Study on Unstationary Vibration of Building Structure with Plastic Deformation of Substructure

by Takuji Kobori* and Ryoichiro Minai**

Abstract: This paper presents a general analytical method to obtain transient responses of a "shear" structural system which consists of main elastic, continuous body at the upper part with arbitrary mass, rigidity, and damping distributions and, at the bottom, a poly-linear lumped system with dampings, subjected to arbitrary ground excitations. The solution of the equations of motion of this system are obtained by successive connection procedures of general solutions in corresponding linear branches. General solution in each linear branch is generally expressed in terms of infinite non-orthogonal complex eigen-function expansion series. In the special case where convergency of the solution corresponding to nonhomogeneous terms of boundary condition becomes poor in the neighbourhood of the boundary, the general solution can be expressed in two parts one of which is the eigen-function expansion series and other is a boundary homogenizing function. Numerical calculations are carried out and applications of these nondimensional responses to dynamic, ultimate aseismic design are considered.

Introduction: Under the action of a violent earthquake, structural system, above all, substructure including soil would not behave elastically but elasto-plastically. It was already made clear that energy dissipation due to hysteresis loop and nonlinearity of structural system extraordinarily restrained stress and strain responses of the structure, and the most destructive element of seismic waves had the period near to the natural period of the structural system under the condition of constant ground velocity or constant ground displacement[1], and these facts would make the so-called dynamic ultimate aseismic design possible.

To establish this design method, it is important to study more in details quantitative and qualitative properties of earthquake responses of structures in ultimate state as well as statistical characteristics of seismic waves depending on various ground conditions, and relative displacement limits of various structures. For this purpose, earthquake responses of the coupled system consisted of upper elastic, continuous, mainstructure and lower elasto-plastic lumped substructure are analized as follows.

Fundamental Differential System: It is assumed that the upper mainstructure is an elastic, continuous shear body which has arbitrarily and independently distributed inertia, rigidity, external damping and internal damping, and the lower substructure is a lumped system which has translational and rotational inertias, poly-linear spring characteristics and viscous dampings.

Transforming fundamental differential system with respect to fixed co-ordinates to moving co-ordinates and nondimensionizing this, following fundamental partial differential system corresponding to any branch of poly-linear system is obtained.

$$\left.\begin{aligned} {}_pL(\eta) &= \left[\frac{\partial}{\partial\xi}\left\{g(\xi)+\gamma_i d_i(\xi)\frac{\partial}{\partial\tau}\right\}\left\{\frac{\partial}{\partial\xi}-\left|\frac{\partial}{\partial\xi}\right|_1\right\}-a(\xi)\frac{\partial^2}{\partial\tau^2}-\gamma_e d_e(\xi)\frac{\partial}{\partial\tau}\right]\eta=\left\{a(\xi)\frac{\partial^2}{\partial\tau^2}+\gamma_e d_e(\xi)\frac{\partial}{\partial\tau}\right\}\bar{\ddot{Y}}/h \\ {}_pW_1(\eta) &= \left[\int_0^1\left\{g(\xi)+\gamma_i d_i(\xi)\frac{\partial}{\partial\tau}\right\}\left\{\frac{\partial}{\partial\xi}-\left|\frac{\partial}{\partial\xi}\right|_1\right\}d\xi-\left\{\varkappa_R+\epsilon_R\frac{d}{d\tau}+\mu_R\frac{d^2}{d\tau^2}\right\}\left|\frac{\partial}{\partial\xi}\right|_1\right]\eta=0 \end{aligned}\right\} \quad (1)$$

*Assistant Professor of Architecture, Engineering, Kyoto University, Japan
**Assistant of Architecture, Engineering, Kyoto University, Japan

$$\left.\begin{aligned}
&{}_pW_2(\eta)=[\{g(0)+\delta_i d_i(0)\tfrac{d}{d\tau}\}\{|\tfrac{\partial}{\partial\xi}|_0-|\tfrac{\partial}{\partial\xi}|_1\}-\{\varkappa_T+\epsilon_T\tfrac{d}{d\tau}+\mu_T\tfrac{d^2}{d\tau^2}\}|1|_0]\eta=\mu_T\tfrac{d^2}{d\tau^2}(\bar{Y}/h)\\
&{}_{py}W_1(\eta)=[\int_0^1\{g(\xi)+\delta_i d_i(\xi)\tfrac{\partial}{\partial\tau}\}\{\tfrac{\partial}{\partial\xi}-|\tfrac{\partial}{\partial\xi}|_1\}d\xi-\{\epsilon_R\tfrac{d}{d\tau}+\mu_R\tfrac{d^2}{d\tau^2}\}|\tfrac{\partial}{\partial\xi}|_1]\eta=\pm\beta_R\\
&{}_{py}W_2(\eta)=[\{g(0)+\delta_i d_i(0)\tfrac{d}{d\tau}\}\{|\tfrac{\partial}{\partial\xi}|_0-|\tfrac{\partial}{\partial\xi}|_1\}-\{\epsilon_T\tfrac{d}{d\tau}+\mu_T\tfrac{d^2}{d\tau^2}\}|1|_0]\eta=\mu_T\tfrac{d^2}{d\tau^2}(\bar{Y}/h)\pm\beta_T
\end{aligned}\right]\quad(2)$$

where, $\xi=x/h$, $\eta=(Y-\bar{Y})/h$, $\tau=(1/h)\sqrt{G_0/\rho A_0}\cdot t$, $g(\xi)=\bar{g}(h\xi)$, $a(\xi)=\bar{a}(h\xi)$, $d_i(\xi)=\bar{d}_i(h\xi)$
$d_e=\bar{d}_e(h\xi)$, ... (3)

$$\left.\begin{aligned}
&\delta_i=D_{i0}/h\sqrt{\rho A_0 G_0},\quad \delta_e=hD_{e0}/\sqrt{\rho A_0 G_0},\quad \varkappa_R=K_R/hG_0,\quad \varkappa_T=hK_T/G_0,\\
&\epsilon_R=E_R/h^2\sqrt{\rho A_0 G_0},\quad \epsilon_T=E_T/\sqrt{\rho A_0 G_0},\quad \mu_R=M_R/h^3\rho A_0,\quad \mu_T=M_T/h\rho A_0,\\
&\beta_R=|M_{yield}|/hG_0,\quad \beta_T=|Q_{yield}|/G_0
\end{aligned}\right]\quad(4)$$

and where, the nondimensional overturning moment is expressed by

$$m_0=M_0/hG_0=-\int_0^1\{g(\xi)+\delta_i d_i(\xi)\tfrac{\partial}{\partial\tau}\}\{\tfrac{\partial\eta}{\partial\xi}-|\tfrac{\partial\eta}{\partial\xi}|_1\}d\xi\quad(5)$$

Operating Laplace transformation, (1), (2) are reduced to the following fundamental ordinary differential system.

$$\left.\begin{aligned}
&L(\phi)=[\tfrac{d}{d\xi}\{g(\xi)+s\delta_i d_i(\xi)\}\{\tfrac{d}{d\xi}-|\tfrac{d}{d\xi}|_1\}-s^2a(\xi)-s\delta_e d_e(\xi)]\phi=F(\xi,s)\\
&W_1(\phi)=[\int_0^1\{g(\xi)+s\delta_i d_i(\xi)\}\{\tfrac{d}{d\xi}-|\tfrac{d}{d\xi}|_1\}d\xi-\{\varkappa_R+s\epsilon_R+s^2\mu_R\}|\tfrac{d}{d\xi}|_1]\phi=F_1(s)\\
&W_2(\phi)=[\{g(0)+s\delta_i d_i(0)\}\{|\tfrac{d}{d\xi}|_0-|\tfrac{d}{d\xi}|_1\}-\{\varkappa_T+s\epsilon_T+s^2\mu_T\}|1|_0]\phi=F_2(s)
\end{aligned}\right]\quad(6)$$

$$\left.\begin{aligned}
&{}_yW_1(\phi)=[\int_0^1\{g(\xi)+s\delta_i d_i(\xi)\}\{\tfrac{d}{d\xi}-|\tfrac{d}{d\xi}|_1\}d\xi-\{s\epsilon_R+s^2\mu_R\}|\tfrac{d}{d\xi}|_1]\phi={}_yF_1(s)\\
&{}_yW_2(\phi)=[\{g(0)+s\delta_i d_i(0)\}\{|\tfrac{d}{d\xi}|_0-|\tfrac{d}{d\xi}|_1\}-\{s\epsilon_T+s^2\mu_T\}|1|_0]\phi={}_yF_2(s)
\end{aligned}\right]\quad(7)$$

where,

$$\left.\begin{aligned}
F(\xi,s)=&\{s^2a(\xi)+s\delta_e d_e(\xi)\}F(s)-\{sa(\xi)+\delta_e d_e(\xi)\}|\bar{Y}/h+\eta|_{\tau=0}\\
&-a(\xi)|\tfrac{1}{h}\tfrac{d\bar{Y}}{d\tau}+\tfrac{\partial\eta}{\partial\tau}|_{\tau=0}+\tfrac{d}{d\xi}[\delta_i d_i(\xi)|\tfrac{\partial\eta}{\partial\xi}-(\tfrac{\partial\eta}{\partial\xi})_1|_{\tau=0}]\\
F_1(s)=&\int_0^1\delta_i d_i(\xi)|\tfrac{\partial\eta}{\partial\xi}-(\tfrac{\partial\eta}{\partial\xi})_1|_{\tau=0}d\xi-(s\mu_R+\epsilon_R)|(\tfrac{\partial\eta}{\partial\xi})_1|_{\tau=0}-\mu_R|(\tfrac{\partial^2\eta}{\partial\xi\partial\tau})_1|_{\tau=0}\\
F_2(s)=&s^2\mu_T F(s)-s\mu_T|\tfrac{\bar{Y}}{h}+(\eta)_0|_{\tau=0}-\mu_T|\tfrac{1}{h}\tfrac{d\bar{Y}}{d\tau}+(\tfrac{\partial\eta}{\partial\tau})_0|_{\tau=0}-\epsilon_T|(\eta)_0|_{\tau=0}+\delta_i d_i(0)|(\tfrac{\partial\eta}{\partial\xi})_0-(\tfrac{\partial\eta}{\partial\xi})_1|_{\tau=0}
\end{aligned}\right]\quad(8)$$

$$\left.\begin{aligned}
&{}_yF_1(s)=\pm\tfrac{\beta_R}{s}+F_1(s),\quad {}_yF_2(s)=\pm\tfrac{\beta_T}{s}+F_2(s),\\
&\phi=\int_0^\infty e^{-s\tau}\eta\,d\tau,\quad F(s)=\int_0^\infty e^{-s\tau}\tfrac{\bar{Y}}{h}d\tau,
\end{aligned}\right].\quad(9)$$

where, subindeces p and y mean the partial differential system and the perfectly yielding of lower substructure, respectively.

Solution of Laplace Transformed Differential System: Four differential systems are possible to exist in the transformed space, however, as the most general case, $\{L;W_1,W_2\}$ is considered herein. Green's functions for this linear system can be defined as in the usual way.[2),3)]

I $G(\xi,\zeta;s)$; $L(G)=0$ for $\xi\neq\zeta$, $W_i(G)=0$ $i=1,2$

$$\lim_{\varepsilon\to0}\{|G_\xi(\xi,\zeta;s)|_{\zeta+\varepsilon}-|G_\xi(\xi,\zeta;s)|_{\zeta-\varepsilon}\}=-1/\{g(\zeta)+s\delta_i d_i(\zeta)\}$$

II $G_i(\xi;s)$; $L(G_i)=0$, $W_i(G_j)=-\delta_{ij}$ $i,j=1,2$

Then, G and G_i can be formulated as follows, by using two independent solutions ϕ_1, ϕ_2 which are easily obtainable from those of equation without boundary operator by means of parameter variation.

$$G(\xi,\zeta;s)=N(\xi,\zeta;s)/[\{g(\zeta)+s\delta_i d_i(\zeta)\}\Delta_w(\zeta,s)\Delta_c(s)\quad(10)$$

$$N(\xi,\zeta;s) = \begin{vmatrix} \phi_1(\xi,s) & \phi_2(\xi,s) & \bar{N}(\xi,\zeta;s) \\ W_1(\phi_1) & W_1(\phi_2) & W_1(\bar{N}) \\ W_2(\phi_1) & W_2(\phi_2) & W_2(\bar{N}) \end{vmatrix}, \quad \bar{N} = \frac{1}{2}\,sgn.(\zeta-\xi)\begin{vmatrix} \phi_1(\xi,s) & \phi_2(\xi,s) \\ \phi_1(\zeta,s) & \phi_2(\zeta,s) \end{vmatrix}$$

$$N^{+}(\xi,\zeta;s) = \begin{vmatrix} \phi_1(\xi,s) & \phi_2(\xi,s) \\ W_1(\phi_1) & W_1(\phi_2) \end{vmatrix}\begin{vmatrix} W_{2\ell}(\phi_1) & W_{2\ell}(\phi_2) \\ \phi_1(\zeta,s) & \phi_2(\zeta,s) \end{vmatrix} - \begin{vmatrix} \phi_1(\xi,s) & \phi_2(\xi,s) \\ W_2(\phi_1) & W_2(\phi_2) \end{vmatrix}\begin{vmatrix} W_{1\ell}(\phi_1) & W_{1\ell}(\phi_2) \\ \phi_1(\zeta,s) & \phi_2(\zeta,s) \end{vmatrix} \qquad \xi>\zeta$$

$$N^{-}(\xi,\zeta;s) = \begin{vmatrix} \phi_1(\xi,s) & \phi_2(\xi,s) \\ W_2(\phi_1) & W_2(\phi_2) \end{vmatrix}\begin{vmatrix} W_{1u}(\phi_1) & W_{1u}(\phi_2) \\ \phi_1(\zeta,s) & \phi_2(\zeta,s) \end{vmatrix} - \begin{vmatrix} \phi_1(\xi,s) & \phi_2(\xi,s) \\ W_1(\phi_1) & W_1(\phi_2) \end{vmatrix}\begin{vmatrix} W_{2u}(\phi_1) & W_{2u}(\phi_2) \\ \phi_1(\zeta,s) & \phi_2(\zeta,s) \end{vmatrix} \qquad \xi<\zeta$$

... (11)

where, $W_i = W_{i\ell} + W_{iu}$, u : operator for upper boundary and $\xi>\zeta$
ℓ : operator for lower boundary and $\xi<\zeta$

$$\Delta_\omega(\zeta,s) = \begin{vmatrix} \phi_1'(\zeta,s) & \phi_2'(\zeta,s) \\ \phi_1(\zeta,s) & \phi_2(\zeta,s) \end{vmatrix}, \quad \Delta_c(s) = \begin{vmatrix} W_1(\phi_1) & W_1(\phi_2) \\ W_2(\phi_1) & W_2(\phi_2) \end{vmatrix} \tag{12}$$

$$G_1(\xi,s) = -\frac{1}{\Delta_c(s)}\begin{vmatrix} \phi_1(\xi,s) & \phi_2(\xi,s) \\ W_2(\phi_1) & W_2(\phi_2) \end{vmatrix} \tag{13}$$

$$G_2(\xi,s) = \frac{1}{\Delta_c(s)}\begin{vmatrix} \phi_1(\xi,s) & \phi_2(\xi,s) \\ W_1(\phi_1) & W_1(\phi_2) \end{vmatrix} \tag{14}$$

Using these Green's functions, the solution of nonhomogeneous ordinary differential system (6) can be expressed as follows.

$$\phi = -\int_0^1 G(\xi,\zeta;s)\chi(\zeta,s)\,d\zeta - \sum_{i=1}^{2} G_i(\xi;s)F_i(s) \tag{15}$$

where, $\chi(\zeta,s)$ is the modified nonhomogeneous term of the differential equation which satisfies the following second kind Volterra-type integral equation.

$$\chi(\xi,s) - \int_\xi^1 K(\xi,\zeta;s)\chi(\zeta,s)\,d\zeta = F(\xi,s)$$

$$K(\xi,\zeta;s) = \frac{g'(\xi) + s\gamma_i d_i'(\xi)}{\{g(\zeta) + s\gamma_i d_i(\zeta)\}\Delta_\omega(\zeta,s)}\cdot\begin{vmatrix} \phi_1'(1,s) & \phi_2'(1,s) \\ \phi_1(\zeta,s) & \phi_2(\zeta,s) \end{vmatrix}$$

i.e. $$\chi(\xi,s) = F(\xi,s) - \int_\xi^1 \Gamma(\xi,\zeta;s)F(\zeta,s)\,d\zeta \tag{16}$$

$$\Gamma(\xi,\zeta;s) = -\sum_{i=1}^{\infty} K^{\circledast i}, \quad \varphi \circledast \psi = \int_\xi^1 \varphi(\xi,\tau)\psi(\tau,\zeta)\,d\tau$$

Substituting (16) into (15) and exchanging order of integration process the solution can be expressed in the transformed space as follows.

$$\phi = -\int_0^1 G(\xi,\zeta;s)F(\zeta,s)\,d\zeta - \sum_{i=1}^{2} G_i(\xi,s)F_i(s) + \int_0^1 F(\lambda,s)\int_0^\lambda G(\xi,\zeta;s)\Gamma(\zeta,\lambda;s)\,d\zeta\,d\lambda \tag{17}$$

The third term in the above equation is produced due to existence of boundary operation in the differential operator $L(\phi)$. When rotational terms do not exist, or when $g'(\zeta) \equiv d_i'(\zeta) \equiv 0$ are valid even if rotational terms exist the boundary operator vanishes and the third term may be eliminated. Moreover, in these cases, symmetricity of Green's function $G(\xi,\zeta;s)$ is valid because following relations are verified.

$$\begin{vmatrix} W_{1l}(\phi_1) & W_{1l}(\phi_2) \\ W_{2l}(\phi_1) & W_{2l}(\phi_2) \end{vmatrix} = \begin{vmatrix} W_{1u}(\phi_1) & W_{1u}(\phi_2) \\ W_{2u}(\phi_1) & W_{2u}(\phi_2) \end{vmatrix} \quad \text{for symmetricity of numerator} \tag{18}$$

$$\{g(\xi) + s\gamma_i d_i(\xi)\}\Delta_w(\xi) = C(s) \quad \text{for denominator} \tag{19}$$

In the case where symmetricity of Green's function is valid corresponding equations can be considerably simplified. (see Appendix I)

<u>Eigen-value, Eigen-function and Modified Orthogonality:</u> In the homogeneous differential system $\{L; W_1, W_2\}$ eigen-value and eigen-function can be defined in complex plane, assuming s be complex parameter. $\{S_\nu\}$ are complex eigen-value series which are successive roots of characteristic equation $\Delta_c(S)=0$. Corresponding to $\{S_\nu\}$, complex eigen-function series $\{\varphi_\nu^c(\xi)\}$ which content homogeneous differential system and are not identically zero, can be obtained as follows.

$$\varphi_\nu^c(\xi) = \begin{vmatrix} \phi_1(\xi,s) & \phi_2(\xi,s) \\ W_1(\phi_1) & W_1(\phi_2) \end{vmatrix}_{s=s_\nu} = \frac{1}{C(S_\nu)} \begin{vmatrix} \phi_1(\xi,s) & \phi_2(\xi,s) \\ W_2(\phi_1) & W_2(\phi_2) \end{vmatrix}_{s=s_\nu} \tag{20}$$

$$C(S_\nu) = \left|\frac{W_2(\phi_1)}{W_1(\phi_1)}\right|_{s=s_\nu} = \left|\frac{W_2(\phi_2)}{W_1(\phi_2)}\right|_{s=s_\nu} \tag{21}$$

As all coefficients of the differential system are real numbers, there exist conjugate sets of complex eigen-value $\{S_\nu, \bar{S}_\nu\}$ and corresponding complex eigen-function $\{\varphi_\nu^c, \bar{\varphi}_\nu^c\}$. Following relation between conjugate eigen-values and corresponding eigen-functions can easily be calculated.

$$s_\nu, \bar{s}_\nu = \alpha_\nu \pm i\beta_\nu = -\frac{D_\nu}{2A_\nu} \pm i\sqrt{\frac{G_\nu}{A_\nu} - \left(\frac{D_\nu}{2A_\nu}\right)^2} \quad \dots \tag{22}$$

where,

$$\left.\begin{aligned} A_\nu &= \int_0^1 a(\xi)|\varphi_\nu^c|^2 d\xi + \mu_R|\varphi_\nu'^c|_1^2 + \mu_T|\varphi_\nu^c|_0^2 > 0 \\ D_\nu &= \gamma_i\int_0^1 d_i(\xi)|\varphi_\nu'^c - (\varphi_\nu'^c)_1|^2 d\xi + \gamma_e\int_0^1 d_e(\xi)|\varphi_\nu^c|^2 d\xi + \epsilon_R|\varphi_\nu'^c|_1^2 + \epsilon_T|\varphi_\nu^c|_0^2 \geqq 0 \\ G_\nu &= \int_0^1 g(\xi)|\varphi_\nu'^c - (\varphi_\nu'^c)_1|^2 d\xi + \varkappa_R|\varphi_\nu'^c|_1^2 + \varkappa_T|\varphi_\nu^c|_0^2 > 0 \end{aligned}\right\} \tag{23}$$

Eigen-values can be classified by sign of following quantity.

$$B = \frac{G_\nu}{A_\nu} - \left(\frac{D_\nu}{2A_\nu}\right)^2 \tag{24}$$

$D = 0$ no damping $\Re(S_\nu, \bar{S}_\nu) = 0$

$B > 0$ conjugate imaginary eigen-values-----periodic

$D > 0$ with damping $\Re(S_\nu, \bar{S}_\nu) = -D_\nu/2A_\nu < 0$

$B > 0$ conjugate complex eigen-values----damped, periodic

$B = 0$ one real negative eigen-value ----damped, non-periodic

$B < 0$ two real negative eigen-values----damped, non-periodic

Cardinal numbers $\{\nu\}$ can be determined by sequence of equivalent absolute values of eigen-values which are arranged from smallest one.

$$|S_\nu|_e = |\bar{S}_\nu|_e = \sqrt{\alpha_\nu^2 + \beta_\nu^2} = \sqrt{G_\nu/A_\nu} \tag{25}$$

Modified orthogonality[4),5)] of the eigen-functions of the above differential system is defined as follows. Defining operation between φ_i^c and φ_j^c as,

$$[\varphi_i^c, \varphi_j^c] = \int_0^1 a(\xi)\varphi_i^c\varphi_j^c d\xi + \mu_R(\varphi_i'^c)_1(\varphi_j'^c)_1 + \mu_T(\varphi_i^c)_0(\varphi_j^c)_0 + \frac{1}{S_i+S_j} \times$$
$$\times\left[\gamma_i\int_0^1 d_i(\xi)\{\varphi_i'^c - (\varphi_i'^c)_1\}\{\varphi_j'^c - (\varphi_j'^c)_1\}d\xi + \gamma_e\int_0^1 d_e(\xi)\varphi_i^c\varphi_j^c d\xi + \epsilon_R(\varphi_i'^c)_1(\varphi_j'^c)_1 + \epsilon_T(\varphi_i^c)_0(\varphi_j^c)_0\right] \tag{26}$$

Then, modified orthogonality is defined by the following relation.

$$\text{if,} \quad s_i^2 - s_j^2 \neq 0 , \qquad [\varphi_i^c, \varphi_j^c] = 0 \tag{27}$$

This relation is generally nonlinear with respect to i or j because of existence of the second term which contains s_i and s_j, therefore, a formal expansion of an arbitrary function can not be easily obtainable. The normalized modified orthogonal eigen-function series can be defined as follows.

$$[n\varphi_i^c, n\varphi_j^c] = \delta_{ij} \tag{28}$$

This system can be obtained from any modified orthogonal system which is not yet normalized.

$$\{n\varphi_\nu^c\} = \{\varphi_\nu^c / [\varphi_\nu^c, \varphi_\nu^c]^{\frac{1}{2}}\} \tag{29}$$

General Solution of Original Differential System: Inversely transforming the solution of transformed differential system (17) into the original space, general solution in the original differential system is expressed by following formula.

$$\eta = \int_0^1 [g(\xi,\zeta;\tau) * f(\zeta,\tau)] d\zeta - \sum_{i=1}^{2} [g_i(\xi,\tau) * f_i(\tau)] + \int_0^1 \int_0^\lambda [f(\lambda,\tau) * \delta(\xi,\zeta,\lambda;\tau)] d\zeta d\lambda \tag{30}$$

where,

$$\begin{aligned} & f(\xi,\tau) \supset F(\xi,s) \qquad , \quad f_i(\tau) \supset F_i(s) \\ & g(\xi,\zeta;\tau) \supset G(\xi,\zeta;s), \quad g_i(\xi,\tau) \supset G_i(\xi,s) \\ & \delta(\xi,\zeta,\lambda;\tau) \supset G(\xi,\zeta;s)\Gamma(\zeta,\lambda;s) \end{aligned} \tag{31}$$

Both hand side terms in (31) are generally related by the following inverse formula.

$$\psi = \lim_{\beta\to\infty} \frac{1}{2\pi i} \int_{\gamma - i\beta}^{\gamma + i\beta} e^{s\tau} \Psi \, ds \qquad \text{where,} \quad \psi \supset \Psi \tag{32}$$

$f(\xi,\tau)$ and $f_i(\tau)$, however, can be obtained formally from (8), (9).

$$\begin{aligned} f(\xi,\tau) = {} & a(\xi)\ddot{F}(\tau) + \gamma_e d_e(\xi)\dot{F}(\tau) + \mathcal{E}\cdot\gamma_i [d_i(\xi)\{m'(\xi) - m'(1)\}]' \\ & - [\{\tfrac{\partial}{\partial\tau} + (1)_o\} a(\xi) + \mathcal{E}\cdot\gamma_e d_e(\xi)] m(\xi) - \mathcal{E}\cdot a(\xi) n(\xi) \end{aligned} \quad \ldots \tag{33}$$

$$\begin{aligned} & f_1(\tau) = \mathcal{E}\cdot\gamma_i \int_0^1 d_i(\xi)\{m'(\xi) - m'(1)\} d\xi - [\{\tfrac{\partial}{\partial\tau} + (1)_o\}\mu_R + \mathcal{E}\cdot\epsilon_R] m'(1) - \mathcal{E}\cdot\mu_R n'(1) \\ & f_2(\tau) = \mu_T \ddot{F}(\tau) + \mathcal{E}\gamma_i d_i(0)\{m'(0) - m'(1)\} - [\{\tfrac{\partial}{\partial\tau} + (1)_o\}\mu_T + \mathcal{E}\cdot\epsilon_T] m(0) - \mathcal{E}\cdot\mu_T n(0) \\ & y f_1(\tau) = \pm\beta_R + f_1(\tau), \qquad y f_2(\tau) = \pm\beta_T + f_2(\tau) \end{aligned} \quad \ldots \tag{34}$$

where, $\{\frac{\partial}{\partial\tau} + (1)_o\}$ is an operator for the objective function in convolution and $(1)_o$ indicates the initial value of that function. $\mathcal{E}$ is unit operator of convolution, i.e. $[\mathcal{E} * f] = f$, and, $\cdot$ and $'$ indicate differentiation with respect to τ and ξ respectively.

$g(\xi,\zeta;\tau)$, $g_i(\xi,\tau)$ and $\delta(\xi,\zeta,\lambda;\tau)$ are directly calculated from (10),(13),(14),(16), using (32). Singular points of Green's function consist of poles $\{s_\nu\}$ and branch points $\{s_b\}$. ϕ_1 and ϕ_2 may be analytic except $\{s_b\}$ so that the poles only occur at zeros of the denominator, and the branch points may occur only in arguments of ϕ_1, ϕ_2. Then, $g(\xi,\zeta;\tau)$ can be expressed as follows.

$$g(\xi,\zeta;\tau) = \sum_\nu R_\nu\{e^{s\tau} G(\xi,\zeta;s)\} - \frac{1}{2\pi i}\sum_b \oint_{cb} e^{s\tau} G(\xi,\zeta;s) ds \tag{35}$$

where, c_b is a cut which is taken so as to avoid a branch point s_b. Zeros of $\{g(\zeta) + s\gamma_i d_i(\zeta)\}\Delta_w(\zeta,s)$ are not poles of $G(\xi,\zeta;s)$, because ϕ_1, and ϕ_2 become dependent each other in this case, then, poles agree to zeros of $\Delta_c(s)$, i.e. infinite eigen-value series $\{s_\nu\}$. As to branch points of argument of ϕ_1 and ϕ_2, it is assumed that they are of the 1st order and finite numbers and

$\lim_{s\to s_k} G(\xi,\zeta;s) \neq \infty$ are valid. Under these conditions, the second term in (35) vanishes. Then, (35) is reduced to following expression.

$$g(\xi,\zeta;\tau)=\sum_{\nu} R_{\nu}\{e^{s\tau}G(\xi,\zeta;s)\} \tag{36}$$

Further, it is assumed here that all eigen-values are simple, i.e.

$$\frac{d}{ds}\Delta_c(s_\nu)=\Delta_c'(s_\nu)\neq 0$$

Then, (36) yields to

$$g(\xi,\zeta;\tau)=\sum_{\nu} e^{s_\nu\tau}R(\xi,\zeta;s_\nu) \tag{37}$$

where, $R(\xi,\zeta;s_\nu)$ is residue of $G(\xi,\zeta;s)$ at $s=s_\nu$.
Considering existence of conjugate sets of eigen-values, following expression is finally obtained for $g(\xi,\zeta;\tau)$.

$$g(\xi,\zeta;\tau)=\sum_{\nu} e^{s_\nu\tau}\lim_{s\to s_\nu}(s-s_\nu)G(\xi,\zeta;s)=2\sum_{\nu}[\mathbb{R}\{e^{s_\nu\tau}\}\mathbb{R}\{R(\xi,\zeta;s_\nu)\}-\mathbb{I}\{e^{s_\nu\tau}\}\mathbb{I}\{R(\xi,\zeta;s_\nu)\}] \tag{38}$$

where,

$$R(\xi,\zeta;s_\nu)=N(\xi,\zeta;s_\nu)/[\{g(\zeta)+s_\nu\gamma_i d_i(\zeta)\}\Delta_w(\zeta,s_\nu)\Delta_c'(s_\nu)]$$
$$=I(\zeta,s_\nu)\varphi_\nu^c(\xi)$$

$$I(\zeta,s_\nu)=\left(\begin{vmatrix} W_{2\ell}(\phi_1) & W_{2\ell}(\phi_2) \\ \phi_1(\zeta) & \phi_2(\zeta)\end{vmatrix}_{s_\nu}-c(s_\nu)\begin{vmatrix} W_{1\ell}(\phi_1) & W_{1\ell}(\phi_2) \\ \phi_1(\zeta) & \phi_2(\zeta)\end{vmatrix}_{s_\nu}\right)\Big/[\{g(\zeta)+s_\nu\gamma_i d_i(\zeta)\}\Delta_w(\zeta,s_\nu)\times$$
$$\times\Delta_c'(s_\nu)]$$

$$I(\zeta,s_\nu)=\left(c(s_\nu)\begin{vmatrix} W_{1u}(\phi_1) & W_{1u}(\phi_2) \\ \phi_1(\zeta) & \phi_2(\zeta)\end{vmatrix}_{s_\nu}-\begin{vmatrix} W_{2u}(\phi_1) & W_{2u}(\phi_2) \\ \phi_1(\zeta) & \phi_2(\zeta)\end{vmatrix}_{s_\nu}\right)\Big/[\{g(\zeta)+s_\nu\gamma_i d_i(\zeta)\}\times$$
$$\times\Delta_w(\zeta,s_\nu)\Delta_c'(s_\nu)] \qquad \ldots (39)$$

Similarly, following expressions are valid for $g_i(\xi;\tau)$ and $\delta(\xi,\zeta,\lambda;\tau)$.

$$g_i(\xi;\tau)=2\sum_{\nu}[\mathbb{R}\{e^{s_\nu\tau}\}\mathbb{R}\{R_i(\xi;s_\nu)\}-\mathbb{I}\{e^{s_\nu\tau}\}\mathbb{I}\{R_i(\xi;s_\nu)\}] \tag{40}$$

where,

$$R_i(\xi;s_\nu)=I_i(s_\nu)\varphi_\nu^c(\xi)$$
$$I_1(s_\nu)=-c(s_\nu)/\Delta_c'(s_\nu)\ ,\quad I_2(s_\nu)=1/\Delta_c'(s_\nu) \qquad \ldots (41)$$
$$\delta(\xi,\zeta,\lambda;\tau)=2\sum_{\nu}\Big(\mathbb{R}\{e^{s_\nu\tau}\}[\mathbb{R}\{\Gamma(\zeta,\lambda;s_\nu)\}\mathbb{R}\{R(\xi,\zeta;s_\nu)\}-\mathbb{I}\{\Gamma\}\mathbb{I}\{R\}]$$
$$-\mathbb{I}\{e^{s_\nu\tau}\}[\mathbb{R}\{\Gamma\}\mathbb{I}\{R\}+\mathbb{I}\{\Gamma(\zeta,\lambda;s_\nu)\}\mathbb{R}\{R(\xi,\zeta;s_\nu)\}]\Big) \qquad \ldots (42)$$

where,

$$\Gamma(\zeta,\lambda;s_\nu)=-\sum_{i=1}^{\infty}\overset{\circledast}{K}{}^{i}(\zeta,\lambda;s_\nu) \tag{43}$$

If generally $\Delta_c^{(j)}(s_\nu)=0\ (j=0\sim n_\nu)$, $\Delta_c^{(n_\nu+1)}(s_\nu)\neq 0$, are valid, s_ν is $(n_\nu+1)$th order pole of $\Delta_c(s)$. In this case, the following expression is replaced to the corresponding one.

$$g(\xi,\zeta;s_\nu)=2\sum_{\nu}\sum_{j=0}^{n_\nu}\frac{\tau^j}{(n_\nu-j)!j!}[\mathbb{R}\{e^{s_\nu\tau}\}\mathbb{R}\{(s-s_\nu)^{n_\nu+1}G(\xi,\zeta;s_\nu)\}_{s_\nu}^{(n_\nu-j)}$$
$$-\mathbb{I}\{e^{s_\nu\tau}\}\mathbb{I}\{(s-s_\nu)^{n_\nu+1}G(\xi,\zeta;s_\nu)\}_{s_\nu}^{(n_\nu-j)}] \tag{44}$$

After all, using (30)~(44), general solution in any linear branch of the substructure is obtained as a real convolution type function which consist of four complex modified orthogonal eigen-function expansion series. Therefore solution in the full domain can be obtained by the successive procedure, connecting solutions in adjacent linear branches.

Numerical Examples: In this section, various nondimensional earthquake responses of simple structural system are calculated for several combinations of nondimensional parameters. It is assumed here that the structural system has no rotational terms and any dampings and that the upper mainstructure has uniform distribution functions and the lower substructure has elasto-plastic characteristics without translational inertia. Concerning to ground excitation, it is assumed that the structural system is subjected to nondimensional quadratic half wave i.e. one rectangular acceleration pulse of duration which agrees to the fundamental period of the system. Nondimensional fundamental differential system is written as follows.

$$_pL(\eta)=\frac{\partial^2\eta}{\partial\xi^2}-\frac{\partial^2\eta}{\partial\tau^2}=f(\tau)=\alpha M(\bar{\tau}/4+\tau,\ \bar{\tau}) \tag{45}$$

$$_pW_1(\eta)=\left|\frac{\partial\eta}{\partial\xi}\right|_1=0\ ,\ {}_pW_2(\eta)=\left|\frac{\partial\eta}{\partial\xi}\right|_0-\mathcal{K}_T|\eta|_0=0 \tag{46}$$

or, $_{py}W_1(\eta)=|\partial\eta/\partial\xi|_1=0\ ,\ {}_{py}W_2(\eta)=|\partial\eta/\partial\xi|_0=\pm\beta_T$ upper sign / lower sign $\sim\eta\gtrless0$ (47)

$$\text{where,}\quad \alpha=\frac{hPA_0}{G_0}C_a\ ,\ \bar{\tau}=\frac{1}{h}\sqrt{\frac{G_0}{PA_0}}\,\bar{T}=\frac{2\pi}{\omega_1}=\bar{\tau}(\mathcal{K}_T) \tag{48}$$

$$(\omega_1\tan\omega_1=\mathcal{K}_T)$$

General solutions for each linear brahch are expressed as follows.
For branch with $\mathcal{K}_T\neq0$ i.e.(45) and (46).

$$\eta=-\sum_{\nu=1}^{\infty}\frac{{}_n\varphi_\nu(\xi)}{\omega_\nu}(1,{}_n\varphi_\nu)[f(\tau)*\sin\omega_\nu\tau]+\sum_{\nu=1}^{\infty}{}_n\varphi_\nu(\xi)\Big\{(m(\zeta),{}_n\varphi_\nu)\cos\omega_\nu\tau+\frac{1}{\omega_\nu}(n(\zeta),{}_n\varphi_\nu)\sin\omega_\nu\tau\Big\} \tag{49}$$

where,

$$\{{}_n\varphi_\nu(\xi)\}=\left\{\sqrt{\frac{2\mathcal{K}_T}{\mathcal{K}_T+\sin^2\omega_\nu}}\cdot\cos\omega_\nu(1-\xi)\right\}\ ,\ \{\omega_\nu\}:\omega_\nu\tan\omega_\nu=\mathcal{K}_T \tag{50}$$

For branch with $\mathcal{K}_T=0$ i.e. (45) and (47).

$$\eta=-[f(\tau)*\tau]+(1,m(\zeta))+(1,n(\zeta))\tau+\sum_{\nu=2}^{\infty}{}_n\varphi_\nu(\zeta)\Big\{(m(\zeta),{}_n\varphi_\nu)\cos\omega_\nu\tau+\frac{1}{\omega_\nu}(n(\zeta),{}_n\varphi_\nu)\sin\omega_\nu\tau\Big\}\pm\Delta\eta \tag{51}$$

$$\text{where,}\ \Delta\eta=\beta_T\Big\{\frac{1}{6}-\frac{1}{2}(1-\xi)^2-\frac{\tau^2}{2}+\sum_{\nu=2}^{\infty}\frac{{}_n\varphi_\nu(0)}{\omega_\nu^2}{}_n\varphi_\nu(\xi)\cos\omega_\nu\tau\Big\} \tag{52}$$

$${}_n\varphi_1(\xi)\equiv1,\ \{{}_n\varphi_\nu(\xi)\}=\{\sqrt{2}\cos\omega_\nu(1-\xi)\}\ \text{for}\ \nu\geq2\ ,\ \{\omega_\nu\}=\{(\nu-1)\pi\} \tag{53}$$

$\Delta\eta$ is calculated using following boundary homogenizing function. (see Appendix II)

$$\bar{\bar{\eta}}=\mp\frac{\beta_T(1-\xi)^2}{2}\ \supset\ \bar{\bar{\phi}}=\mp\frac{\beta_T(1-\xi)^2}{2s} \tag{54}$$

and associated initial condition.

$$\bar{\bar{m}}(\xi)=\mp\frac{\beta_T(1-\xi)^2}{2} \tag{55}$$

General solution (49) in the linear branch $\mathcal{K}_T\neq0$ has the normalized orthogonal eigen-function expansion part only; on the other hand, general solution (51) in the linear branch $\mathcal{K}_T=0$ has the normalized orthogonal eigen-function expansion part and the boundary homogenizing function part.

Other responses, for example, shear-strain, nondimensional instantaneous lateral force, velocity and shear-strain velocity responses in any linear branch are easily obtained by differentiation of the general solutions (49) or (51) with respect to ξ , τ . Whole earthquake responses are obtained by connecting solutions of successive linear branches $\{i\}$ of which odd numbered branches $\{2m-1\}$ m=1,2,··· indicate elastic system and even numbered branches $\{2m\}$ m=1,2,··· indicate system with perfectly plastic substructure. Transition from odd branch to even branch is determined by the following condition and nondimensional transition time τ_{2m} is determined as root of the transcendental equation.

$$\left|\frac{\partial \eta_{2m-1}}{\partial \xi}\right|_0 = \varkappa_T |\eta_{2m-1}|_0 = \pm \beta_T \quad at\ \bar{\tau}_{2m} \tag{56}$$

and transition from even branch to odd branch is determined by the condition,

$$\left|\frac{\partial \eta_{2m}}{\partial \tau}\right|_0 = 0 \quad at\ \bar{\tau}_{2m+1} \tag{57}$$

Co-ordinate systems are chosen for the set of branches $\{2m-1,\ 2m\}$ m=1,2,⋯. On the other hand, the time co-ordinate is chosen for each linear branch $\{i\}$ i=1,2,⋯ . Between these adjacent co-ordinate systems, there exist following relations.

$$\begin{aligned}\eta_{2m-1} &= \eta_{2(m-1)-1} - \left\{ |\eta_{2(m-1)-1}|_{0,\bar{\tau}_{2m-1}} - |\eta_{2(m-1)-1}|_{0,\bar{\tau}_{2(m-1)}} \right\} \\ &= \eta_{2(m-1)-1} - \left\{ m^{2m-1}_{2(m-1)-1}(0) - m^{2(m-1)}_{2(m-1)-1}(0) \right\} = \eta_{2(m-1)-1} - \Delta_p^{2(m-1)}\end{aligned} \tag{58}$$

where the superscript indicates system number and the subscript indicates the adopted co-ordinate system. And $\Delta_p^{2(m-1)}$ is a quantity of plastic deformation of substructure in 2(m-1) branch. Therefore, transforming to original co-ordinate system, the following relations are obtained.

For odd branches:

$$\eta = \eta_1 = \eta^{2m-1}_{2m-1} + \sum_{m=2}^{m} \Delta_p^{2(m-1)} = \eta^{2m-1}_{2m-1} + \sum_{m=1}^{m-1} \Delta_p^{2m} = \eta^{2m-1}_{2m-1} + \Delta_p \tag{59}$$

For even branches:

$$\eta = \eta_1 = \eta^{2m}_{2m-1} + \sum_{m=2}^{m} \Delta_p^{2(m-1)} = \eta^{2m}_{2m-1} + \sum_{m=1}^{m-1} \Delta_p^{2m} = \eta^{2m}_{2m-1} + \Delta_p \tag{60}$$

For any branches:

$$\tau = \tau_1 = \tau_i + \sum_{i=2}^{i} \bar{\tau}_i \qquad (\bar{\tau}_1 = 0) \tag{61}$$

where, Δ_p is the permanent set of the substructure.

Using the above procedure, several earthquake responses which seem to be important in view of earthquake engineering, that is, the nondimensional displacement η, the shear-strain $\partial\eta/\partial\xi$, nondimensional elasto-plastic behaviour of substructure, $(\partial\eta/\partial\xi)_0$ to $(\eta)_0$, the nondimensional overturning moment m_0 and the distribution of the maximum absolute shear-strain $|\partial\eta/\partial\xi|_{max.}$ are calculated for following combinations of nondimensional parameters.

(1)	$\varkappa_T=5$	$c_T \leq 0.25$	(5)	$\varkappa_T=10$	$c_T \leq 0.25$
(2)	$\varkappa_T=5$	$c_T=0.50$	(6)	$\varkappa_T=10$	$c_T=0.50$
(3)	$\varkappa_T=5$	$c_T=1.00$	(7)	$\varkappa_T=10$	$c_T=1.00$
(4)	$\varkappa_T=5$	$c_T=1.50$	(8)	$\varkappa_T=10$	$c_T=1.50$

Physical significance of these parameters are as follows.

$$\varkappa_T = \frac{hK_T}{G_0} = \frac{K_T}{G_0/h} \quad \text{: ratio of spring constant of substructure to equivalent spring constant of mainstructure.} \tag{62}$$

$$c_T = \frac{\alpha}{\beta_T} = \frac{h\rho A_0 C_a}{Q_{yield}} \quad \text{: ratio of static base shear force to yield force of substructure.} \tag{63}$$

Concerning to choice of nondimensional parameters of ground excitation, nondimensional period $\bar{\tau}$ is determined as a function $\varkappa_T$ only, and the nondimensional amplitude of rectangular pulse can be arbitrarily chosen because of validity of similitude for fixed combination of $(\varkappa_T,\ c_T)$. Therefore, in this calculation, α is chosen equal to 10 conventionally.

Results of the numerical calculation are shown in Figs. 4 ~ 10. in which the nondimensional time is multiplied by $20/\bar{\tau}(\varkappa_T)$ so as to be convenient for comparison.

Applications of Nondimensional Responses to Aseismic Design: General applications of the nondimensional responses obtained by the above analytical method are briefly considered. There are eleven independent physical quantities and four independent distribution functions concerning to any branch of original structural system, that is,

$$\{h; G_o(P_o,S_o); \rho A_o; D_{io}; D_{eo}; K_T; K_R; (|M_{yield}|, |Q_{yield}|); M_T; M_R; E_R; E_T\}$$
$$\{\bar{g}(x); \bar{di}(x); \bar{a}(x); \bar{de}(x)\} \qquad \ldots (64)$$

On the other hand, any branch of nondimensional structural system has eight independent parameters and four independent distribution functions, that is,

$$\{\gamma_e; \gamma_i; \varkappa_T; \varkappa_R; (\beta_T; \beta_R); \mu_T; \mu_R; \epsilon_T; \epsilon_R\}$$
$$\{g(\xi); di(\xi); a(\xi); de(\xi)\} \qquad \ldots (65)$$

These parameters and distribution functions are theoretically independent each other, however, there will be several conditions between these parameters in the engineering sense. Consequently, as the first step, for appropriate combinations of these parameters, earthquake responses due to the most dangerous nondimensional ground excitation, of which scale is arbitrarily chosen, are numerically calculated. On the other hand, ultimate deformations and strains should be determined theoretically and experimentally. Then, selecting safety factors in an adquate standard, allowable deformations and allowable strains are determined.

For the upper mainstructure:

$$\left(\frac{\partial\eta}{\partial\xi}\right)_{allow.} = \frac{1}{\Lambda_u}\left(\frac{\partial\eta}{\partial\xi}\right)_{ult.} \qquad \ldots (66)$$

For the lower substructure:

$$\left(\frac{\partial\eta}{\partial\xi}\right)_{o,allow.} = \frac{1}{\Lambda_{LR}}\left(\frac{\partial\eta}{\partial\xi}\right)_{o,ult.}, \quad (\eta)_{o,allow.} = \frac{1}{h}(y)_{o,allow.} = \frac{1}{h}\frac{1}{\Lambda_{LT}}(y)_{o,ult.} \qquad \ldots (67)$$

In the next step, selecting rigorous condition of destructive seismic waves depending mainly to ground conditions and type of structural system, for instance, constant ground acceleration C_a, constant ground velocity C_v, otherwise constant ground displacement C_d, magnification factors λ_s of nondimensional responses can be determined as functions of original physical quantities and selected condition of seismic waves. Thus, the following three equations are obtained.

$$\left(\frac{\partial\eta}{\partial\xi}\right)_{allow.} = \frac{1}{\lambda_s}\left(\frac{\partial\eta}{\partial\xi}\right)_{response\ max.}, \quad \left(\frac{\partial\eta}{\partial\xi}\right)_{o,allow.} = \frac{1}{\lambda_s}\left(\frac{\partial\eta}{\partial\xi}\right)_{o,response\ max.}$$

$$(\eta)_{o,allow.} = \frac{1}{\lambda_s}(\eta)_{o,response\ max.} \qquad \ldots (68)$$

where λ_s is comprehensive symbol representing λ_a, λ_v, or λ_d.
Therefore, three quantities of original physical quantities can be determined by above equations, and others are to be arbitrarily chosen from another requirements. For the structural design of the upper elastic mainstructure, it would be useful to calculate equivalent lateral force coefficient distribution $\frac{\partial}{\partial\xi}\left|\frac{\partial\eta}{\partial\xi}\right|_{max.}$ which is adaptable directly to the present convention-

al design method. But, for application, it is necessary to multiply this coefficient by $G_o/\lambda_s h$ and to gain equivalent lateral force distribution as follows.

$$w(x) = \frac{G_o}{\lambda_s h}\frac{\partial}{\partial \xi}\left|\frac{\partial \eta}{\partial \xi}\right|_{max.} = \frac{1}{\bar{\lambda}_s}\frac{\partial}{\partial \xi}\left|\frac{\partial \eta}{\partial \xi}\right|_{max.} \tag{69}$$

$$\bar{\lambda}_s = \lambda_s h/G_o$$

Discussions of the Results: In the numerical calculations, expansion series in the solution are considered up to the first four terms. Convergency of series and the accordance of responses on both sides of each connection are considerably good, as shown as Table 1.

Generally, in elasto-plastic domain, there are two parameters κ_T, C_T corresponding to structural system, which influence the characteristics of nondimensional earthquake responses to a rectangular acceleration pulse with resonant period to fundamental vibration of the structural system. When C_T is not larger than 0.25, the structural system behave only in elastic domain and then, there exists only one parameter κ_T. In this elastic domain, responses show the so-called resonance phenomena. The nondimensional displacement η decreases as κ_T increases, but the nondimensional relative displacement of the upper mainstructure $\eta-(\eta)_o$, nondimensional overturning moment m_o, shear-strain $\partial\eta/\partial\xi$ are in agreement irrespective of κ_T in full domain $(o,\infty]$. In this case the maximum absolute shear-strain $|\partial\eta/\partial\xi|_{max}$ has linear distribution and the value at $\xi = 0$ is nearly 40 as nondimensional quantity. In comparison to corresponding static base shear-strain i.e.

$$\left|\frac{\partial \eta}{\partial \xi}\right|_{max.\,static,\,o} = \frac{h\rho A_o C_a}{G_o} = \alpha = 10 \tag{70}$$

this dynamic base shear-strain due to one pulse reaches four times as large as the static one. This means that the dynamic base shear force is four times of corresponding static one under the condition of constant ground acceleration and these are determined independently on G_o, κ_T, i.e.

$$Q_{dynamic,o} = 4\,h\rho A_o C_a = 4\,Q_{static,o} \tag{71}$$

However, if conditions of constant ground velocity or constant ground displacement are taken, these relations reduce to following expressions respectively.

$$Q_{dynamic,o} = \frac{2}{\pi}\sqrt{\rho A_o G_o}\,\bar{\tau}(\kappa_T)C_v\ ,\quad Q_{dynamic,o} = \frac{2}{\pi}\frac{G_o}{h}\bar{\tau}(\kappa_T)C_d \tag{72}$$

and,

$$Q_{static,o} \gtreqless Q_{dynamic,o}\quad ;\quad \frac{\pi}{2}\sqrt{\rho A_o/G_o}\cdot h/\bar{\tau}(\kappa_T) \gtreqless C_v/C_a$$

$$Q_{static,o} \gtreqless Q_{dynamic,o}\quad ;\quad \frac{\pi}{2}(\rho A_o/G_o)\cdot h^2/\bar{\tau}(\kappa_T) \gtreqless C_d/C_a \qquad \ldots (73)$$

These mean that the dynamic base shear force would be smaller than the so-called static one in the structural system with a comparably long period.

When C_T increases larger than 0.25, the structural system behaves elasto-plastically. It is shown in Figs. 4 ~ 10 and Tables 2~3 that characteristics of the nondimensional earthquake responses are chiefly influenced by parameter C_T which gives the base shear-strain of the upper mainstructure in the plastic domain, i.e.

$$(\partial\eta/\partial\xi)_o = \beta_T = \alpha/C_T = 10/C_T \tag{74}$$

and represents transferability to plastic domain. Strictly speaking, responses are influenced by $\varkappa_T$ as well as C_T, however, parameter C_T has decisive influence on elasto-plastic behaviours of responses in an appropriate range of parameter $\varkappa_T$. Successive series of ratios of nondimensional transition time to standard one $\{\tau_i/\tau_{standard}\}$ which represents allocation of elasto-plastic domains are determined by almost C_T alone. Concerning to nondimensional displacement η, the translational element as a rigid body dominates in the plastic domain, and nondimensional maximum absolute displacement $|\eta|_{max}$ and plastic deformations of the substructure Δ_p^{2m} and Δ_p increase as $\varkappa_T$ decreases and C_T increases up to a certain value which lies between 1.00 and 1.50. $|\eta|_{max}$, however, rather decreases gradually as C_T is larger than this value. And, it is seemed that $|\eta|_{max}$ and Δ_p in this range of parameter C_T, are rapidly increase and then tend to almost stabilized value as C_T increases from 0.25, and this would be the nondimensional ground displacement,

$$(\overline{Y}/h) = \alpha\,\overline{\tau}^2/16 \tag{75}$$

when C_T tends to infinity. Shear-strain $\partial\eta/\partial\xi$ is mainly controlled by C_T and scarcely influenced by $\varkappa_T$ as shown in Figs. 6~7. The maximum absolute shear-strain $|\partial\eta/\partial\xi|_{max}$ considerably decreases as C_T increases, especially, in the lower part of the upper mainstructure. And it increases uniformly a little as $\varkappa_T$ increases because of occurence of higher modes due to an abrupt changes of the system. Distribution of the maximum absolute shear-strain looks like a straight line when $C_T \leqq 0.25$ almost independently on $\varkappa_T$. As C_T increases, it becomes a convex curve similar to quadratic curve with the peak located at $0<\xi<1$, and as C_T increases more, it uniformly decreases and approaches rather a rectangular distribution. When C_T tends to infinity, it should tend to the zero line as shown in Fig. 10. As to the nondimensional overturning moment m_0, same discussions as shear-strain can be made.

These facts show that energy dissipation due to elasto-plastic loop of the substructure, limitation of transmition of the shear-strain due to yielding of the substructure--plastic flow of soil or yielding of the stiffness member of the substructure--and non-synchronized property due to nonlinearity of the structural system restrain remarkably development of earthquake responses of the upper mainstructure, although abrupt changes of the system bring out higher modes in the elasto-plastic domain.

Conclusions: The problem treated in this paper is mathematically reduced to a complex non-orthogonal eigen-value problem in which eigen-values and eigen-functions are complex numbers and nonlinear modified orthogonality is valid in general. General solution of the problem is obtained in terms of complex eigen-function expansion series and in special case, a boundary homogenizing function is introduced to improve the convergency of the general solution in the neighbourhood of the boundary.

Nondimensional earthquake responses of the structural system with the elasto-plastic substructure are calculated by a successive procedure using the above general solution for appropriate combinations of nondimensional parameters. And it is found out that earthquake responses of the upper mainstructure are remarkably restrained by elasto-plastic characteristics of the substructure in comparison with those of the purely elastic system. This fact would be more notable as the number of seismic pulses increases because of resonance in the elastic system. For the elasto-plastic system, however, the number of seismic pulses is not so important since earthquake responses of the upper mainstructure are remarkably decreased when one pulse comes to end.

Applications of nondimensional responses are considered. The maximum absolute shear-strain distributions shown in Fig.10 and nondimensional plastic deformations of the substructure shown in Fig.8 and Table 2 would be most useful. For instance, the equivalent lateral force coefficient distribution obtained by differentiation of the maximum absolute shear-strain distribution corresponds to the value of c_T as follows. When c_T is not larger than 0.25, this is uniform distribution. As c_T increases from 0.25 this is transfered to a straight line changing sign at the peak of the maximum absolute shear-strain distribution. If c_T increases further, distribution remains only on the upper part of the mainstructure, for example, for c_T=1.50, this is a rectangular distribution on the upper one third of the mainstructure and its value is about 20 in the nondimensional value. When c_T tends to infinity, this would tend to zero. Therefore, for some values of $\mathcal{K}_T$, an equivalent lateral force distribution $w(x)$ can be obtained by dividing the equivalent lateral force coefficient distribution above obtained by the following magnification factor determined from the given condition of seismic waves.

$$\tilde{\lambda}_a=\frac{10}{\rho A_o C_a}\ ,\quad \tilde{\lambda}_v=\frac{20\pi h}{\sqrt{\rho A_o G_o}\,\bar{\tau}(\mathcal{K}_T)C_v}\ ,\quad \tilde{\lambda}_d=\frac{20\pi h^2}{G_o\bar{\tau}(\mathcal{K}_T)C_d} \tag{76}$$

Using this equivalent lateral force distribution, following procedure of aseismic design may be considered. At the first step, corresponding to conditions of the substructure, K_T and Q_{yield} are mainly determined by dead and live loads of the structural system. At the second step, assuming appropriate G_o, and selecting condition of seismic waves, the magnification factor is determined. At the third step, calculating the value of c_T, the equivalent lateral force distribution is obtained. And then, members of the mainstructure can be designed using the present conventional design criteria. Where, S_o should be determined so that $G_o=S_o-P_o$ is near the previously assumed one. Concerning to the plastic deformation of the substructure, it is checked by a given allowable displacement.

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Nomenclature

Y ; lateral displacement in fixed co-ordinate.

x ; axial co-ordinate.

$y=Y-\bar{Y}$; lateral displacement in moving co-ordinate.

t ; time.

$\bar{Y}$; ground motion.

g ; gravitational acceleration.

For the upper mainstructure:

h ; height.

ρ ; density.

$A(x)=A_0\bar{a}(x)$, $\bar{a}(x)$; distribution function of area.

$P(x)=P_0\bar{p}(x)=\rho g\int_x^h A(x)dx$, $\bar{p}(x)$; distribution function of axial force.

$S(x)=S_0\bar{s}(x)$, $\bar{s}(x)$; distribution function of shear rigidity.

$G(x)=S(x)-P(x)=G_0\bar{g}(x)$, $\bar{g}(x)$; distribution function of equivalent shear rigidity.

$D_i(x)=D_{i0}\bar{d}_i(x)$, $\bar{d}_i(x)$; distribution function of internal damping.

$D_e(x)=D_{e0}\bar{d}_e(x)$, $\bar{d}_e(x)$; distribution function of external damping.

For the lower substructure:

K_R ; rotational spring constant.

K_T ; translational spring constant.

E_R ; rotational viscous damping coefficient.

E_T ; translational viscous damping coefficient.

M_R ; rotational inertia.

M_T ; translational inertia.

M_{yield}; yield force of rotational spring.

Q_{yield}; yield force of translational spring.

For the others:

M_o ; overturning moment.

$\overline{T}$; period of equivalent seismic wave.

C_a ; maximum acceleration of equivalent seismic wave.

C_v ; maximum velocity of equivalent seismic wave.

C_d ; maximum displacement of equivalent seismic wave.

Λ_U ; safety factor of the upper mainstructure.

Λ_{LR}; safety factor of rotational spring of the lower substructure.

Λ_{LT}; safety factor of translational spring of the lower substructure.

λ_a ; magnification factor of nondimensional responses for constant acceleration.

λ_v; magnification factor of nondimensional responses for constant velocity.

λ_d; magnification factor of nondimensional responses for constant displacement.

<u>Figure Captions</u>

Fig. 1 Convergency of Non-orthogonal Eigen-function Expansions ($\mu_T \neq 0$).

Fig. 2 Convergency of Orthogonal Eigen-function Expansions I ($\mu_T = 0$).

Fig. 3 Convergency of Orthogonal Eigen-function Expansions II ($\mu_T = 0$).

Fig. 4 Nondimensional Displacement η for $\kappa_T = 5$ & various values of c_T.

Fig. 5 Nondimensional Displacement η for $\kappa_T = 10$ & various values of c_T.

Fig. 6 Shear-strain $\partial\eta/\partial\xi$ for $\kappa_T = 5$ & various values of c_T.

Fig. 7 Shear-strain $\partial\eta/\partial\xi$ for $\kappa_T = 10$ & various values of c_T.

Fig. 8 Elasto-plastic Behaviour of Substructure.

Fig. 9 Nondimensional Overturning Moment.

Fig. 10 Comparison of Maximum Absolute Shear-strain Distribution $|\partial \eta / \partial \xi|_{max}$ for various values of $\varkappa_T$ & c_T.

notes:

Number of curves in Figs. 1~3 corresponds to upper limit λ of summation.

Nondimensional transition times without star, with one star and with two stars in Figs. 4~9 correspond to $c_T = 0.50$, $c_T = 1.00$ and $c_T = 1.50$ respectively.

Table 1 Nondimensional Responses on Both Sides of Nondimensional Transition Time.

Table 2 Nondimensional Plastic Deformations of Substructure.

Table 3 Nondimensional Maximum Absolute Values of Responses.

Appendix I: Linear Modified Orthogonal Differential System: Modified orthogonal relations (27) and (28) can be reduced to linear relations with respect to i or j, under certain conditions. For example, assuming,

$$g(\xi) = d_i(\xi) = b(\xi), \quad a(\xi) = d_e(\xi) = \bar{c}(\xi) \tag{a.1}$$

$$\varkappa_R \gamma_i + \mu_R \gamma_e - \epsilon_R = 0, \quad \varkappa_T \gamma_i + \mu_T \gamma_e - \epsilon_T = 0 \tag{a.2}$$

modified normalised orthogonality (28) yields to

$$[{}_n\varphi_i, {}_n\varphi_j] = \int_0^1 \bar{c}(\xi)\, {}_n\varphi_i\, {}_n\varphi_j\, d\xi + \mu_R ({}_n\varphi_i')_1 ({}_n\varphi_j')_1 + \mu_T ({}_n\varphi_i)_0 ({}_n\varphi_j)_0 = \delta_{ij} \tag{a.3}$$

In this case, the differential system in the transformed space is reduced to following system.

$$\left.\begin{aligned} L(\phi) &= \left[\frac{d}{d\xi}\left[b(\xi)\left\{\frac{d}{d\xi} - \left|\frac{d}{d\xi}\right|_1\right\}\right] + \omega^2 \bar{c}(\xi)\right]\phi = 0 \\ W_1 &= \left[\int_0^1 b(\xi)\left\{\frac{d}{d\xi} - \left|\frac{d}{d\xi}\right|_1\right\} d\xi + (\omega^2 \mu_R - \varkappa_R)\left|\frac{d}{d\xi}\right|_1\right]\phi = 0 \\ W_2 &= \left[b(0)\left\{\left|\frac{d}{d\xi}\right|_0 - \left|\frac{d}{d\xi}\right|_1\right\} + (\omega^2 \mu_T - \varkappa_T)|1|_0\right]\phi = 0 \end{aligned}\right] \tag{a.4}$$

$$\left.\begin{aligned} {}_yW_1 &= \left[\int_0^1 b(\xi)\left\{\frac{d}{d\xi} - \left|\frac{d}{d\xi}\right|_1\right\} d\xi + \omega^2 \mu_R \left|\frac{d}{d\xi}\right|_1\right]\phi = 0 \\ {}_yW_2 &= \left[b(0)\left\{\left|\frac{d}{d\xi}\right|_0 - \left|\frac{d}{d\xi}\right|_1\right\} + \omega^2 \mu_T |1|_0\right]\phi = 0 \end{aligned}\right] \tag{a.5}$$

Replacing parameter S by ω using the following relation,

$$\omega^2 = -\frac{S^2 + S\gamma_e}{\gamma_i S + 1} \tag{a.6}$$

above system can be easily obtained from (6) and (7) under the conditions of (a.1) and (a.2). In this system, there exist real and positive eigen-value series $\{\omega_\nu^2\}$ and the corresponding real eigen-function series $\{{}_n\varphi_\nu\}$, as understanding from relation (22).

Here, the more explicit and concrete inverse transformations of Green's functions are to be considered, by assuming symmetricity of Green's function $G(\xi, \zeta; s)$. This assumption means that there exist no rotational terms under the condition $b'(\xi) \neq 0$, otherwise $b'(\xi) \equiv 0$ under existence of rotational terms. In this case, following expressions can be easily obtained.

$$G_1(\xi, S) = G_\zeta(\xi, 1; S), \qquad G_2(\xi, S) = G(\xi, 0; S) \tag{a.7}$$

$$\Gamma(\zeta,\lambda;s)=0$$

Therefore, it is important to express the inverse transformation $g(\xi,\zeta;\tau)$, because $G_i(\xi,S)$ i=1,2 can be obtained by considering (a.7). This is obtained directly from (38) and (39), however, it will be more convenient to use the following expression of eigen-function.

$$\begin{aligned}\varphi_\nu(\xi) &= -2\omega_\nu \lim_{\omega\to\omega_\nu}(\omega-\omega_\nu)\Big[\int_0^1 G(\xi,\zeta;\omega)\overline{c}(\zeta)\varphi_\nu(\zeta)d\zeta+\mu_R G_\zeta(\xi,1;\omega)(\varphi_\nu')_1 \\ &\qquad +\mu_T G(\xi,0;\omega)(\varphi_\nu)_0\Big] \\ &= -2\omega_\nu \lim_{\omega\to\omega_\nu}(\omega-\omega_\nu)[G(\xi,\zeta;\omega)\varphi_\nu(\zeta)] \\ &= \frac{\delta_i S_\nu^2+2S_\nu+\delta_e}{(1+\delta_i S_\nu)^2}\lim_{s\to S_\nu}(s-S_\nu)[G(\xi,\zeta;s)\varphi_\nu(\zeta)]\end{aligned} \qquad \text{... (a.8)}$$

Using symmetricity of Green's function, the second formula of (38) yields to following expression.

$$R(\xi,\zeta;S_\nu)=\lim_{s\to S_\nu}(s-S_\nu)G(\xi,\zeta;s)=I(S_\nu)\varphi_\nu(\zeta)\varphi_\nu(\xi) \qquad \text{(a.9)}$$

Operating linear operator $[\ ,\varphi_\nu(\zeta)]$ to (a.9) and using (a.8), following expression can be obtained for the original system (6), (7).

$$\begin{aligned}[\varphi_\nu(\zeta),\varphi_\nu(\zeta)]\bar{I}(S_\nu) &= \frac{\delta_i S_\nu+1}{\delta_i S_\nu^2+2S_\nu+\delta_e}, \qquad \bar{I}(S_\nu)=I(S_\nu)/(\delta_i S_\nu+1) \\ &= -i\frac{1}{\sqrt{4\omega_\nu^2-(\delta_e+\delta_i\omega_\nu^2)^2}}=-i\frac{1}{\sqrt{D(\omega_\nu^2)}}\end{aligned} \qquad \text{(a.10)}$$

Then, $g(\xi,\zeta;\tau)$ is finally formulated as follows, from the first equation of (38), using normalized linear modified orthogonal eigen-function.

$$\begin{aligned}g(\xi,\zeta;\tau) &= [g(\xi,\zeta;\tau)]_{D>0}+[g(\xi,\zeta;\tau)]_{D=0}+[g(\xi,\zeta;\tau)]_{D<0} \\ &= \sum_\nu \frac{2}{\sqrt{D(\omega_\nu^2)}}e^{-\frac{\delta_e+\delta_i\omega_\nu^2}{2}\tau}\cdot\sin\frac{\sqrt{D(\omega_\nu^2)}}{2}\tau\cdot{}_n\varphi_\nu(\xi)_n\varphi_\nu(\zeta)+\sum_\nu e^{-\omega_\nu\tau}\tau\cdot{}_n\varphi_\nu(\xi)_n\varphi_\nu(\zeta) \\ &\quad +\sum_\nu \frac{2}{\sqrt{-D(\omega_\nu^2)}}e^{-\frac{\delta_e+\delta_i\omega_\nu^2}{2}\tau}\cdot\sinh\frac{\sqrt{-D(\omega_\nu^2)}}{2}\tau\cdot{}_n\varphi_\nu(\xi)_n\varphi_\nu(\zeta)\end{aligned} \qquad \text{... (a.11)}$$

<u>Appendix II; Convergency of General Solution and Boundary Homogenizing Function:</u> Under condition of convergency of the infinite eigen-function expansion series (30) and its continuity, one-to-one correspondence between the transformed function and the inverse transformed function is valid. This problem can be reduced to investigate the validity of eigen-function expansion of a given function. In general, nonlinear modified orthogonal eigen-function expansion of any function $\tilde{F}(\xi)$ which is real and continuous and which has a continuous derivative, is obtained from (30) by substituting following relations into this.

$$F(\tau)=0, \quad n(\xi)=0, \quad m(\xi)=\tilde{F}(\xi) \quad at\ \tau=0 \qquad \text{(a.12)}$$

When linear modified orthogonality is valid, this can be reduced to following expression.

$$\tilde{F}(\xi)=\sum_\nu \frac{[\tilde{F}(\xi),\varphi_\nu(\xi)]}{[\varphi_\nu(\xi),\varphi_\nu(\xi)]}\varphi_\nu(\xi)=\sum_\nu[\tilde{F}(\xi),{}_n\varphi_\nu(\xi)]_n\varphi_\nu(\xi) \qquad \text{(a.13)}$$

This expansion formula agrees to the expansion treated in bibliography 4) and 5), in which the convergency of such infinite eigen-function expansion series has been proved in the closed interval [0, 1].

However, the convergency in the neighbourhood of boundaries 0 or 1, is likely to be poor in certain problems, especially in the case where $\mu_T=0, \mu_R=0$, and there exist non-homogeneous terms of boundary condition.

In other words, convergency of solutions which concern to non-homogeneous terms of boundary conditions, becomes poor in case where the substructure has not inertia. For example, in case where the system has uniform distribution functions and has not rotational terms nor any damping, the eigen-function expansions of $\widetilde{F}(\xi)\equiv 1$ are calculated and shown in Figs. 1~3. It is easily verified that for the validity of the general solution such as (30), following expressions should be valid.

$$\sum_{\nu=1}^{\infty}[1, {}_n\varphi_\nu]\,{}_n\varphi_\nu(\xi) = A_\infty(\xi) + B_\infty(\xi) \equiv 1$$

$$A_\lambda(\xi) = \sum_{\nu=1}^{\lambda}\int_0^1 \overline{C}(\xi)\,{}_n\varphi_\nu(\xi)\,d\xi\cdot{}_n\varphi_\nu(\xi) = \sum_{\nu=1}^{\lambda}(1, {}_n\varphi_\nu)\,{}_n\varphi_\nu(\xi),\quad B_\lambda(\xi) = \mu_T\sum_{\nu=1}^{\lambda}{}_n\varphi_\nu(0)\,{}_n\varphi_\nu(\xi)$$

$$\lim_{\lambda\to\infty}A_\lambda(0) = 0 \qquad \lim_{\lambda\to\infty}B_\lambda(0) = 1 \qquad \text{for } \xi = 0 \tag{a.14}$$

$$\lim_{\lambda\to\infty}A_\lambda(\xi) = 1 \qquad \lim_{\lambda\to\infty}B_\lambda(\xi) = 0 \qquad \text{for } 0<\xi\leqq 1 \tag{a.15}$$

$$\lim_{\lambda\to\infty}A_\lambda(\xi) + B_\lambda(\xi) = 1 \quad \text{for } 0\leqq\xi\leqq 1 \tag{a.16}$$

Validity of (a.14) and (a.15) are necessary for the solution corresponding to non-homogeneous terms of boundary. So long as μ_T does not vanish, these are valid. If μ_T tends to zero, however, the second equation of (a.14) is not valid as shown in Fig. 3. i.e.

$$\lim_{\lambda\to\infty}B_\lambda(0) = \lim_{\substack{\mu_T\to 0\\ \lambda\to\infty}}\mu_T\widetilde{B}_\lambda(0) = \text{undefined value} \tag{a.17}$$

In these case, the general solution such as (30) is not adaptable, because the solution corresponding to non-homogeneous terms of boundary condition does not converge in this boundary.

In the sense of earthquake engineering, it is most important to study stress and strain responses in the neighbourhood of the boundary. For this purpose, boundary homogenizing function can be introduced to improve the convergency of the solution in the neighbourhood of the boundary, in above mentioned special case. This function may be chosen arbitrarily so as to eliminate non- homogeneous terms of boundary, instead of using $g_i(\xi,\tau)$ represented by infinite series which does not converge at boundary.

$${}_pW_i[\overline{\overline{\eta}}] = f_i \supset W_i[\overline{\overline{\Phi}}] = F_i \tag{a.18}$$

These functions are easily determined in the transformed space in separable type with respect to ξ and s.

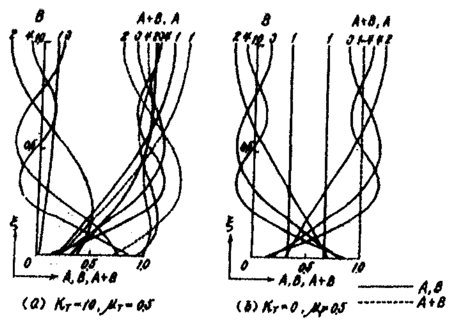
$$\overline{\overline{\eta}} = \sum_j q_j(\xi)\,r_j(\tau) \supset \sum_j q_j(\xi)\,R_j(s) = \overline{\overline{\Phi}} \tag{a.19}$$

where, q_j can be chosen conventionally in terms of polynomials.
Using these boundary homogenizing functions, and transforming η to $\overline{\eta}$ by

$$\overline{\eta} = \eta - \overline{\overline{\eta}} \supset \overline{\Phi} = \Phi - \overline{\overline{\Phi}} \tag{a.20}$$

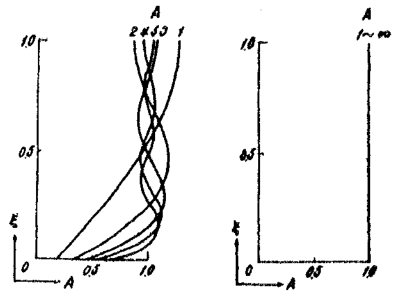
the fundamental differential system concerning to $\overline{\eta}$ or $\overline{\Phi}$ with homogeneous boundary conditions is obtained, and of which general solution is convergent in the fully closed interval. Inversely transforming, after all, the general solution $\eta \supset \Phi$ which consists of an eigen-function expansion part and a boundary homogenizing function part is obtained.

$$\eta = \overline{\eta} + \overline{\overline{\eta}} \tag{a.21}$$



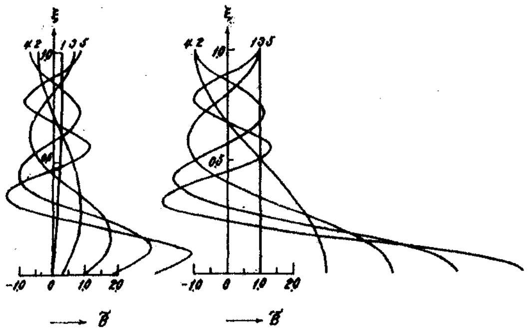
(a) $K_T=10, \mu_T=0.5$ (b) $K_T=0, \mu_T=0.5$

Fig. 1



(a) $K_T=10, \mu_T=0$ (b) $K_T=0, \mu_T=0$

Fig. 2



(a) $K_T=10, \mu_T=0$ (b) $K_T=0, \mu_T=0$

Fig. 3

Table 1

(a) $K=10$ $\varphi=0.50$

τ_i	$(\eta)_0$	$(\eta)_{0.3}$	$(\eta)_{0.7}$	$(\eta)_{1.0}$	m_0	$(-\frac{\partial\eta}{\partial\xi})_0$	$(\frac{\partial\eta}{\partial\xi})_{0.3}$	$(\frac{\partial\eta}{\partial\xi})_{0.7}$	i, domain
$\tau_1=0$	0	0	0	0	0	0	0	0	
$\tau_2=2.9035$	+ 1.9997 + 2.0082	+ 7.5794 + 7.5842	+12.0384 +12.0330	+12.9067 +12.9029	-10.9071 -10.8947	+20.0010 +20.0000	+16.2825 +16.3003	+ 6.0056 + 5.9922	1, elastic domain
$\tau_3=3.0518$	+ 7.1983 + 7.1747	+12.3093 +12.3285	+16.9821 +16.9818	+18.7281 +18.7326	-11.5298 -11.5579	+20.0000 +19.7637	+14.0951 +14.0815	+ 9.8477 + 9.8876	2, plastic domain
$\tau_4=4.9415$	+ 3.1464 + 3.0693	- 1.8387 - 1.8237	- 5.9945 - 5.9738	- 7.3890 - 7.4013	+10.5354 +10.4706	-20.5189 -20.0000	-12.6016 -12.9356	- 8.4098 - 8.2905	3, elastic domain
$\tau_5=6.0002$	- 0.1478 - 0.1222	- 4.5379 - 4.5437	- 7.7058 - 7.7421	- 8.5632 - 8.5237	+ 8.4154 + 8.4016	-20.0000 -19.7482	- 9.4159 -10.0366	- 5.4986 - 5.5925	4, plastic domain
									5, elastic domain

(b) $K=10$ $\varphi=1.50$

τ_i	$(\eta)_0$	$(\eta)_{0.3}$	$(\eta)_{0.7}$	$(\eta)_{1.0}$	m_0	$(\frac{\partial\eta}{\partial\xi})_0$	$(\frac{\partial\eta}{\partial\xi})_{0.3}$	$(\frac{\partial\eta}{\partial\xi})_{0.7}$	i, domain
$\tau_1=0$	0	0	0	0	0	0	0	0	
$\tau_2=0.7775$	- 0.6069 - 0.6812	- 2.2673 - 2.2030	- 3.0197 - 3.0152	- 2.9745 - 3.0149	+ 2.3076 + 2.3337	- 6.6702 - 6.6667	- 3.7411 - 3.8108	- 0.3434 - 0.3135	1, elastic domain
$\tau_3=1.5216$	- 2.4922 - 2.4806	- 4.4859 - 4.4958	- 7.0451 - 7.0293	- 7.9204 - 7.9346	+ 5.4282 + 5.4541	- 6.6667 - 6.5520	- 6.6867 - 6.7917	- 5.3057 - 5.3215	2, plastic domain
$\tau_4=2.4560$	- 1.1593 - 1.1503	+ 0.6096 + 0.6075	+ 1.8769 + 1.8741	+ 2.0232 + 2.0229	- 3.1825 - 3.1732	+ 6.6638 + 6.6647	+ 4.8973 + 4.9367	+ 1.5858 + 1.3162	3, elastic domain
$\tau_5=4.1165$	+17.9512 +17.9442	+18.8274 +18.8228	+18.4922 +18.5294	+18.5295 +18.4923	- 0.5784 - 0.5481	+ 6.6667 + 6.5976	- 0.1090 - 0.4757	- 0.2135 - 0.1256	4, plastic domain
									5, elastic domain

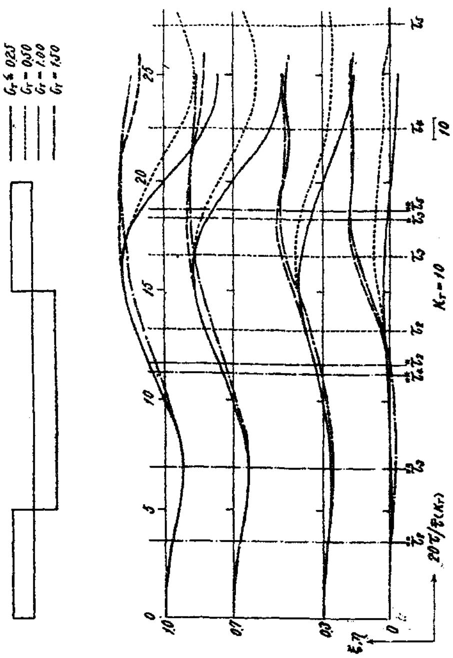


Fig. 5

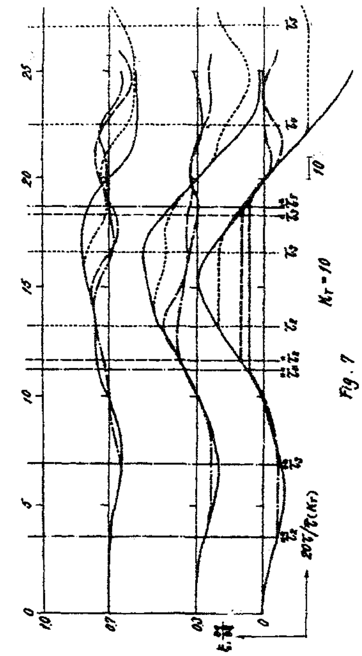


Fig. 7

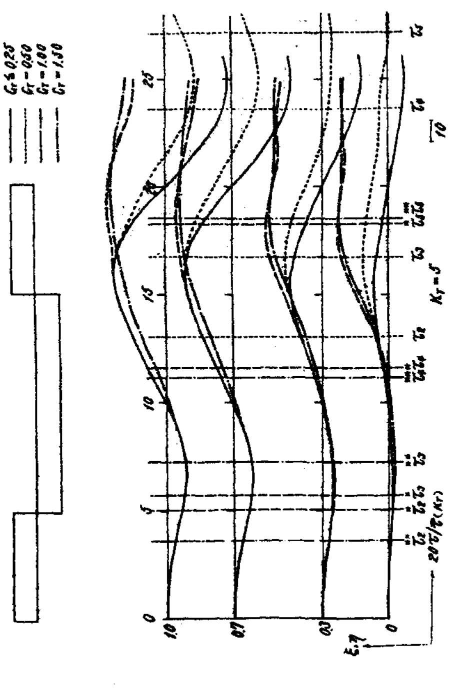


Fig. 4

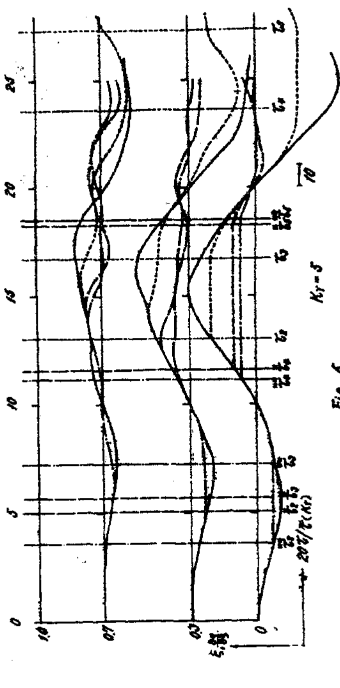


Fig. 6

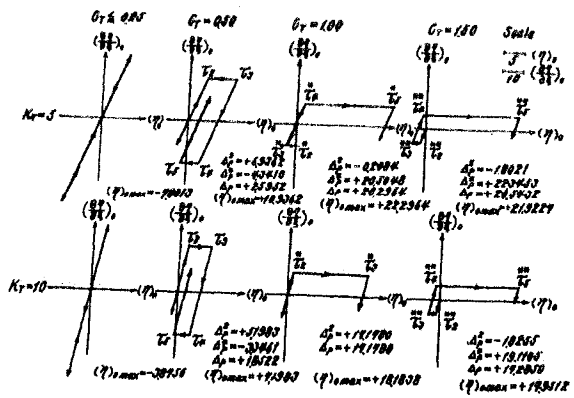


Fig. 8

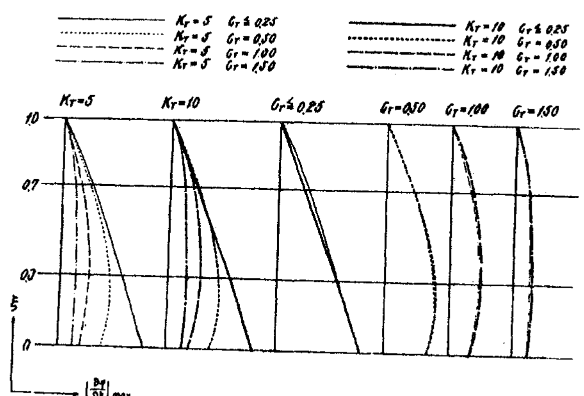


Fig. 10

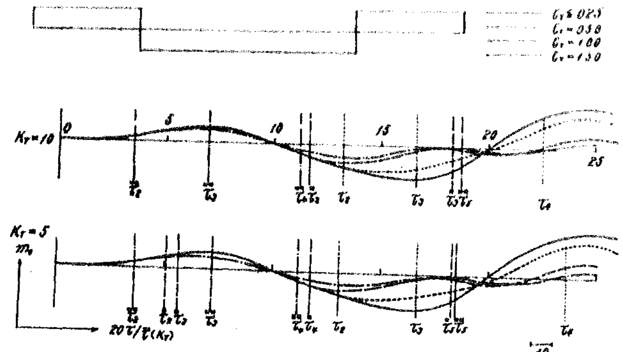


Fig. 9

Table 2

parameters	Δ_p^e	Δ_p^k	Δ_p
$\kappa=5$ $c_T=0.25$	0	0	0
$\kappa=5$ $c_T=0.50$	+ 6.9362	− 4.3410	+ 2.5952
$\kappa=5$ $c_T=1.00$	− 0.2084	+20.5048	+20.2964
$\kappa=5$ $c_T=1.50$	− 1.8021	+22.3453	+20.5432

parameters	Δ_p^e	Δ_p^k	Δ_p
$\kappa=10$ $c=0.25$	0	0	0
$\kappa=10$ $c=0.50$	+ 5.1983	− 3.3461	+ 1.8522
$\kappa=10$ $c=1.00$	+17.1780	0	+17.1780
$\kappa=10$ $c=1.50$	− 1.8255	+19.1105	+17.2849

Table 3

parameters	$\|\eta\|_0$	$\|\eta\|_{0.3}$	$\|\eta\|_{0.7}$	$\|\eta\|_{1.0}$	$\|m_0\|$	$\left\|\frac{\partial\eta}{\partial\xi}\right\|_0$	$\left\|\frac{\partial\eta}{\partial\xi}\right\|_{0.3}$	$\left\|\frac{\partial\eta}{\partial\xi}\right\|_{0.7}$
$\kappa=5$ $c_T=0.25$	7.8013	18.1190	26.2640	28.0630	20.2617	38.9780	27.4880	12.0250
$\kappa=5$ $c_T=0.50$	10.9362	16.0709	20.5497	22.0956	15.7555	20.0000	22.3419	12.2626
$\kappa=5$ $c_T=1.00$	22.2964	24.3147	24.4436	24.8552	6.3592	10.0000	13.2488	9.2597
$\kappa=5$ $c_T=1.50$	21.9227	22.6094	23.1034	23.3507	5.8612	6.6667	7.1385	6.4218
$\kappa=10$ $c_T=0.25$	3.8756	14.2176	22.1527	23.9742	20.0986	39.1150	28.2360	12.1954
$\kappa=10$ $c_T=0.50$	7.1983	12.3285	17.2946	19.1989	15.5034	20.0000	23.2993	12.2198
$\kappa=10$ $c_T=1.00$	18.1838	20.3645	19.9842	19.8603	8.8757	10.0000	14.4018	10.3962
$\kappa=10$ $c_T=1.50$	17.9512	18.8274	18.8123	19.0145	6.1681	6.6667	8.5420	6.7658

$\kappa=5$ $c_T=\infty$ $|\eta|_{0\,max}=\Delta_p=14.2921$ $\kappa=10$ $c_T=\infty$ $|\eta|_{0\,max}=\Delta_p=12.0846$