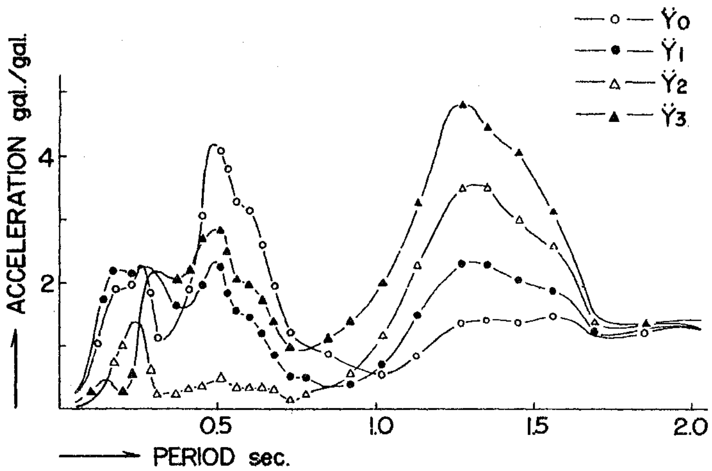

Fig.10 Acceleration resonance curve of BLDG-NO.2 EW component

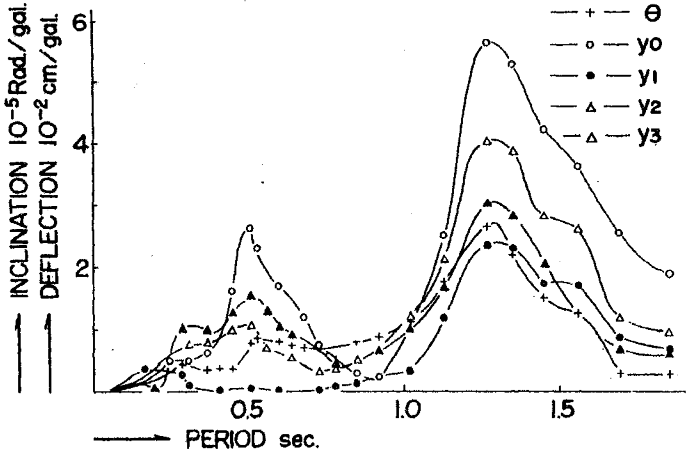

Fig.11 Deflection resonance curve of BLDG-NO.2 EW component

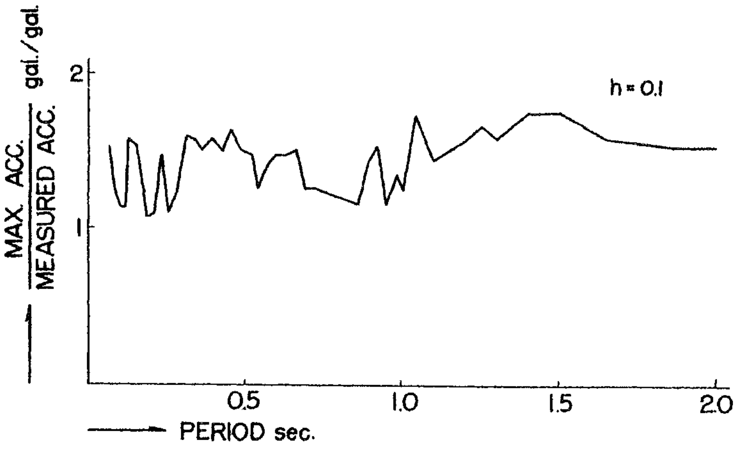

Fig.12 Acceleration response spectrum of the basement motion of BLDG-NO.2 EW component

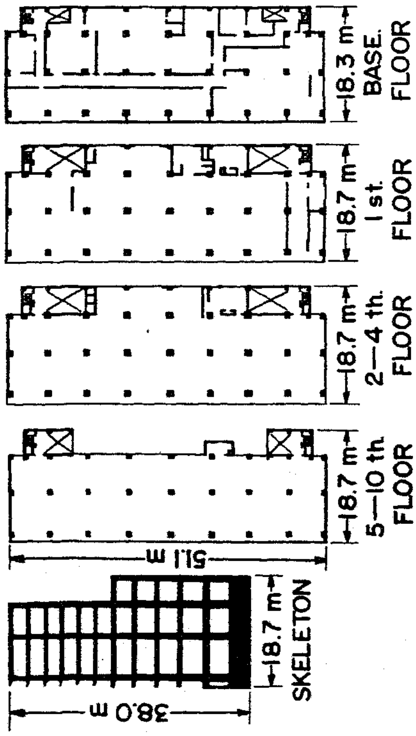

Fig.14 Plans and skeleton of BLDG-NO.3

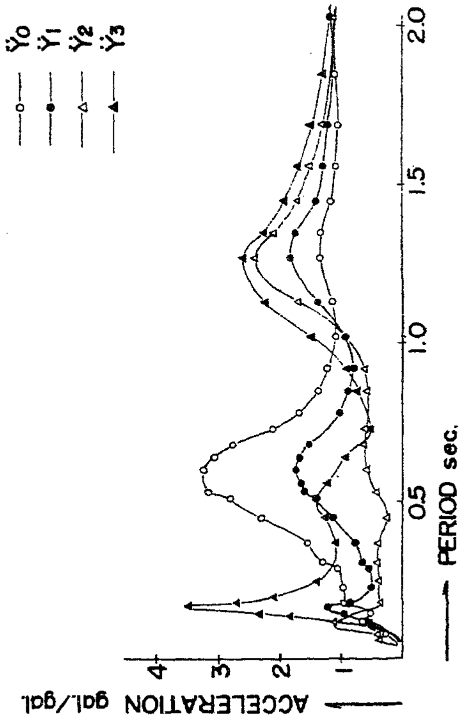

Fig.15 Acceleration resonance curve of BLDG-NO.3 NS component

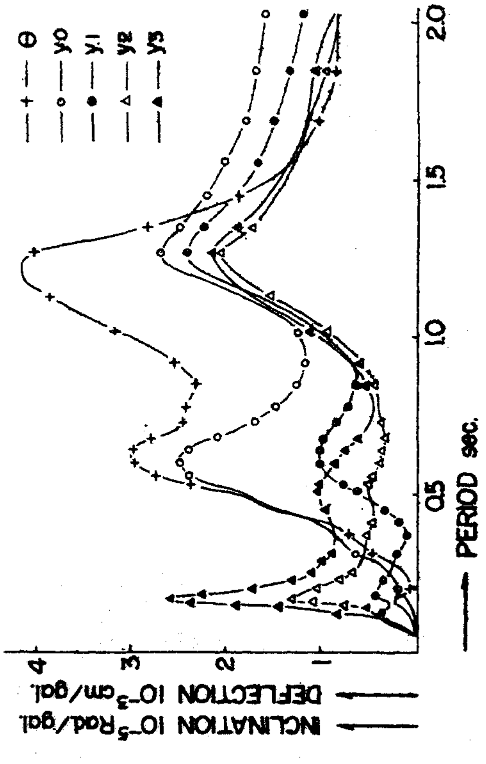

Fig.16 Deflection resonance curve of BLDG-NO.3 NS component

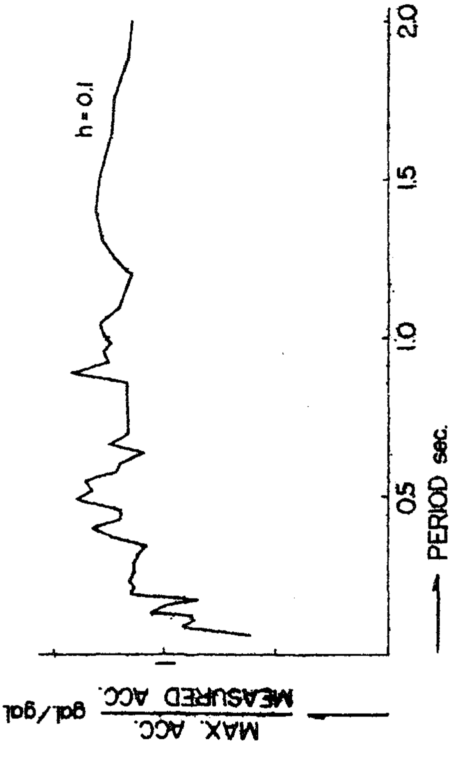

Fig.17 Acceleration response spectrum of the basement motion of BLDG-NO.3 NS component

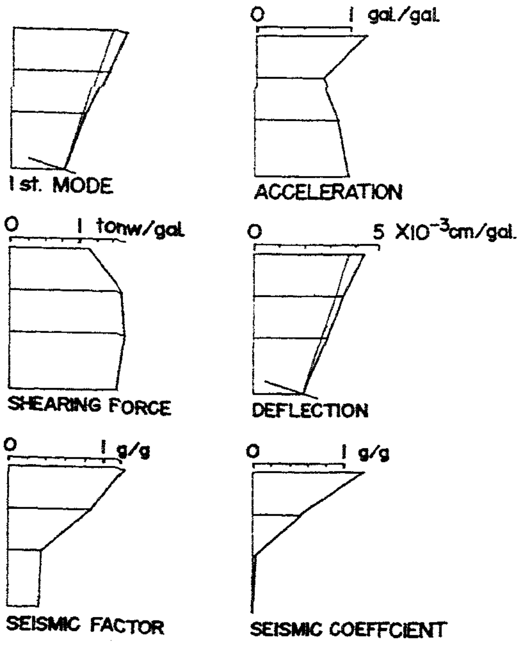

Fig.8 Dynamic properties of BLDG-NO.1 NS component

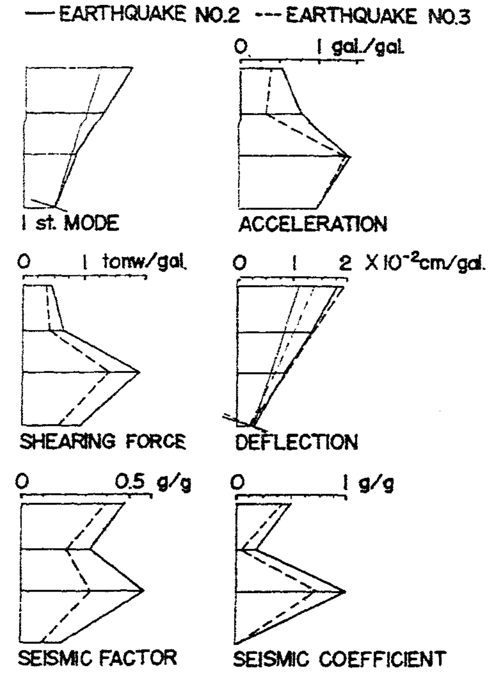

Fig.13 Dynamic properties of BLDG-NO.2 EW component

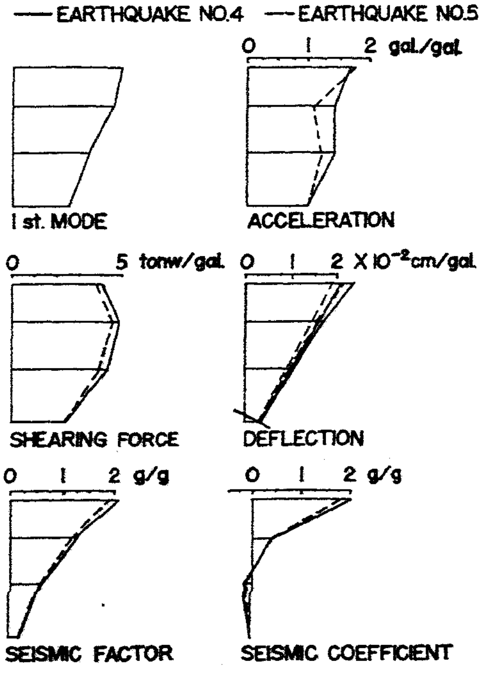

Fig.18 Dynamic properties of BLDG-NO.3 NS component

Table 1 Dynamic properties of BLDG-NO.1 NS component

Floor			roof	3rd	2nd	base	θ
Mass		ton	1086	1021	1418	1804	
Height		m	12.88	9.00	5.20	0	
Storey height		m	3.88	3.80	5.20		
Moment of inertia		10^5ton·m^2					4.225
Rigidity	k	tonw/cm				903	
	k'	10^6tonw/radian	1.93	3.77	5.15		
	k_θ	10^6tonw·m/radian					9.92
Frequency of storey	p	1/sec	66.9	97.4	82.7	22.2	151.8
Period of storey	T	sec	0.094	0.064	0.076	0.283	0.041
Damping ratio	h		0.096	0.085	0.098	0.074	0.046
Earthquake NO.1							
Acceleration		gal/gal	1.17	0.71	0.89	1.00	
Shearing force		tonw/gal	1.32	1.84	1.88	1.74	
Deflection		10^{-4}cm/gal	2.71	1.86	1.90	19.3	
Overturning moment		tonw·m/gal					13.8
Inclination		10^{-6}radian/gal					1.39
Seismic factor		g/g	1.21	0.87	0.35	0.32	
Seismic coefficient		g/g	1.21	0.51	0.03		

Mode			1	2	3	4
Period	by analogue circuit	sec.	0.62	0.35	0.28	0.09
	by response spectrum	sec.	0.68	0.37	0.26	0.12

Table 2 Dynamic properties of BLDG-NO.2 EW component

Floor		R,9,8,7	6,5,4	3,2,1	B-1,B-2	θ
Mass	ton	964	1023	1256	1642	
Height	m	30.9	20.7	11.7	0	
Moment of inertia	10^6ton·m^2					1.607
Rigidity k	tonw/cm				331	
k'	10^5tonw/radian	2.24	2.56	7.73		
k_θ	10^7tonw·m/radian					1.32
Frequency of storey p	1/sec	15.0	16.5	22.7	14.0	89.7
Period of storey T	sec	0.40	0.38	0.28	0.45	0.07
Damping ratio h		0.083	0.167	0.150	0.290	0.054
Earthquake NO.2						
Acceleration	gal/gal	0.518	0.768	1.425	1.00	
Shearing force	tonw/gal	0.473	0.655	1.895	0.946	
Deflection	10^{-3}cm/gal	2.15	2.31	2.86	2.84	
Overturning moment	tonw·m/gal					26.2
Inclination	10^{-6}radian/gal					1.98
Seismic factor	g/g	0.481	0.323	0.572	0.190	
Seismic coefficient	g/g	0.481	0.174	1.01		
Earthquake NO.3						
Acceleration	gal/gal	0.378	0.335	1.36	1.00	
Shearing force	tonw/gal	0.385	0.418	1.394	0.582	
Deflection	10^{-3}cm/gal	1.75	1.47	2.11	1.76	
Overturning moment	tonw·m/gal					55.7
Inclination	10^{-6}radian/gal					4.28
Seismic factor	g/g	0.391	0.206	0.321	0.097	
Seismic coefficient	g/g	0.391	0.032	0.76		

Mode		1	2	3	4	5	6
Period	by analogue circuit s.	1.30	0.60	0.48	0.31	0.23	0.15
	by response spectrum s.	1.45	0.63	0.45	0.31	0.23	0.13

Table 3 Dynamic properties of BLDG-NO.3 NS component

Floor		R,10,9	8,7,6	5,4,3,2	1,B	Θ
Mass	ton	1939	1966	3348	4064	
Height	m	31.70	22.74	12.09	0	
Moment of inertia	10^6ton·m^2					3.937
Rigidity k	tonw/cm				796	
k'	10^6tonw/radian	2.23	5.28	8.01		
k_θ	10^7tonw.m/radian					0.80
Frequency of storey p	1/sec	35.4	49.7	44.0	13.86	44.6
Period of storey T	sec	0.177	0.126	0.143	0.453	0.141
Damping ratio h		0.143	0.197	0.260	0.115	0.173
Earthquake NO.4						
Acceleration	gal/gal	1.68	1.44	1.46	1.00	
Shearing force	tonw/gal	4.01	4.84	4.28	2.42	
Deflection	10^{-3}cm/gal	1.63	0.98	0.65	3.04	
Overturning moment	tonw.m/gal					46.3
Inclination	10^{-6}radian/gal					5.79
Seismic factor	g/g	2.03	1.21	0.58	0.21	
Seismic coefficient	g/g	2.03	0.41	-0.16		
Earthquake NO.5						
Acceleration	gal/gal	1.78	1.10	1.21	1.00	
Shearing force	tonw/gal	3.64	4.38	3.89	2.20	
Deflection	10^{-3}cm/gal	1.46	0.88	0.58	2.77	
Overturning moment	tonw.m/gal					42.0
Inclination	10^{-6}radian/gal					5.25
Seismic factor	g/g	1.84	1.10	0.53	0.19	
Seismic coefficient	g/g	1.84	0.37	-0.14		

Mode	1	2	3	4	5	6
Period by analogue circuit s.	1.28	0.66	0.60	0.50	0.17	0.12
by response spectrum s.	1.40	0.65	0.60	0.52	0.19	0.12