A Comparison of Theoretical and Experimental Determinations of Building Response to Earthquakes

by

D. E. Hudson*

Introduction. Applications of classical normal mode theory to the earthquake engineering problem have been discussed in a number of notable theoretical papers (1-7)** Such theory has been difficult to use in practice because of: 1. insufficient information as to such dynamic properties as natural periods and damping of typical buildings; 2. lack of knowledge of the nature of the ground motion associated with strong earthquakes; and 3. the complexity of typical building structures and of earthquake ground acceleration records. It is only in recent years that ground acceleration records for strong earthquakes have become available for some areas, and the current development of high speed computing techniques has already placed such dynamic problems within the reach of the structural engineer. Some work has been done on the experimental study of the dynamic properties of buildings by means of shaking machines and ground motions excited by explosive blasts. Much more work of this type is needed, as well as increased instrumentation which will lead to actual measured responses of real buildings to strong earthquake ground motions.

Because of the difficulty of carrying out realistic artificial tests of building structures, an important part of the strong-motion accelerograph program of the United States Coast and Geodetic Survey has been devoted to the measurement of building responses during strong earthquakes. At present eight of the standard USCGS strong-motion accelerographs are located in the upper floors of buildings in which basement accelerographs are installed to record simultaneously the ground acceleration. The Alexander Building in San Francisco is particularly suitable for studies of this kind. It has strong-motion accelerographs in the basement, the 11th floor, and the 16th floor (roof), and its properties have been extensively studied in the past so that the dynamic characteristics are relatively well known. Over a period of years Mr. J. A. Blume and his associates have made numerous theoretical and experimental investigations of the dynamic properties of this building so that the basic values of the mass and stiffness distributions are available. (8-10)

A number of simultaneous measurements of ground motion and building response have been obtained in California for small earthquakes. The San Francisco Earthquake of March 22, 1957 made it possible for the first time to compare directly such ground and building vibrations for a strong earthquake. During this earthquake the three accelerographs in the Alexander Building produced excellent records, and it is the purpose of the present paper to show the extent to which the measured values of the building response verify the calculations based on classical normal mode theory using the experimentally determined dynamic properties of the building.

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** Numbers in parentheses indicate references in bibliography.

Normal Mode Theory Applied to the Earthquake Problem. Consider the lateral displacements of an n-story building, as represented by Fig. 1. The horizontal ground acceleration is $\ddot{y}(t)$; the absolute displacements of the floors of concentrated mass m_1, $m_2 \ldots m_n$ are x_1 $x_2 \ldots x_n$. The relative displacements between the floors and the ground are $z_n = x_n - y(t)$. No special types of connections will be assumed; the elastic properties will be described by the mode shapes associated with the natural frequencies $\omega_1, \omega_2 \ldots \omega_n$ or the natural periods T_1, $T_2, \ldots T_n$, where $\omega_r = 2\pi/T_r$. These mode shapes will be described by the relative displacements $A_1^{(r)}$, $A_2^{(r)}, \ldots A_n^{(r)}$ where the subscripts refer to the individual masses, and the superscripts refer to the mode of vibration. Thus the fundamental mode would be given by $A_1^{(1)}$, $A_2^{(1)}, \ldots A_n^{(1)}$. Since only the shape of a normal mode is defined, it is the ratios between the A's which are to be determined and not the values of the individual A's.

Each mode will have associated with it some energy dissipation, as is indicated by the internal and external dashpots of Fig. 1. It will be supposed that the total effect of all of these energy dissipating mechanisms is described by an equivalent viscous damping coefficient for each mode of vibration, which is expressed in terms of the critical damping for that mode, $n_r = (c/c_c)_r$. The analysis will be limited to small damping, $(n_r < 0.20)$, and hence "damped normal modes" will exist having the same shape as the undamped modes.

Each of the n simultaneous equations of motion for the system will in general involve several z's, through various coupling terms. It is always possible, under the above assumptions, to make a transformation of coordinates from the original z coordinates to the normal coordinates ξ:

$$z_i = \sum_{r=1}^{n} A_i^{(r)} \xi_i \qquad \ldots (1)$$

In terms of these normal coordinates the original equations will be reduced to n independent equations, each involving only one unknown, so that the general equations of motion have the form:

$$\ddot{\xi}_r + 2\omega_r n_r \dot{\xi}_r + \omega_r^2 \xi_r = f_r \qquad \ldots (2)$$

where f_r is a "generalized force" exciting the rth mode of vibration.

The solution of eq. (2) can be written in terms of the superposition integral, which for small damping, $\sqrt{1-n_r^2} \approx 1$, gives:

$$\xi_r = \frac{1}{\omega_r} \int_0^t f_r(\tau)\, e^{-\omega_r n_r (t-\tau)} \sin \omega_r (t-\tau)\, d\tau \qquad \ldots (3)$$

The generalized forces f_r may be expressed in terms of applied external lateral forces acting on each mass, F_1, $F_2, \ldots F_n$ by requiring that the same amount of work be done by each set of forces during a virtual displacement in the rth mode. In this way it is found that:

$$f_r = \frac{\sum_{j=1}^{n} F_j A_j^{(r)}}{\sum_{j=1}^{n} m_j A_j^{(r)2}} \qquad \ldots (4)$$

For the earthquake excitation, the external forces are inertia forces which are a consequence of the ground acceleration:

$$F_j = - m_j \ddot{y}(t) \qquad \ldots (5)$$

The final expression for the relative displacements z_i is obtained by combining eqs. (1), (3), (4), and (5):

$$z_i = - \sum_{r=1}^{n} \frac{1}{\omega_r} A_i^{(r)} \frac{\sum_{j=1}^{n} m_j A_j^{(r)}}{\sum_{j=1}^{n} m_j A_j^{(r)2}} \int_0^t \ddot{y}(\tau) e^{-\omega_r n_r (t-\tau)} \sin\omega_r(t-\tau) d\tau \qquad \ldots (6)$$

The response in the rth mode is:

$$z_i^{(r)} = - \frac{A_i^{(r)}}{\omega_r} \frac{\sum_{j=1}^{n} m_j A_j^{(r)}}{\sum_{j=1}^{n} m_j A_j^{(r)2}} \int_0^t \ddot{y}(\tau) e^{-\omega_r n_r (t-\tau)} \sin\omega_r(t-\tau) d\tau \qquad \ldots (7)$$

For simplicity, we introduce the notation:

$$\frac{1}{\bar{A}_r} = \frac{\sum_{j=1}^{n} m_j A_j^{(r)}}{\sum_{j=1}^{n} m_j A_j^{(r)2}} \qquad \ldots (8)$$

By definition, the maximum relative velocity response spectrum is given by:

$$S_v^{(r)} = \left[\int_0^t \ddot{y}(\tau) e^{-\omega_r n_r (t-\tau)} \sin\omega_r(t-\tau) d\tau \right]_{max} \qquad \ldots (9)$$

Hence the maximum response in each mode can be expressed in terms of the response spectrum:

$$[z_i^{(r)}]_{max} = - \frac{1}{\omega_r} \frac{A_i^{(r)}}{\bar{A}_r} S_v^{(r)} \qquad \ldots (10)$$

Other quantities of interest can be derived directly in terms of these relative displacements. For example, the base shear in each mode, $V_B^{(r)}$, is given by $k_n z_n^{(r)}$:

$$V_B^{(r)} = \frac{k_n A_n^{(r)}}{\omega_r \bar{A}_r} S_v^{(r)} \qquad \ldots (11)$$

The absolute acceleration of any mass $\ddot{x}_i$ can be found by differentiating eq. (6), noting that $\ddot{x} = \ddot{z} + \ddot{y}$. Making the usual approximations in differentiating the S_v integral, which assume small damping and earthquake-like excitation, we obtain:

$$\ddot{x}_i^{(r)} = \omega_r \frac{A_i^{(r)}}{\bar{A}_r} S_v^{(r)} \qquad \ldots (12)$$

Equations (10), (11), and (12) express the quantities of major interest for the earthquake structural response problem in terms of the natural frequencies and mode shapes, the damping in each mode, and the response spectrum values for each mode. We shall now show how each of these quantities can be evaluated for one particular situation.

Periods and Mode Shapes of the Alexander Building. The Alexander Building is a typical office building constructed in the 1920's. It is a rectangular 15 story building 60 ft. x 68 ft. x 197 ft. height, with a riveted structural steel frame with concrete fireproofing on the beams and interior columns and brick fireproofing on the exterior columns. The floors are reinforced concrete beam and slab construction, and the building has brick curtain walls and hollow tile partitions. The spread footing foundations rest on a sand having an allowable bearing value of 8000 lb/ft^2.

The results of extensive vibration tests are reported by Blume (9), and additional information is included in the original U. S. Coast and Geodetic report (8). Fig. 2 is a resonance curve for transverse vibrations parallel to the 68 ft. dimension of the building. Similar curves were obtained for the perpendicular direction, but since the most complete mode shape data was obtained for this 68 ft. (N9°W) direction, only this motion will be calculated. The data of Fig. 2 were obtained by forced oscillation tests using the U. S. Coast and Geodetic Building and Ground Vibrator (11). It should be noted that the amplitudes of the motions involved in these vibration tests were considerably smaller than would be expected to occur in a damaging earthquake. The main consequence of this is that somewhat higher values of damping would be expected for strong earthquake motions than were indicated by the vibration tests.

The damping values in the various modes were obtained from the resonance curves of Fig. 2 by measuring the width of the resonance peaks at a prescribed fraction of the peak amplitude. The period and damping data as used in the calculations are summarized in Table I. Also shown in Table I are the ratios between the natural periods in the first 3 modes as measured, and as they would be for a uniform pure shear structure, and a uniform cantilever pure bending structure. It is evident that the Alexander Building behaves approximately as a pure shear structure in this respect.

The mode shapes for the first three modes were experimentally determined during the forced vibration tests by measuring the displacement amplitudes at the various floors. In Fig. 3 this amplitude data (8) has been replotted, and smoothed mode shape curves have been drawn through the points, keeping in mind the theoretical shapes for a uniform shear type structure. Since the building as constructed has an approximately uniform distribution of mass and stiffness, it is assumed that local departures from a smooth mode shape curve are measurement errors, and the calculations have been based on the smoothed curves.

The mode shape coefficients $A_i^{(r)}$ as measured from the curves of Fig. 3 are given in Table II, along with some other pertinent information for the building. These $A_i^{(r)}$ coefficients are expressed in arbitrary length units since they appear in all of the expressions as ratios.

The San Francisco Earthquake of March 22, 1957. The ground acceleration measured in the N9°W direction in the basement of the Alexander Building during the San Francisco Earthquake of March 22, 1957 is shown in Fig. 4, along with the accelerations obtained in the same direction at the upper floor locations. The Alexander Building was located about 11 miles from the epicenter of this shock of Gutenberg-Richter Magnitude 5.3, which was the strongest earthquake felt in the San Francisco Bay area since 1906. In Fig. 5 are shown the maximum relative velocity response spectrum S_v curves as calculated from the ground acceleration-time records by the electric analog response spectrum analyzer of the California Institute of Technology (12). These response spectrum curves do not indicate anomolous behavior at periods corresponding to the natural periods of the building, and it may thus be concluded that the basement motions are essentially those of the earthquake ground motion (13).

For the calculations outlined above, values of S_v are required corresponding to the natural periods and damping of the first three modes. Using the period and damping values of Table I, and the spectrum curves of Fig. 5, the following values were arrived at: $S_v^{(1)} = 0.18$ ft/sec; $S_v^{(2)} = 0.17$ ft/sec; $S_v^{(3)} = 0.18$ ft/sec. These values of S_v were taken as average values in the neighborhood of the spectrum point.

Calculated Building Response. With the above information accelerations in each mode at each floor were calculated from eq. (12), with the results given in Table III. From these accelerations and the known masses, the lateral inertia forces acting at each floor can be determined. By summing these inertia forces from the top of the building the total shear force $V_i^{(r)}$ at each floor for each mode can be computed. These total shear values have been tabulated in Table III, and are plotted in Fig. 6. As a check on the base shear $V_B = V_{ir}$, equation (11) can be used, which gives: $V_B^{(1)} = 434,000$ lb, $V_B^{(2)} = 130,000$ lb, and $V_B^{(3)} \doteq 110,000$ lb. Comparing these values with those given in Table III which were calculated by summing the inertia forces, it will be seen that a discrepancy of some 30% exists in the first and third modes. This is not surprising in view of the difficulty in determining the mode shapes with the desired accuracy. The values given in Table III would be expected to be more accurate as they are not as dependent on mode shape accuracy. Also given in Table III and plotted in Fig. 7 are the equivalent static accelerations for each floor, obtained by dividing the total shear at each floor by the total weight of the building above that floor.

Comparison with Measured Building Acceleration. During the earthquake acceleration-time records were obtained from U. S. C. G. S. Strong-Motion Accelerographs installed at the 11th floor and the 16th floor (roof) of the Alexander Building. The acceleration records for the N9oW direction are shown in Fig. 4 along with the ground acceleration records. It is of interest to note that the second and third mode periods are much more prominent in the acceleration records than the fundamental.

The measured and calculated values of maximum acceleration are compared in Table IV. As would be expected, the assumption that the maximum value of the acceleration in each mode would be reached simultaneously gives a total maximum acceleration which is higher than that measured. It is of interest to note that the square root of the sum of the squares of the individual modes gives a maximum acceleration which is lower than the measured value. This RSS value would be the one which would be expected to occur most frequently if a large number of different earthquakes and structures could be considered, and if each earthquake could be represented as a series of random pulses (13), (14). The present calculations suggest that in individual particular cases the actual accelerations may be somewhat higher than this RSS value. Until further measurements are available, it will not be possible to judge the extent to which the present tests are typical.

The acceleration measurements in the Alexander Building also revealed an interesting condition of vertical vibrations. In Fig. 8 are shown the vertical accelerations as measured at the ground, the 11th, and the 16th floor. It is unlikely that the upper story motions are the result of the ground excitation shown in Fig. 8. It appears that the ground acceleration has been modified by the vertical vibration of the building, which has evidently been initiated by a large acceleration pulse occurring before the start of the accelerograph. It is surprising that a vertical ground acceleration of sufficient magnitude to excite such large vertical accelerations in the upper stories is not accompanied by more horizontal disturbance. The calculations of the present paper have been limited to lateral vibrations, but it is evident from Fig. 8 that vertical accelerations could also be of structural significance in some instances.

No general conclusions are to be expected from the present investigation, which has involved just one particular structure and one particular earthquake. It is believed, however, that conditions in this special case were typical of many earthquake excited structural structural situations. The analysis does clearly reveal the nature of the data that will be needed for such dynamic studies, and indicates that if such data is available the classical methods of dynamic analysis will lead to calculations of engineering significance. There are at present several buildings in San Francisco, Los Angeles, and Tokyo that are instrumented with sufficient completeness so that as future strong earthquakes occur in these regions additional studies of the kind outlined here can be carried out. Since strong earthquakes are likely to be rare, it would be very desirable that additional buildings in other seismic regions of the world be so instrumented, to increase the chance of acquiring useful data.

Acknowledgements. Thanks are expressed to Mr. W. K. Cloud, Chief of the Seismological Field Survey, United States Coast and Geodetic Survey, and his staff, for their excellent work in carrying out the strong-motion accelerograph program, and to Mr. J. A. Blume, Consulting Engineer, for information and data on his vibration tests of the Alexander Building.

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Nomenclature

m_i	=	concentrated mass of the individual floor.
k_i	=	equivalent linear spring constant, lb/in., for lateral deflections of individual floors.
c_i	=	equivalent viscous damping coefficient. c_{io} relative to ground; c_{ii} relative to next floor.
c_c	=	critical damping in any mode of vibration.
n_r	=	$(c/c_c)_r$ = fraction of critical damping in the rth mode of vibration.
n	=	number of degrees of freedom; number of individual floors.
x_i	=	absolute horizontal displacement of the mass m_i.
$y(t)$	=	absolute horizontal displacement of the ground, a function of time.
$\ddot{y}(t)$	=	absolute horizontal acceleration of ground.
$z_i = x_i - y(t)$	=	relative lateral displacement of the mass m_i with respect to ground.
$A_i^{(r)}$	=	amplitude of the mass m_i in the rth mode of vibration.
ξ_i	=	normal coordinate defined by eq. (1).
T_r	=	natural period of the rth mode, sec.
ω_r	=	natural frequency of the rth mode, rad/sec.; $\omega_r = 2\pi/T_r$.
F_j	=	external applied horizontal force on the mass m_j.
f_r	=	generalized force for the rth mode, defined by eq. (4).
$\bar{A}_r$	=	displacement coefficient defined by eq. (8).
S_v	=	maximum relative velocity response spectrum, ft/sec., defined by eq. (9).
$V_i^{(r)}$	=	lateral shear force at m_i in the rth mode, lb.
$V_B^{(r)} = V_{15}^{(r)}$	=	base shear in the rth mode, lb.

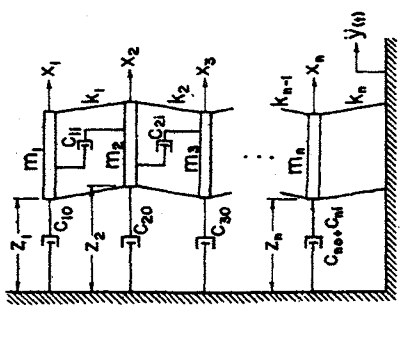

FIG. 1. MULTI-STORY BUILDING WITH HORIZONTAL GROUND MOTION

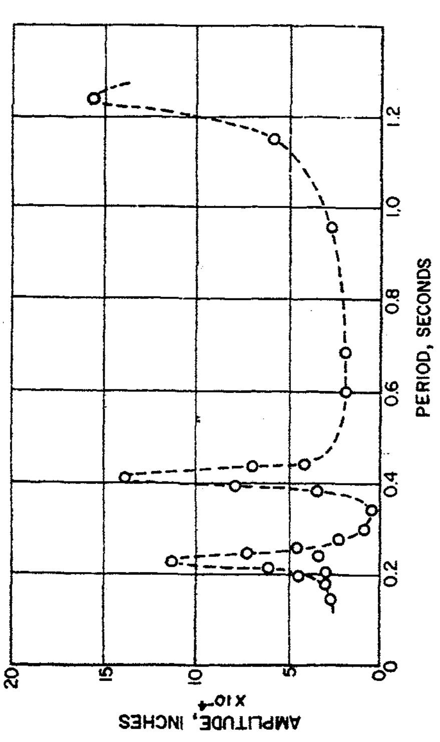

FIG. 2. RESONANCE TEST OF THE ALEXANDER BUILDING

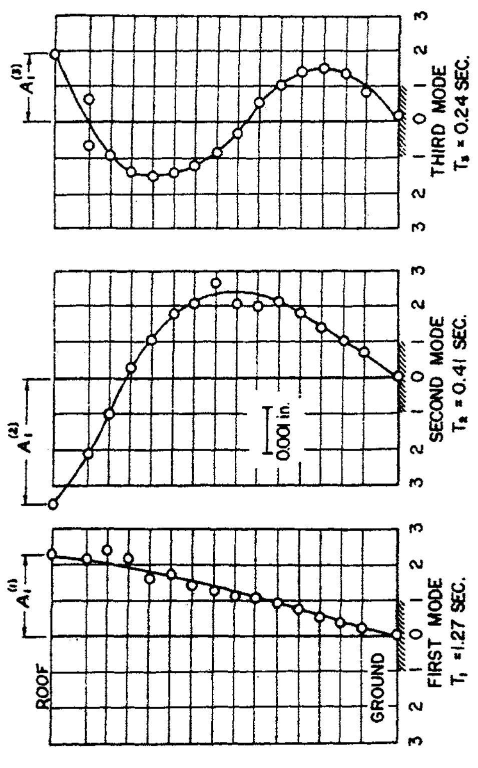

FIG. 3. MODE SHAPES FOR ALEXANDER BUILDING

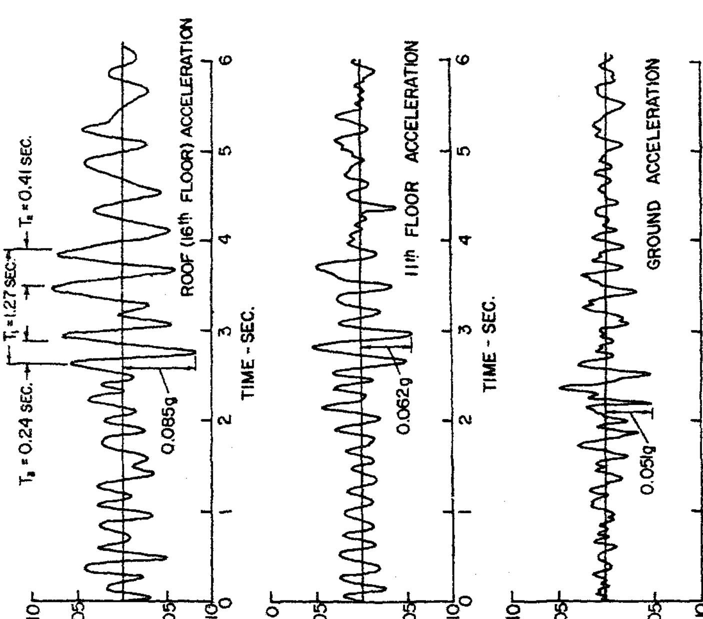

FIG. 4. HORIZONTAL ACCELERATIONS OF ALEXANDER BUILDING DURING EARTHQUAKE OF MARCH 22nd, 1957

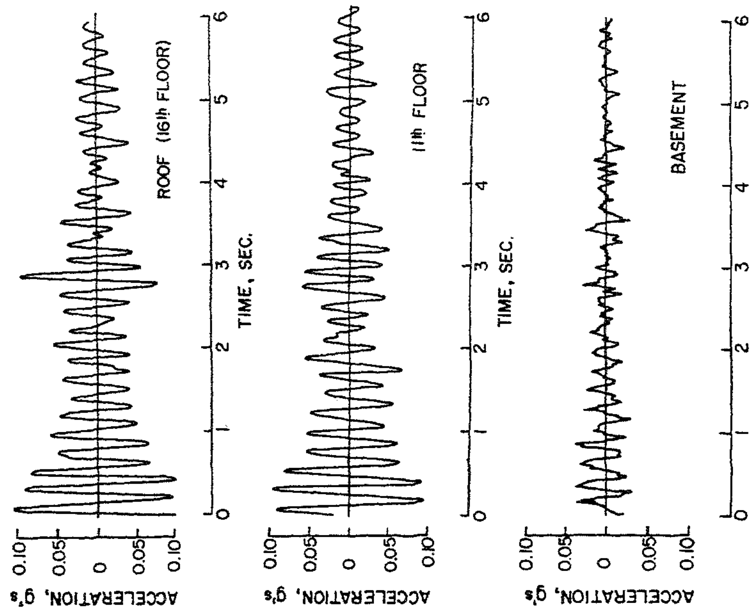

FIG. 8. VERTICAL ACCELERATIONS IN ALEXANDER BUILDING DURING EARTHQUAKE OF MARCH 22nd, 1957

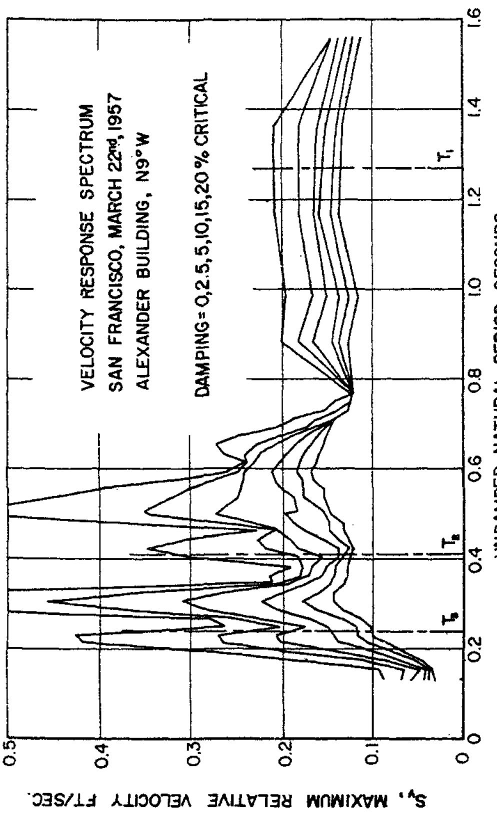

FIG. 5. RESPONSE SPECTRUM CURVES - ALEXANDER BUILDING, EARTHQUAKE OF MARCH 22nd, 1957

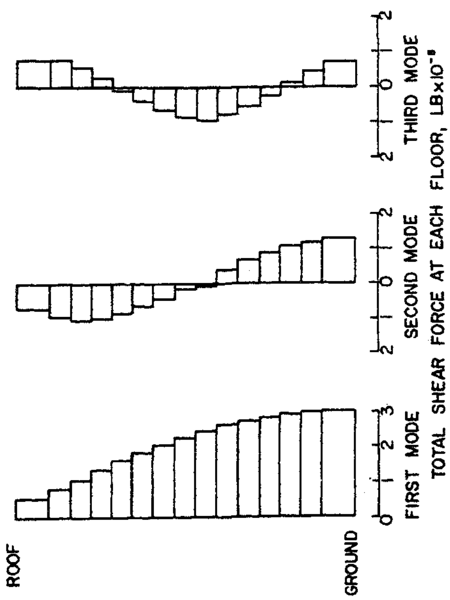

FIG. 6. TOTAL SHEAR FORCE - ALEXANDER BUILDING

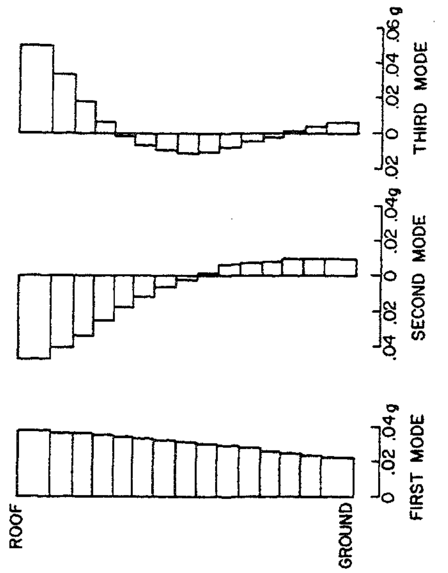

FIG. 7. EQUIVALENT STATIC ACCELERATION - ALEXANDER BUILDING

Table I. Dynamic Properties of the Alexander Building. N9°W Direction

Mode	Natural Period (sec)	Damping	Period Ratio	Shear Period Ratio	Bending Period Ratio
1	1.27	0.02	1	1	1
2	0.41	0.04	3.1	3	6.28
3	0.24	0.04	5.3	5	17.6

Table II. Alexander Building Properties, N9°W

Mass No.	Story Height (ft)	Weight (lb) x 10^6	Stiffness k (lb/in) x 10^6	Mode Amplitude Numbers $A_j^{(1)}$	$A_j^{(2)}$	$A_j^{(3)}$
1	20	1.550	4.9	2.22	-3.48	1.90
2	12	0.749	6.8	2.10	-2.10	0
3	12	0.809	8.6	2.02	-1.00	-0.95
4	12	0.850	8.7	1.92	0.30	-1.40
5	12	0.869	8.1	1.80	1.08	-1.50
6	12	0.841	8.1	1.68	1.80	-1.40
7	12	0.841	9.3	1.50	2.10	-1.20
8	12	0.847	9.3	1.38	2.35	-0.85
9	12	0.856	10.6	1.20	2.45	-0.30
10	12	0.865	10.6	1.05	2.32	0.55
11	12	0.863	12.6	0.90	2.10	1.05
12	12	0.886	12.6	0.75	1.80	1.40
13	12	0.918	17.2	0.55	1.40	1.50
14	12	0.931	14.2	0.38	1.00	1.35
15	20	1.173	8.2	0.20	0.70	0.90

$$\frac{1}{\bar{A}_1} = \frac{\sum_{j=1}^{n} m_j A_j^{(1)}}{\sum_{j=1}^{n} m_j A_j^{(1)2}} = 0.604 \; ; \; \frac{1}{\bar{A}_2} = 0.169 \; ; \; \frac{1}{\bar{A}_3} = 0.181$$

Table III. Calculated Accelerations and Shears. Alexander Building

Mass No.	First Mode			Second Mode			Third Mode		
	$\ddot{x}_i^{(1)}$ (g's)	$V_i^{(1)}$ (LB)	$(Eq.g)_1$	$\ddot{x}_i^{(2)}$ (g's)	$V_i^{(2)}$ (LB)	$(Eq.g)_2$	$\ddot{x}_i^{(3)}$ (g's)	$V_i^{(3)}$ (LB)	$(Eq.g)_3$
1	.037	57,400	.037	-.048	-73,700	-.048	.050	78,000	.050
2	.035	83,600	.036	-.029	-95,100	-.042	0	78,000	.034
3	.034	110,800	.036	-.014	-106,100	-.034	-.025	57,700	.019
4	.032	138,000	.035	.004	-102,600	-.026	-.037	26,200	.007
5	.030	164,100	.034	.015	-89,800	-.019	-.040	-8,300	-.002
6	.028	187,700	.033	.025	-69,100	-.012	-.037	-39,500	-.007
7	.025	208,800	.032	.029	-45,000	-.007	-.032	-66,300	-.010
8	.023	228,300	.031	.032	-17,800	-.002	-.023	-85,400	-.012
9	.020	245,400	.030	.033	10,800	.001	-.008	-92,200	-.011
10	.018	260,600	.029	.032	38,200	.004	.015	-79,600	-.009
11	.015	273,600	.028	.029	63,000	.006	.028	-55,600	-.006
12	.013	284,700	.026	.025	84,800	.008	.037	-22,700	-.002
13	.009	293,100	.025	.019	102,300	.009	.040	13,800	.001
14	.006	299,000	.024	.014	115,000	.009	.036	47,200	.004
15	.003	302,900	.022	.010	126,200	.009	.024	75,200	.005

Table IV. Comparison of Measured and Calculated Maximum Accelerations

	Max. Acceleration, g's	
	11th Floor	16th Floor
First Mode	.028	.037
Second Mode	.024	.047
Third Mode	.037	.050
Abs. Sum 3 Modes	.089	.134
R. S. S. 3 Modes	.052	.078
Measured	.062	.085

DISCUSSION

R. N. Joshi, S. B. Joshi & Co. Private Ltd., India:

A comparison of the Resonance Curve of Alexander Building in Fig. 2 and the Velocity Response Spectrum obtained from the ground acceleration - time relation recorded by the accelerograph in the basement of the building, in Fig. 5, would suggest that the latter is partly influenced by the vibration characteristics of the building, since it shows peaks at periods corresponding to the second & third modes, and is partly independent of the vibration characteristics since it also shows peaks for periods which are not related to the mode periods.

It is not understood how the calculated accelerations for the second & third modes are higher than that for the fundamental mode.

M. P. White, University of Massachusetts, U. S. A.:

The author has assumed damping forces to be proportional to displacements. Has we investigated damping dependent on the relative motions between floors?

D. E. Hudson:

Since the equivalent viscous damping was directly measured in each mode of vibration, it was not necessary to assume any particular type of damping. In fact, the relationship between the measured damping values in the three modes was different than would be obtained by either absolute or relative viscous damping acting separately. The basic assumption in the calculations is that the damped mode shapes are the same as the undamped mode shapes. For the relatively small damping present during the tests, this assumption must be close to the truth. For large damping such as might be associated with pronounced plastic deformations, the influence of damping on mode shape would have to be examined in more detail.