A RESERVE ENERGY TECHNIQUE FOR THE EARTHQUAKE DESIGN AND RATING OF STRUCTURES IN THE INELASTIC RANGE

By

JOHN A. BLUME*

ABSTRACT

A simplified technique of designing, analyzing, or comparing structures of any height, type, or combination of materials and elements in the inelastic range up to failure is described and illustrated. The procedure utilizes the energy absorption capacity of all building elements and considers change of natural periods, damping, energy value, etc., under increasing or repeated oscillations and deterioration from major earthquakes. The procedure also provides a practical means of reconciling static building code design coefficients with the much greater coefficients obtained from actual earthquake record analysis by elastic spectrum techniques. Simple expressions for spectrum velocity and acceleration are also provided.

A proposed system of rating structures for inelastic energy value, drift, and the damage risk factor for a "standard" enveloping earthquake spectrum or for selected specific earthquake records is also presented.

Previous work in this area by the author is considerably extended herein and examples of procedures and results for actual structures are shown. Comparisons are made between various types of buildings in several stages of elastic and inelastic response up to failure and with consideration of energy absorption, drift, permanent set, damage potential, cumulative damage, and multi-paths of earthquake resistance.

INTRODUCTION

Research efforts in recent years have demonstrated that earthquake history including lack of damage as well as damage, cannot usually be reconciled with normal static design coefficients, elastic unit stresses, and current procedures. Inelastic action must be considered together with the capacity of the structures to dissipate energy in emergencies. Unfortunately, this tends to inject further complexity into a problem which should be simplified to obtain broad application of aseismic design for public safety. Moreover, an approach to earthquake-resistant design has already been established that is difficult to change, and probably should not be changed if advances made to date are not to be lost.

In view of these considerations and the urgent need to include all basic parameters in design and analysis procedures, the technique to be described below was developed in such a way as to avoid complex mathematical

* President, John A. Blume & Associates, Engineers, San Francisco, California U.S.A.

expressions and to deal in terms which the structural designer uses in his daily work. Considerable effort and thought, as well as some carefully considered empirical relationships and approximations, were employed in order to reduce the complex problem of inelasticity and energy to what is hoped will be more general and more immediate application. The lack of a more rigorous approach and waiting perhaps years for complete scientific justification is excusable, it is hoped, because of the urgent need to design much closer to the way things really are rather than how we would like them to be for convenience and to suit traditional procedures.

The Reserve Energy Technique was presented in the Proceedings of the American Society of Civil Engineers as part of a broad consideration of structural-dynamics(1)(2), and certain elements of same have been discussed further(3)(4). This is the first presentation solely on the technique and its application to actual and hypothetical buildings. Additional data and refinements have also been developed since the original paper and are included herein.

THE ANOMOLIES OF "ELASTICITY"

Every "elastic" analysis of accurately recorded earthquake records conducted in the United States and New Zealand to date, regardless of any reasonable damping assumption or the number of masses involved, indicates that the responses are much greater than any static code or design procedure requires. The differences are so great as to negate reconciliation of the gap by "safety factors" or other conventional explanations. Fig. 5 illustrates elastic spectral accelerations for the "1940 El Centro (N-S) earthquake". Numerical considerations of the energy dissipation capacity of complex structures have been quite limited to date(1)(2)(5)(6) as have inelastic spectral analyses of actual earthquake records. The latter have thus far been limited to elasto-plastic systems, a special case of inelasticity(7)(8)(9)(10). The indications from these investigations are, however, that inelasticity is a most realistic approach to the earthquake problem. It can explain most, if not all, of the anomalies which elasticity presents including why a "weak" building has a good earthquake history and a "strong" building may not.

Modern elastic design procedures and code requirements do not, directly at least, satisfactorily consider the following important items:

(1) The deflections necessary to develop energy resistance to severe earthquakes.

(2) The effect of these deflections on the structure - its unit stresses, yield excursions, cracking, permanent set, damage or possible collapse.

(3) Changes of natural period with change in stiffness.

(4) The fact that for a majority of buildings the stiffness is not constant, perhaps even at low amplitudes; that with severe earthquakes

the characteristics change with not only deflection but the number of severe excursions beyond "design" values.

(5) Buildings have many and highly variable elements of structural and energy resistance including frames, walls, partitions, stairways, foundations, and soil.

(6) There may be two or more basic "plateaus" of resistance depending upon the design, the construction and the earthquake exposure.

(7) In addition to unit stress, there are four other characteristics or indices of measurement or comparison in earthquake resistance; namely, (1) energy development, (2) maximum deflection or drift, (3) permanent set, and (4) damage (structural and non-structural).

With the advent of buildings without traditional "filler" walls and heavy partitions which were either "non-calculated" elements[1] or were in whole or in part considered structural, it becomes even more vital and urgent that the anomalies of elastic design become recognized and that they be eliminated from design procedures. As one who has been privileged to work over the years in not only structural design but in earthquake and dynamic research, the writer holds little hope of this being done practically except with procedures and terms familiar to building designers. The Reserve Energy Technique and the Earthquake Rating System are suggested as practical means of reconciling the anomalies of elasticity listed above and of improving communication in inelasticity.

THE RESERVE ENERGY TECHNIQUE - BASIC CONSIDERATIONS

(1) <u>The energy demands</u> of a vibrating or moving system must balance as well as the static and dynamic forces. It is convenient to consider "feed-in" energy or kinetic energy from the ground motion effect on the building, strain energy (elastic), energy dissipation or work capacity (strain and/or damage), internal energy loss (to heat) without damage, and energy "feed-back" or radiation from the structure back to the soil. At peak demands "feed-in" kinetic energy less energy "feed-back" must equal strain energy plus energy lost to heat plus energy dissipation in work or damage done. The strain energy is often a small part of the total energy requirement or capacity. (See Fig. 1)

(2) <u>The force-deflection</u> diagram of a structure or basic element of same can be used graphically or numerically as a measure of strain energy and work capacity, as well as a convenient means of adjusting period, estimating permanent yield or damage and of visualizing the basic problem. Fig. 2 illustrates the general construction and use of the diagram[2] and Fig. 3 illustrates a diagram for a story of a complex traditional building[1] and how the various elements contribute to resistance and energy capacity, whether or not so designed. The elastic strain energy is often difficult to isolate in buildings due to the lack of a definite "yield point". Therefore the design energy, D, up to force P_D, corresponding to a certain static coefficient C is used as a basis.

The value U_i as shown on Fig. 2 is total work capacity on the virgin application of severe force. In order to allow for "deterioration" or "softening" of the resistance under repeated excursions beyond the yield, a factor H_i is introduced and half of which "hump area" is arbitrarily deducted from the capacity. Jacobsen(5) has considered deterioration under repeat cycles based upon various tests by Japanese and American investigators. The force-deflection diagram of Fig. 2 is for the initial loading on a static (or slow-loading) basis. This is done for the practical reasons that (a) only such data are available today in reasonable quantities, and (b) this eliminates complications as to the various speeds of rapid loading and the history of prior ycles. In general, the additional values of rapid loading tend to compensate for the deterioration of initial excursions into the damaging or straining range.* To any who may suggest that there are inadequate data to construct a force-deflection diagram for a complete story of a building, there can only be one answer, namely, that unless the response characteristics of the building are known, or can be estimated reasonably well, up to failure no method of design or analysis no matter how rigorous, mathematical, or involved can consistently or properly provide for economical and predictable earthquake resistance. Actually, considerable data are available and diagrams have been developed for very complex as well as simple structures(1)(6). The procedure can be relatively simple for personnel experienced in structural design who have basic test data available.

(3) Earthquake exposure - The technique is based upon using the elastic acceleration spectral response for any specific earthquake or for a standard or averaged earthquake spectrum (see Fig. 5). In some cases, two or more earthquakes may be "tried" on the same structure to predict results. Spectral diagrams are now available for many United States earthquakes(11)(12)(4) and it is hoped that many more will become available for various parts of the world. It is not necessary to have a spectral diagram nor to become involved with their many peaks and valleys, however, since simple formula have been developed(4) to provide basic information closely approximating the spectral results provided thus far by Housner and others(12). For 5% of critical damping the elastic spectral velocity, v, may be expressed as

$$v_i = F(T_i)^{1/4} \quad \ldots\ldots\ldots (1)$$

for periods of 0.30 seconds and over, up to 3 seconds (see Fig. 4). Although this value would apply to much shorter periods, for reasons

---

* A hypothetical example would be one of a composite material in moment with the unit stresses such that the dynamic values would be 15% greater than the static values. However the softening effect or loss of some resistance due to reversals and additional cycles under earthquake reversals might reduce the dynamic values by 13%. Thus the effective values would be (1 + 0.15) X (1 - 0.23) = 1.00 = Static Value. Although these values are variable they do indicate the general order of magnitude.

later to be discussed, a different relationship is used in the short period range. The elastic spectral acceleration, α, for the same period values may be expressed

$$\alpha = 0.194\, F (T)^{-3/4} \quad . \; . \; . \; . \; . \; . \; . \; . \; (2)$$

Fig. 5 indicates various acceleration values. Estimated values for F and the number of equal or greater occurrences somewhere in California per 100-year period* are:

	F	Occurrences*
"Spectrograph"	2.77	2
El Centro 1940 N-S	1.83	20
El Centro 1940, av. of 2 components	1.50	32
El Centro 1934, av. of 2 components	1.00	70

*Approximations only, by analogy to Ref. (11) data.

The range of periods shorter than 0.3 seconds was investigated in considerable detail in a recent research effort(6). There are many low, rigid buildings of one or two stories in this range. A typical acceleration response vs. period plot such as Fig. 5 indicates some similarity with forced vibration resonance curves. An analogy can be drawn between magnification factor or transmissibility of forced vibration in the steady state to the spectral acceleration response. It can be argued that for the major earthquakes thus far recorded the period range of 0.2 to 0.3 could be considered a resonant response band where the dominant ground motion for all practical purposes was close to that of the natural period of the system being forced into vibration. Thus in this range the structure would be expected to vibrate in or close to its own natural frequency during the worst part of the earthquake. For longer periods it would be expected that the structure would tend to move in or return to its own natural mode of vibration except when interrupted by severe ground disturbances of other periods. It would thus seem that for the range of 0.2 to 0.3 seconds and up to 3 seconds, it is proper to enter the spectral curves with the natural period of the structure. However, for periods below the 0.2 to 0.3 second range, it would be more logical to enter the spectrum with the period of the ground motion since the structure would be responding directly with such motion. Finite values of velocity and acceleration are thus maintained, even at the situation where the structure's period is 0. Fig. 5 has been constructed in accordance with formula (2) for T values over 0.3 seconds but with values in the short period range determined as follows. At T = 0, the velocity values are as noted on Fig. 4. Between these points and the 0.3 second values determined by formula (1), a straight line variation is arbitrarily assumed. The acceleration and velocity values for corresponding F factors are interchangeable between Fig.4 and Fig.5. The value of ground period assumed for the very short period movement was 0.18 seconds.

There are some indications (Fig. 5) that spectral values obtained by various investigators in the long period range may not agree even after allowances for the different responses of single and multi-mass systems. The results of the energy analysis for certain buildings also indicate that for periods of 2.5 seconds and over the ONR values provide greater energy demands than good buildings may have capacity to meet even with damping and energy considerations. Whether more energy is "fed-back" to the ground than can now be assumed (about 10%), whether the ONR spectral data are too conservative, whether the buildings are inadequate, or whether the assumption of story participation proposed herein is conservative, remains to be demonstrated with future research. It seems logical, however, that velocity must approach zero at some period T. There must be a terminal point for ground energy and therefore the expression $v = F(T)^{1/4}$ should be considered applicable only for the period range shown. It may even be conservative for periods over 2 or 2.5 seconds. This expression, however, is independent of the Reserve Energy Technique.

(4) Damping - The Reserve Energy procedure can be used for any assumed damping value but 5% of critical damping is recommended because it is considered a reasonable, nominal value of damping in the elastic range; since it is adequate to iron out many of the extreme peaks and valleys of spectral response for lesser damping; and since the use of more than 5% would be considered unsafe in view of the fact that the Reserve Energy procedures provides otherwise for any greater values or types of damping which may be due to more than the normal hysteritic (heat loss) values in the elastic range for internal viscous damping. Determinations of damping in actual buildings to date reveal values under small amplitudes in the order of 5 to 10% of critical with some less than 5%[13] and average values closer to 5%[6]. The question of response - transient or steady state - is also important in this consideration and will be discussed below. Contrary to some opinion, damping in earthquake spectral studies does not decrease response directly, in fact its relative value decreases with not only the damping factor[1][6] but there are indications that damping may be less effective for close-epicenter shocks than for longer distances[11]. Fig. 6 illustrates both of these relationships for several recorded earthquakes.

(5) The nature of inelastic response seems to be one of a few occasional lurches or excursions beyond "elastic" conditions rather than a steady state vibration phenomena. There is not only a transient condition but a chaotic one, not easily treated by mathematical formulae. There are somewhat limited but none the less consistent reasons for this theory including (a) observations of the type and degree of damage and permanent set; (b) observations of lack of damage and set; (c) sound and sensation in buildings during earthquakes; (d) study of earthquake records; (e) calculations of energy demand versus energy dissipation and strain, and (f) results to date from digital and analog research in elasto-plastic systems exposed to earthquake spectra.[7][8][9].

The work at Illinois[9] indicated only a few half-cycles of excursions beyond the yield. For the entire El Centro 1940 N-S record of some thirty seconds only 5 half-cycle excursions were noted for one elasto-plastic system and another had only 3 excursions in the first 10 seconds of the record, the worst part. In spite of these few "lurches", the response was greatly attenuated. Similar results were obtained at the New Zealand Dominion Physical Laboratory[7] where the total of all yield excursions at peak period values for El Centro 1940 N-S divided by an arbitrary 2/3 of the maximum yield excursion (by the writer) provides an average of only four excursions beyond yield in the total earthquake

It may thus be postulated that (1) only a few severe energy demands are required for a structure of reasonable value in a major earthquake. The condition is not steady-state or even a transient "build-up" - instead there are a few "lurches" preceded by and followed by elastic response; (2) the equivalent viscous damping concept[5] seems not as applicable to abrupt lurches as it is to steady-state vibration phenomena; (3) the balancing of peak kinetic energy demand with potential energy capacity as in the Reserve Energy Technique, seems appropriate in the inelastic problem without recourse to integration of the whole earthquake record; (4) the use of initial static resistance values with the "hump"deterioration factor for a few to several possible repeat cycles is also appropriate to the problem of inelastic behavior.

ELASTO-PLASTIC BEHAVIOR - A SPECIAL CASE

Elasto-plastic systems are special cases of inelastic response and as such are also susceptible to Reserve Energy analysis. Whether or not a system or a structure truly behaves elasto-plastically is beyond the scope of this paper. However, many cases do occur where mild steel yields either alone or as the controlling element of composite construction without the crushing of non-ductile materials. Modern buildings without walls have little participation of collateral elements and may approach elasto-plastic behavior. Elasto-plastic systems have been used experimentally because of their simplicity. The results of three investigations of such systems were compared[3] to those of Reserve Energy as shown on Fig. 7. The reduction factor, R_i, reduces to the simple expression $R_i = \sqrt{D/U_i}$ since Ti is considered equal to T_D, and if it be postulated for comparative purposes that the elasto-plastic system is of such character as to allow H_i to be zero; i.e., there is no deterioration under repeat cycles. From these comparisons (Fig. 7) it is to be noted that for the limits of the available data (damping from 3% to 20% and period from 0.2 seconds to 0.6 seconds):

(1) Period has little or no relationship in agreement to the R curve,

(2) Damping seems to have little or no relationship in agreement to the R curve.

(3) The agreement is so good in this case at least, as compared to the many other uncertainties in practical earthquake design, as to suggest that the simple Reserve Energy procedure might be applied directly to elasto-plastic energy design without the need for computer or analog analysis. More test data are needed, however.

(4) An elastic acceleration response spectrum might be reduced to equivalent (inelastic) design values by simply multiplying α_i by R_i, determined for any limiting deflection ratio, μ, for an elasto-plastic system. (This is an inverse statement of (3) above.) For example, from Fig. 7 assume a single-mass design problem requires El Centro 1940 "standard" resistance at no safety factor, the limiting μ is 4, T_D is 0.5 seconds. The R factor is then 0.29 and the elastic value for the "standard" El Centro 1940, 5% damping and a period of 0.5 seconds is 0.60g from Fig. 5. Then the equivalent code or design value is simply 0.29 x 0.60g or 0.17g, with no energy safety factor.

MULTI-MASS SYSTEMS

The procedure shown in Fig. 2 is for a one-mass system such as a single-story building, a multi-story building that can reasonably be represented by a single-mass system, or any other structure which can be represented by the conventional lumped mass and spring analogy. A building of a few stories wherein the first story is much more flexible or vulnerable than the upper stories may be represented by a one-mass system in many cases, but with, of course, all the weight, W, considered.

Multi-storied buildings which are better represented by many lumped masses and weightless springs acting in series require additional considerations in the Reserve Energy procedure as follows:

(1) The fundamental elastic period of the entire structure, T, is determined by approximate[14], Rayleigh[15] or other procedures[13].

(2) Multi-mass spectral data should be used story by story, if available. If such are not available, up to 10% of the single-mass spectral acceleration α_i, can be deducted from the base shear as an allowance of energy used higher in the building in other mode response. This reduction should prorate to an addition of plus 10% to the shear of the top story.

(3) Work done anywhere in the structure assists other parts by draining energy, However, because of mode shapes and structural values, all stories do not participate in energy absorption simultaneously, or to their full capacity. A square root relationship for probable capacity has been assumed pending further research data.

For tall, slender units without floors or other massive discontinuities such as chimneys, arbitrarily assume Z = the number of 30-foot units of height. In determining the ($U_i - 0.5 H_i$) values for each story or unit, the absolute positions of all units should be proportional to the mode shape of the structure. Only the net story deflection is used, however, to obtain $\overline{U}_i$ and H_i. Thus, Δ_L could vary between different stories under the same trial, i, unless a straight line fundamental mode shape is assumed, as is often justified. $R_{i_z}\alpha_i$ is compared to the story design coefficient C_z, as per Fig. 2. In all cases C_z is the coefficient used to determine shear, not force, at any level (shear = C_zW). It is basic to check the lowest story and preferably all, or several typical upper stories as well.

(4) If multi-mass story-by-story spectral values are not available, important tall or slender buildings should be checked as well for second and third mode response. These mode periods can be calculated[13][15] or be estimated by ratios to the fundamental period, and the spectral curve entered at each period. Each particular mode shape must be used as a basis for obtaining ($\overline{U}_L - 0.5 H_L$) values. The results for each mode can be weighted[16] or combined by root mean square or other indicated probability values[17][18].

EXAMPLE OF RESERVE ENERGY ANALYSIS

Fig. 8 and Table 1 illustrate the analysis for a rigid one-story wood frame California building[6] which although designed to a C value of 0.133 had, because of its geometry, an actual C' value of 0.19. Fig. 8 indicates the results for several F factors or "earthquakes" to illustrate their various effects on deflection and damage. The cross marks on the diagrams indicate the points of compliance or deflections where the particular earthquake would be expected to reach under the most severe lurches or excursions. Table 1 indicates the tabular form of computation to obtain the $R_i\alpha_i$ values for the F = 1.83 El Centro "standard" earthquake which values are plotted on Fig. 8. The procedures are set forth on Fig. 2. Short period buildings such as this tend to "climb" the spectrum peak from the low period side under damage or yield and lengthening periods. Unlike the case of high buildings with T values greater than the spectral peak, the lengthening of natural periods of low, rigid buildings induces greater response under continued shaking.

Fig. 3 indicates the results of a reserve energy analysis of a traditional fifteen-story steel frame and brick filler wall building[1] which was not directly designed for earthquake but has a considerable amount of resistance inherent from its design for wind forces and its "non-calculated" masonry walls and concrete stairways.

Figs. 9 and 10 illustrate the results of energy analysis of several other buildings which are briefly described in Table 2. The characteristics of types of buildings as well as their number of stories vary over a considerable range. In some cases, especially for the low, rigid buildings, considerably more design static strength C' is provided than the codes provided for. This is usually the result of walls being included in the structure which walls, because of minimum thickness, have extra capacity. It is obvious that the variations between spectral data in the long period range of multi-story buildings must be reconciled and that reliable numerical data for ground "feed-back" must be obtained.

The Reserve Energy procedure clearly demonstrates the variations in resistance and deflection between traditional and "bareframe" buildings and the fact that reliance merely upon earthquake "history" may be very misleading as building types change.

All of the "buildings" of this investigation have been kept confidential and are only identified herein by numbers. They were not selected as being particularly good or bad examples but rather to illustrate the range of results that can be expected over a very wide scope of building types. These structures are either existing or in the late design stages, but in most cases the values as constructed or as to be constructed have been altered in view of various considerations and are not necessarily the same as shown herein.

The fact that various types of structures can have considerable difference in energy-based capacity even though designed for the same or similar static capacities illustrates that the static approach does not include all of the basic parameters of the problem. It is not feasible to attempt to generalize these results between types of buildings or their basic materials since the problem is too involved and can best be approached by individual consideration of the various factors which are automatically brought out in the Reserve Energy analysis. Moreover, it is undesirable to examine a particular structure only on the basis of safety factor since the amount of damage that might be entailed in reaching these safety factors could vary tremendously between structures.

Fig. 11 indicates the amount of $H_1/2U_1$ which is one of the rating indices to be presented below (actually a function of half of the "hump" area to the total energy capacity of an indication of the amount of damage or yield that might be expected) plotted against the Drift Index which is the story slope from vertical at maximum deflection. The figure illustrates the variations in building response when resisting a moderate earthquake. The capacities shown are not necessarily the ultimate for the "buildings" which are all merely "meeting" the energy demands in this figure.

A STANDARD EARTHQUAKE RATING SYSTEM FOR STRUCTURES

A basic requirement in scientific and engineering endeavor is to have a standard means of measuring or comparison. This is very difficult in the earthquake field because of the many and complex parameters and the lack of adequate data. There are many comparisons of earthquakes per se but none for structures except static lateral coefficients which do not include important factors. A system has been proposed based upon the Reserve Energy Technique(2). It is shown here in slightly revised form to include the earthquake characteristic F and with 10% reduction in energy demand for ground feed-back.

Fig. 12 illustrates the method of rating and the manner of reporting. From a total resistance curve for the lower story as per Fig. 2

and for another typical story (if more than 4 stories) the ratings are determined as per the formulae shown. The weakest direction for the lower story and same direction for the other story should be reported.

Three standard F values are shown. When the energy index is determined for one F value the others are proportional to their respective squares and are easily obtained. In general terms these F values may be considered to represent catastrophic, severe (El Centro 1940), and mild earthquakes respectively--with California probability of occurrence as shown above. They may also be severe for long periods, as discussed above, but have the advantage of being a definite basis for comparison.

The ratings consider 3 types of earthquakes, 4 key conditions of structural deflection and the 3 important indices: energy capacities compared to demand, drift (slope), and damage. The reported numbers alone, which can easily be typed or recorded by non-technical personnel give a vast amount of information for those who consider their significance. High BFI values indicate good relative energy capacity for the corresponding conditions of drift (or slope of the structure from the vertical) II, and of damage and/or permanent set, III. Not to consider all these various factors leads to inadequate comparisons and evaluations of risk and poor statistical history.

For the hypothetical example reported, the El Centro 1940 N-S (F = 1.83) energy requirement would be met (BE = 1.00) at a slope of 0.0023h from the vertical (II) and only 6% "damage" (III). The mild shock (F = 1.00) has no appreciable affect. The "catastrophic" (F = 2.77) earthquake could be resisted with 30% "damage". This is a good "building".

CONCLUSIONS AND RECOMMENDATIONS

It is hoped that the Reserve Energy Technique and the proposed Earthquake Rating System for Structures will provide convenient and simple devices for the urgently needed design consideration of inelastic response to earthquake motion. Some empirical and judgment factors have been combined with theoretical considerations. However, such has been done with care and the consideration of many factors too lengthy for inclusion in this paper. Although further research is needed, particularly to provide a library of more lateral force test data up to complete failure, there is no danger or harm (and it is believed, a great deal of good) to be done in using the method as a check and refinement of design for special structures first designed under current codes, or for the research analysis of existing structures.

Several things have been presented in this paper. Some are general in nature such as the Reserve Energy Technique, the Earthquake Rating System, and the expressions for spectral velocity and acceleration. These are independent items and each can be utilized without regard to the others. However, the various common terms and relationships are useful. There were also presented several specific situations or special cases of the general procedures such as the elasto-plastic system analysis or direct inelastic design with the use of the R factor and the deflection or ductility factor. Other special matters have to do with the specific F values proposed for various types of earthquakes. The recommended values and the 5% recommended damping factor are believed reasonable and consistent for California conditions shown to date especially up to 2 or 2.5 seconds. If these spectral values should be found to change in the future, or in areas where the conditions are basically different, naturally the particular values of α should be used in the general equations for the procedures of energy analysis, earthquake rating, or spectral responses.

With the procedure outlined the anomalies of a great deal of "baffling" earthquake history can be explained as can the gap between elastic spectral data and the capacity to resist earthquakes. The advantages of ductility, indeterminate construction, "plateaus" of resistance, period change, damping, energy loss due to cracking, friction, yield, etc., can be approached numerically as well as philosophically and in terms familiar to "static" designers who are even now producing billions of dollars of more structures for future earthquake exposure. Although many of the buildings analyzed thus far have adequate resistance from code design values, all do not, and the amount of drift and damage seem to be highly variable for different building types even though meeting static code requirements. A more realistic approach to earthquake resistant design is indicated, at least for important and unusual structures.

Although the rating system for structures is not a part of the design analysis technique it is considered an important device for better communication and recording as well as rating of structures on the important parameters of inelastic resistance, extreme deflection, and damage necessary to develop the resistance. Unfortunately, it is unrealistic to speak of earthquake resistance without also considering the results, if any, of developing this resistance in the form of drift and damage or permanent set.

It is not intended that the Reserve Energy Technique should replace modern codes or the judgment of structural engineers thoroughly familiar with the earthquake problem in all of its important aspects. It is felt, however, that the technique would sharpen the judgment of the engineers and designers and stimulate the research worker, and perhaps lead to code improvements. One of its basic objectives, of course, is to help to bridge the gap between earthquake history and research and practical structural design.

BIBLIOGRAPHY

1. Blume, John A., "Structural Dynamics in Earthquake-Resistant Design", Paper No. 1695, Proc. A.S.C.E., (July 1958).

2. Blume, John A., Closing discussion to "Structural Dynamics in Earthquake-Resistant Design", Proc. A.S.C.E., Structural Div. (Sept. 1959).

3. Blume, John A., Discussion of "Electrical Analog for Earthquake Yield Spectra", Proc. A.S.C.E., Eng. Mech. Div. (1960).

4. Blume, John A., Discussion of "Behavior of Structures During Earthquakes", Proc. A.S.C.E., Eng. Mech. Div. (1960).

5. Jacobsen, L. S., "Frictional Effects in Composite Structures Subjected to Earthquake Vibrations", Stanford Univ. March 9, 1959 (California State Division of Architecture, Sponsor).

6. Blume, Sharpe, and Elsesser, "A Structural-Dynamic Investigation of Fifteen School Buildings Subjected to Simulated Earthquake Motion", for the State of California, Dept. of Public Works, Division of Architecture (being printed).

7. Bycroft, Murphy and Brown, "Electrical Analog for Earthquake Yield Spectra", Proc. A.S.C.E., Eng. Mech. Div. (October 1959).

8. Berg and Thomaides, Discussion of "Electrical Analog for Earthquake Yield Spectra", Proc. A.S.C.E., Eng. Mech. Div. (1960).

9. Sheth, R. M., "Effect of Inelastic Action on the Response of Simple Structures to Earthquake Motions", M.S. thesis under Dr. N. M. Newmark, University of Illinois, Department of Civil Engineering, 1959.

10. (Deleted)

11. Housner, G. W., "Intensity of Ground Motion During Strong Earthquakes", Office of Naval Research, Contract N6ONR-244 (August 1952).

12. Housner, G. W. "Behavior of Structures During Earthquakes", Proc. A.S.C.E., Eng. Mech. Div. (October 1959).

13. Blume, John A., "Period Determinations and Other Earthquake Studies of a Fifteen-Story Building", Chapter 11, Proc. World Conf. on Earthquake Engineering, June 1956; E.E.R.I., Room 1039, 465 California St., San Francisco, Calif.

14. "Recommended Lateral Force Requirements", Seismology Committee, Structural Engineers Association of California (July 1959).

15. Blume, John A., Discussion of "Earthquake Stresses in Shear Buildings", Trans. A.S.C.E., Vol. 119, p. 171 ff (1954).

16. Anderson, Blume, Degenkolb, Hammill, Knapik, Merchand, Powers, Rinne, Sedgwick, Sjoberg, "Lateral Forces of Earthquake and Wind", Trans. A.S.C.E., 117: p. 732-735 (1952).

17. Hudson, D. E., "A Comparison of Theoretical and Experimental Determinations of Building Response to Earthquakes", 2nd World Conf. on Earthquake Engineering, Tokoyo, Japan, 1960.

18. Clough, Ray W., "On the Importance of Higher Modes of Vibration in the Earthquake Response of a Tall Building", Bulletin Seis. Soc. of America, Vol. 45 (October 1955).

NOMENCLATURE

i = a subscript referring the main symbol to its appropriate trial deflection ($i = 1, 2, 3$, etc.) or to the actual deflection of the trial deflection such as 0.5", 1.2", etc.

y = a subscript referring the main symbol to the yield point condition.

γ = a subscript referring the main symbol to its appropriate story or mass number, starting from the top story as $\gamma = 1$.

n, d, e, f, = subscripts for specific i values as defined in Fig. 12.

B = a safety factor based upon reserve energy, defined as $B_i = \frac{C'}{R_i \alpha_i}$;

BN, BD, BE, BF = earthquake rating symbols as per Fig. 12.

C = the static elastic (usually code specified) design coefficient, as a ratio of gravity.

C' = the actual static elastic value a structure may have (same designation as for C); for a structure barely meeting the code, $C = C'$.

D = the area under the force-deflection curve to the code design force P_D, where $P_D = C'W$; in foot-pounds. As a subscript, D refers to the design or initial elastic condition.

E = the energy requirement $= \frac{W}{2g} v^2$; for Ti from 0.3 to 3.0 seconds, $E = 0.0155\, WF^2\, Ti$; (Note: E may be reduced for the ground "feed-back" energy but not more than 10% without further research data. For use of single mass spectral values for multi-mass structures, see text.)

F = a factor to express earthquake spectral values.

g = acceleration of gravity: 32.2 ft/sec^2

G = the amount of energy assumed "fed-back" to the ground; maximum G without detailed supporting data = 0.10 E.

h = the height of a building story, feet

H_i = the total "hump" area, or area above a straight line from the origin of the P-Δ curve to any point on the curve directly above Δ_i; in foot-pounds.

K_D = initial (elastic) spring factor or stiffness = $\frac{P_D}{\Delta_D}$

K_i = stiffness or spring factor at or after having reached deflection Δ_i; see Fig. 2.

P_D = C'W, the design shear, pounds

P = lateral force or shear, pounds; (for multi-storied buildings P would be the story shear).

R = the Reserve Energy coefficient (as defined in Fig. 2) which adjusts elastic spectral acceleration to values providing for inelastic response with static design procedures.

T = natural period of the whole structure before earthquake; normally fundamental mode; seconds.

T_D = a hypothetical period = $1.11\sqrt{\Delta_D/C'}$

T_i = assumed natural period at, and after having reached, Δ_i; for elasto-plastic systems $T_i = T_D$; for other systems, see Fig. 2.

U_i = the total area under the initial, static force deflection curve up to any trial deflection, Δ_i; in foot-pounds.

v = velocity in ft/second

W = the total weight of the structure and fixed contents at and above the story or level under consideration, pounds.

Z = total number of stories, panels, or units acting in series above and including the one under consideration.

α_i = the spectral or analog response acceleration at period T_i; normally for 5% of critical damping, in ratio to gravity.

β = ratio of design force to yield force.

Δ = the static shear deflection of the story or unit under construction, consistent units.

Δ_{max} = the deflection point beyond which the P- Δ curve never again becomes level or slopes upwards with increasing deflection, or the point of collapse from secondary (local) buckling or other reasons, whichever is of lesser deflection; Δ_{max} also = Δ_f.

ϕ, θ = angles, as shown on Fig. 2.

μ = ductility factor for elasto-plastic system, or the ratio of total deflection Δ_i to yield deflection Δ_y.

ρ = a factor as shown on Figs. 2 and 13.

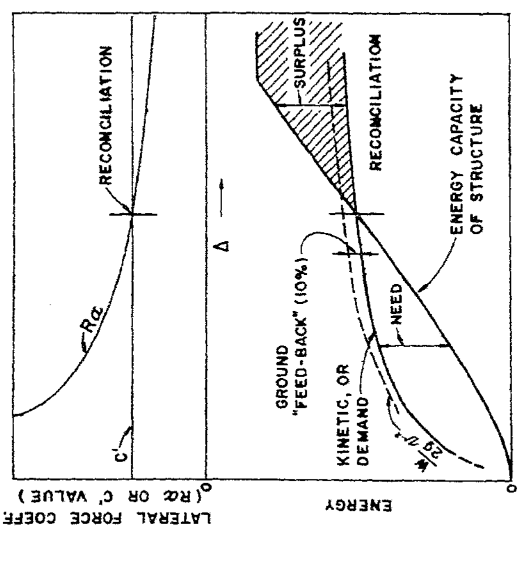


FIG. 1 — ENERGY DEVELOPMENT WITH SHEAR DEFLECTION

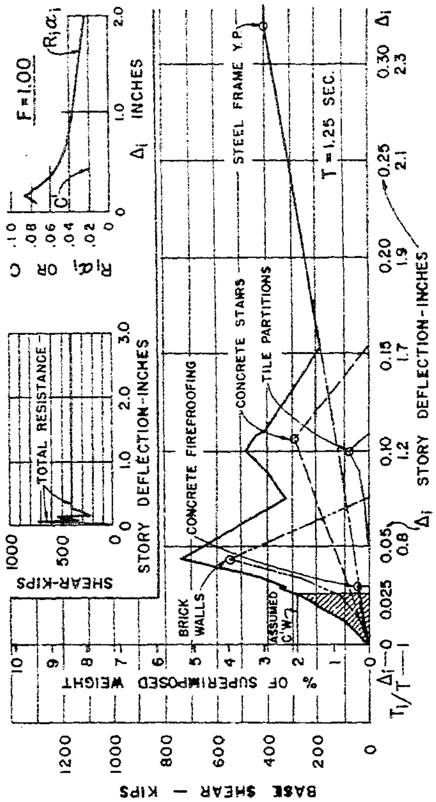


FIG. 3 — RESISTANCE OF 1ST STORY, (N-S) TRADITIONAL BLDG.

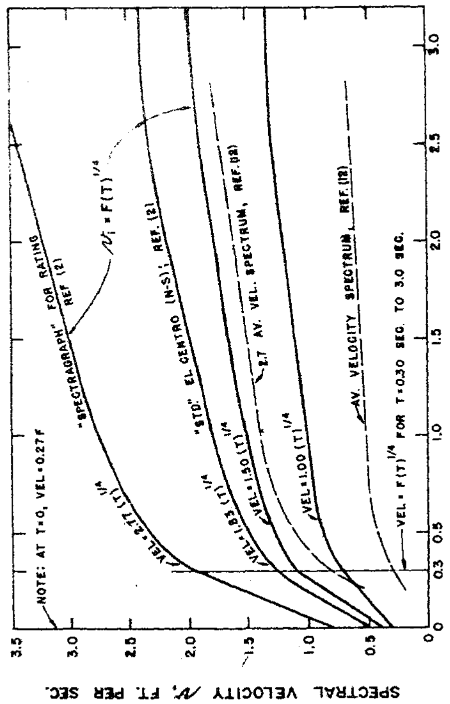


FIG. 4 — ELASTIC VELOCITIES, DAMPING 5% OF CRITICAL

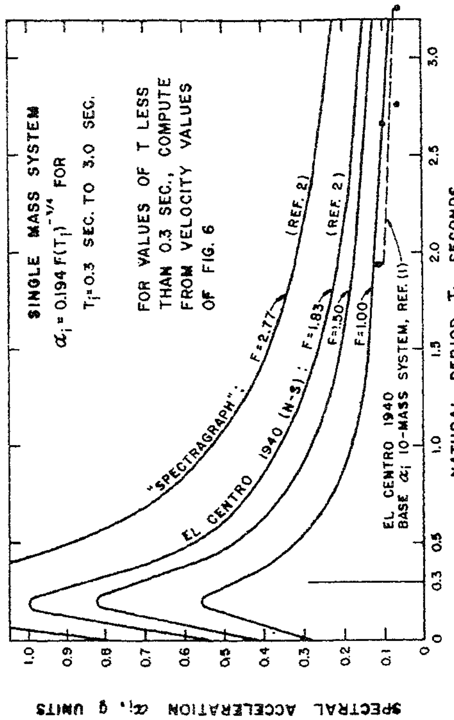
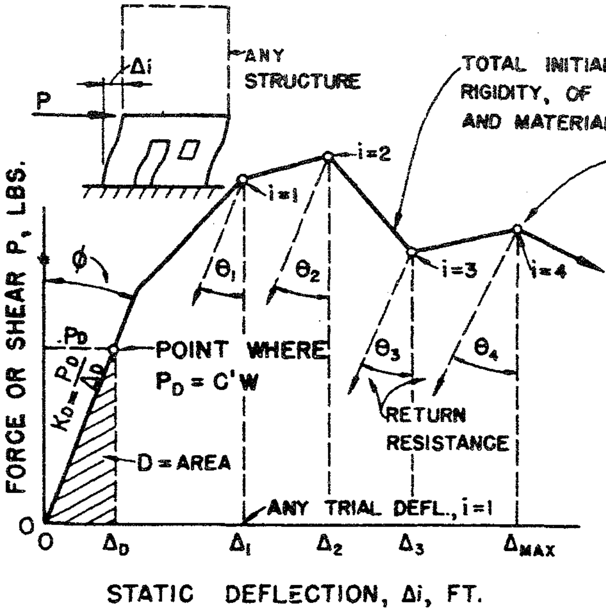


FIG. 5 — ELASTIC ACCELERATIONS, DAMPING 5% OF CRITICAL



Δ_{MAX} = POINT BEYOND WHICH CURVE NEVER BECOMES LEVEL OR SLOPES UPWARDS: OR, THE POINT OF COLLAPSE FROM LOCAL BUCKLING.

U_i = TOTAL AREA UNDER CURVE UP TO TRIAL POINT $\Delta i = 1, 2$, ETC.; FT. LBS.

H_i = "HUMP" AREA ABOVE A STRAIGHT LINE FROM ORIGIN TO ANY POINT, $i = 1, 2$, ETC.; FT. LBS.

α_i = SPECTRAL OR ANALOG ACCELERATION, RATIO TO GRAVITY, AT PERIOD T_i AND 5% DAMPING.

T = FUNDAMENTAL (ELASTIC) PERIOD OF STRUCTURE

$T_D = 1.11\sqrt{W/K_D} = 1.11\sqrt{\Delta_D/C'}$

T_i = ASSUMED PERIOD AT Δi.

$T_i = T \times \sqrt{K_D/K_i}$

IF ANGLE θ_i IS BETWEEN 0.9 ϕ AND 1.1 ϕ, LET $K_i = K_D$; OTHERWISE $K_i = \dfrac{2U_i - H_i}{\Delta i^2}$

R_i = "RESERVE ENERGY REDUCTION FACTOR"

$$R_i = T_i C' \sqrt{\frac{W\sqrt{Z}}{2.44\,\Sigma(U_i - 0.5H_i)}} \quad \text{OR:} \quad R_i = \rho\,\frac{T_i}{T}\sqrt{\frac{D\sqrt{Z}}{\Sigma(U_i - 0.5H_i)}}\ *$$

PROCEDURE FOR "SINGLE MASS STRUCTURES": (SEE TEXT FOR MULTI-STORY UNITS)

1. DESIGN OR ANALYSE STRUCTURE AS PER ELASTIC SEISMIC CODE IN EFFECT.
2. PLOT THE TOTAL RESISTANCE DIAGRAM AS INDICATED.
3. FOR TRIAL Δi, COMPUTE U_i, H_i, K_i, AND T_i; LET $i=1$.
4. OBTAIN α_i FOR PERIOD T_i AND COMPUTE $R_i\alpha_i$; USE SELECTED EARTHQUAKE.
5. IF $R_i\alpha_i$ IS EQUAL TO CODE DESIGN COEFFICIENT, C, THE STRUCTURE (OR PORTION OF SAME UNDER CONSIDERATION) IS ADEQUATE (NO SAFETY FACTOR) AT DEFLECTION Δi AND WITH DAMAGE OR YIELD INDICATED BY H_i. IF $R_i\alpha_i$ EXCEEDS C, THE STRUCTURE IS INADEQUATE AND THE CORRECTIVE CHOICES ARE:
   - (A) ASSUME A GREATER DEFLECTION OR DEFLECTIONS AND REPEAT.
   - (B) ADD MATERIAL TO THE STRUCTURE AND/OR MODIFY THE DESIGN.
   - (C) REDUCE α_i ON THE BASIS OF ANY RELIABLE DATA FOR ENERGY LOSS TO THE GROUND, OR FOR GREATER (NON—DESTRUCTIVE) DAMPING THAN 5%, (IN NO CASE REDUCE α_i MORE THAN 20% TOTAL).
   - (D) COMBINATIONS OF (A), (B) AND (C).
6. IF $R_i\alpha_i$ IS EQUAL TO OR LESS THAN C BUT THE DAMAGE OR DEFORMATION AT Δi AS INDICATED BY H_i IS NOT ACCEPTABLE, THE PROCEDURE WOULD BE AS IN (5) EXCEPT THAT LESSER DEFLECTIONS WOULD BE TRIED, PROBABLY WITH A MODIFIED DESIGN.

* $\rho = 1$ IF $T = 2\pi\sqrt{\dfrac{W}{gK_D}}$; OTHERWISE $\rho = 0.90T\sqrt{\dfrac{C'}{\Delta_D}}$

FIG. 2 – A "RESERVE ENERGY" TECHNIQUE FOR INELASTIC ASEISMIC DESIGN

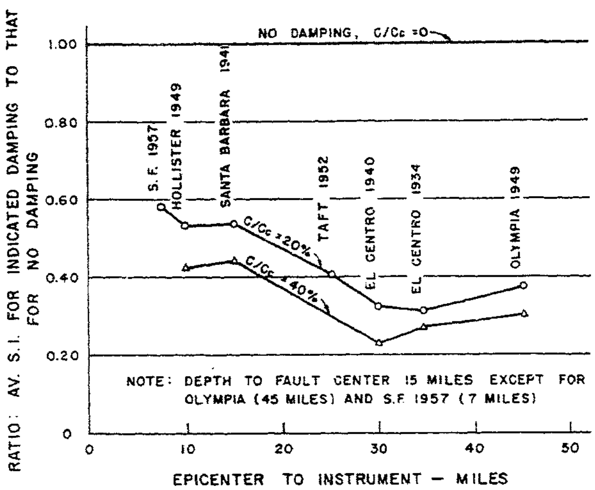


FIG. 6 — EFFECT OF DAMPING ON SPECTRAL INTENSITY

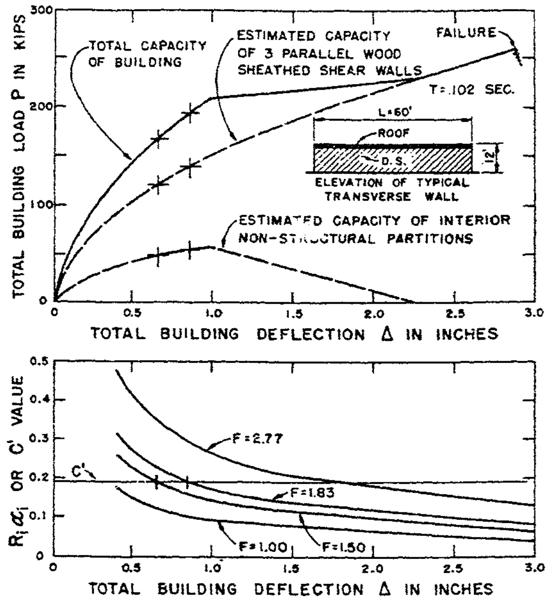


FIG. 8 — RESERVE ENERGY ANALYSIS ONE-STORY WOOD BUILDING, W-1

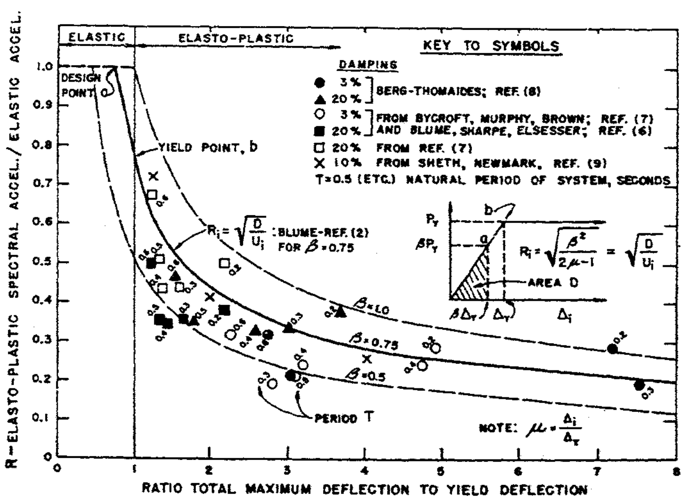


FIG. 7 — SINGLE-MASS ELASTO-PLASTIC SYSTEMS; EL CENTRO 1940

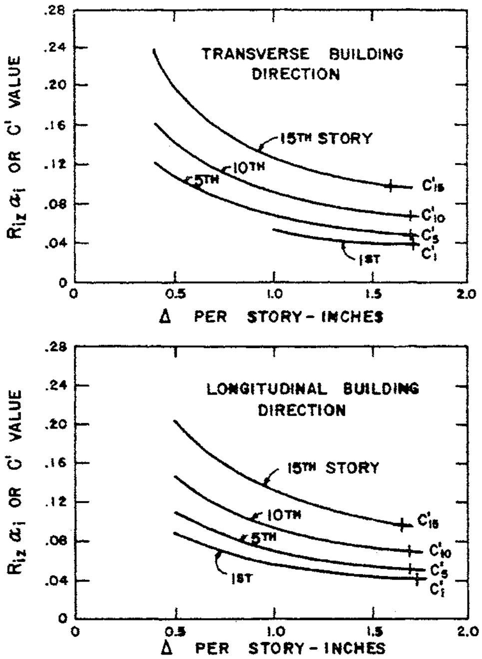


FIG. 9 — RESERVE ENERGY ANALYSIS BUILDING M-1, F=1.00

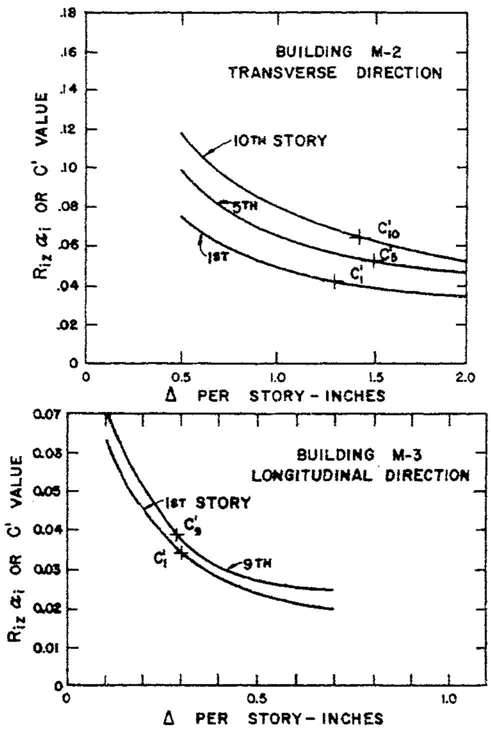


FIG. 10 — RESERVE ENERGY ANALYSIS "BUILDINGS" M-2 & M-3, F=1.00

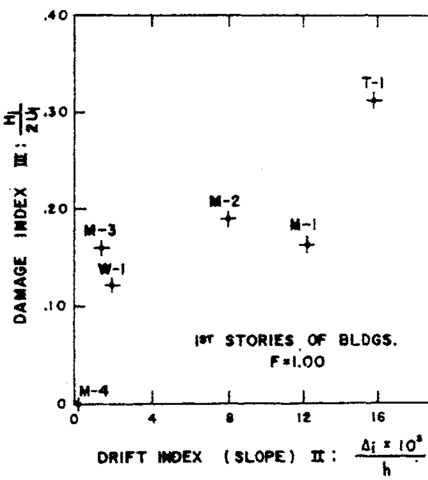


FIG. 11 - COMPARISON OF DAMAGE & DRIFT WHEN ENERGY DEMAND IS RECONCILED

SYMBOL	RATING NAME AND DEFLECTION POSITION	INDEX CONSIDERATION: I. ENERGY $= \frac{\Sigma(U_i - 0.5 H_i)}{0.014 W F^2 \sqrt{T_i Z}}$ *	II. DRIFT $= \frac{\Delta_i \times 10^3}{h}$	III. DAMAGE $= \frac{H_i}{2 U_i}$
"BN" (i=n)	"NO DAMAGE" Δ_n AT THE FIRST PEAK OR Δ BEYOND WHICH $H_n > 0.1 U_n$	$\frac{\Sigma(U_n - 0.5 H_n)}{0.014 W F^2 \sqrt{T_n Z}}$	$\frac{\Delta_n \times 10^3}{h}$	$\frac{H_n}{2 U_n}$
"BD" (i=d)	"DRIFT"-(SLOPE) Δ_d AT 0.004h	$\frac{\Sigma(U_d - 0.5 H_d)}{0.014 W F^2 \sqrt{T_d Z}}$	$\frac{\Delta_d \times 10^3}{h}$ (=4)**	$\frac{H_d}{2 U_d}$
"BE" (i=e)	"ENERGY" Δ_e SUCH THAT BE I =1.0	$\frac{\Sigma(U_e - 0.5 H_e)}{0.014 W F^2 \sqrt{T_e Z}}$ (=1.00)**	$\frac{\Delta_e \times 10^3}{h}$	$\frac{H_e}{2 U_e}$
"BF" (i=f)	"FAILURE" $\Delta_f = \Delta_{MAX}$	$\frac{\Sigma(U_f - 0.5 H_f)}{0.014 W F^2 \sqrt{T_f Z}}$	$\frac{\Delta_f \times 10^3}{h}$	$\frac{H_f}{2 U_f}$

* FOR T_i = 0.30 TO 3.0 SEC.; FOR OTHER VALUES THE GENERAL FORM IS APPLICABLE: $I = \frac{2.72\, \Sigma(U_i - 0.5 H_i)}{W \alpha_i^2 T_i^2 \sqrt{Z}}$; 10% ALLOWANCE FOR G.

** BY DEFINITION

F SHALL BE REPORTED IN THREE VALUES: 2.77, 1.83 & 1.00

SAMPLE REPORT:

BUILDING X - 1ST STORY (N-S) (10 STORIES TOTAL)

ANALYSIS BY ________________ DATE ________

	I			II	III
	2.77	1.83	1.00		
BN	0.67	1.53	5.17	3.2	0.10
BD	0.94	2.15	7.23	4.0	0.20
BE	1.00	—	—	4.8	0.30
	—	1.00	—	2.3	0.06
	—	—	1.00	0.6	0
BF	1.22	2.79	9.40	7.0	0.40

FIG. 12 — EARTHQUAKE RATING SYSTEM FOR STRUCTURES

FIG. 13 — PRINCIPAL DERIVATIONS

(ALSO REFER TO NOMENCLATURE AND FIGURES)

LET $R_{iz}\alpha_i$ = THE EQUIVALENT "STATIC" DESIGN COEFFICIENT

SAFETY FACTOR FOR COEFFICIENT $= B = \frac{C'}{R_{iz}\alpha_i}$ ——— (a)

ASSUME TOTAL STRAIN ENERGY AND ENERGY DISSIPATION VALUE $= U_i - 0.5H_i$

KINETIC ENERGY (GROSS) $= E = \frac{W}{2g} v_i^2 = \frac{Wg}{8\pi^2} \alpha_i^2 T_i^2$ ——— (b)

FOR T = 0.3 SEC. TO 3 SEC., $E = \frac{W}{2g} \times F^2 \sqrt{T_i}$ ——— (c) *

"I" INDEX (GENERAL FORM) $= \frac{U_i - 0.5H_i}{E} = \frac{2.44\,\Sigma(U_i - 0.5H_i)}{W\alpha_i^2 T_i^2 \sqrt{Z}}$ ——— (d)

FOR T = 0.3 SEC. TO 3 SEC., $\text{"I"} = \frac{64.4\,\Sigma(U_i - 0.5H_i)}{W F^2 \sqrt{T_i Z}}$ ——— (e) *

ASSUME C' AND $R_{iz}\alpha_i$ VARY AS $\sqrt{U_i - 0.5H_i}$

THEN SAFETY FACTOR $B = \frac{C'}{R_{iz}\alpha_i} = \sqrt{\text{"I"}}$ ——— (f)

AND $B = \sqrt{\frac{2.44\,\Sigma(U_i - 0.5H_i)}{W\alpha_i^2 T_i^2 \sqrt{Z}}}$ ——— (g)

FROM ABOVE AND GEOMETRY OF FIG. 2:

EQ. (1): $R_{iz} = T_i C' \sqrt{\frac{W\sqrt{Z}}{2.44\,\Sigma(U_i - 0.5H_i)}}$ ——— (h)

EQ. (2): $R_{iz} = T_i \sqrt{\frac{C'}{\Delta_D}} \sqrt{\frac{D_z\sqrt{Z}}{1.22\,\Sigma(U_i - 0.5H_i)}}$ ——— (i)

EQ. (3): $R_{iz} = \rho^* \frac{T_i}{T} \sqrt{\frac{D_z\sqrt{Z}}{\Sigma(U_i - 0.5H_i)}}$ ——— (j)

*NOTE $\rho = 1$ IF $T = 2\pi\sqrt{\frac{W}{g K_D}}$; OTHERWISE $\rho = 0.90T\sqrt{\frac{C'}{\Delta_D}}$ ——— (k)

ALL UNITS IN FEET, POUNDS, SECONDS.

* NOTE: THESE (SPECIAL) EQUATIONS ARE VALID ONLY WHEN $v = F(T)^{1/4}$ IS EMPLOYED

TABLE 1

BUILDING W-1, RESERVE ENERGY ANALYSIS; REF. (6)

Trial No. i	Δ_i (inch)	U_i, (ft-lb)	H_i, (ft-lb)	$K_i = \frac{2U_i - H_i}{\Delta_i^2}$	R_i	$T_i = T\sqrt{\frac{K_D}{K_i}}$	α_i (F = 1.83)	$R_i \alpha_i$
1	0.4	2,900	736	4.56×10^6	0.340	0.262	0.95	0.322
2	0.55	4,860	1,270	3.99×10^6	0.295	0.297	0.88	0.258
3	0.80	8,180	2,070	3.22×10^6	0.250	0.328	0.82	0.205
4	1.2	14,750	3,420	2.61×10^6	0.206	0.365	0.77	0.160
5	1.6	22,000	7,440	2.05×10^6	0.197	0.412	0.70	0.138
6	2.0	30,400	10,890	1.80×10^6	0.180	0.440	0.66	0.119
7	2.4	38,400	14,400	1.56×10^6	0.173	0.474	0.62	0.108

See Fig. 2 for basis and Fig. 8 for data. $P_D = 54^k$; $\Delta_D = 0.05$ ins.; $K_D = 13 \times 10^6$ lb/ft; D = 112 ft-lb.; T = 0.164; C' = 0.19.

TABLE 2

COMPARATIVE DATA FOR LOWER STORIES OF BUILDINGS INVESTIGATED

Bldg. No.	Brief General Description	Static Coefficient Value C^1	Average "Seismic" Weight in Lbs/Cu.Ft.	Av. Elastic Stiffness K_D, Lbs. per in., per sq. ft. of Floor Area	Deflection of 1st Story to Develop Energy for El Centro 1940 N-S (F = 1.00), Inches
W-1	A single-story wood frame building, diagonal sheathing	0.19	2.4	250	0.4
T-1	A multi-story traditional type building, steel frame, unreinforced masonry walls.	Wind design	17	2,400	2.4
M-1	A modern multi-story "glass-skin" type office building, steel framing, no walls.	0.041	10.2	161	1.7
M-2	A modern multi-story "glass-skin" type office building, steel frames, no walls.	0.041	8.1	150	1.3
M-3	A modern multi-story office building with frame encased in concrete walls.	0.035	14.8	8,560	0.4
M-4	A 2-story heavy commercial type building, concrete frame and shear walls	0.24	21	12,700	0.02