EXTREME VALUE THEORY AND EARTHQUAKES

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ABSTRACT

Extreme value theory is applied to New Zealand earthquake data to give estimates of probable maximum magnitudes within specified regions and periods of time.

The purpose of this paper is to demonstrate that the application of extreme value theory can be used to estimate the probable maximum magnitude of an earthquake occurring in a given locality over a period of, say, 100 years.

Because the theory of extreme values has been frequently described (see, for example, reference 1), no attempt will be made to describe it in this paper.

The data used were obtained from the Seismological Observatory of the Geophysics Division, D.S.I.R. Unfortunately, it is only in the last few years that there has been a complete seismograph network covering the whole of the country. Consequently, the shocks used in this paper come from a restricted area (see figure 1), chosen in such a way that the great majority of shocks of magnitude 4.5 or greater on the Richter scale would have been detected in that area over the period, January 1, 1942 to September 30, 1961.

For each shock, the date of occurrence, the latitude and longitude of the location of the epicentre, the magnitude and an estimate of the focal depth were available. In this study only shocks of magnitude 4.5 or greater, occurring at focal depths of less than 100 Kms, were used.

The shocks were arranged in chronological order of occurrence and divided into successive periods of 182 days, i.e., approximately half-yearly periods. For the first attempt, all shocks in the area were included, that is, no attempt was made to distinguish between various localities in the detection area. Within each half-yearly period, the maximum magnitude was noted, there being 39 such maxima altogether. When these maxima are plotted on Gumbel probability paper, they are seen to fall reasonably well on a straight line (figure 2). The solid straight line drawn on the graph is the Gumbel line for these New Zealand shocks.

The Gumbel line would suggest that the magnitude of the probable maximum shock to be expected in the detection area in 100 years would be of magnitude 8, a quite realistic figure. A check back into historical times is in order. From the graph we find that the recurrence interval of a shock of magnitude 6.75 is seven years. This means that if the theory is correct then one would expect to find 14.3 shocks of magnitude 6.75 or greater occurring in the detection area over the last 100 years.

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Reference to the list of major New Zealand earthquakes with focal depth of less than 100 Kms compiled by Hayes(2) shows that from October 19, 1848 to June 24, 1942, sixteen shocks of magnitude 6.75 or greater occurred within the detection area. Hence, depending on just which 100 consecutive years of New Zealand settlement one cares to select from 1848 to 1964, one finds that the number of observed shocks is either 14, 15, or 16, an excellent agreement with the predicted 14.3

The reference point, magnitude 6.75, has been taken because in Hayes' list, two shocks have been described as having magnitudes 6.5 - 7. It is not thought likely that any significant shocks of this magnitude or greater in the detection area would have been unnoticed in period 1848 to the present day. As a better check we can, perhaps, consider shocks of magnitude 7 or more. Then over a period of 120 years, we would expect to find about ten such shocks while Hayes lists 14, again a reasonable agreement.

It must be noted that because of the finite number of observations, the magnitude of the probable maximum shock in 100 years is subject to sampling uncertainties. To a first approximation we may say that there is a probability of about 2/3 that magnitude of the probable maximum shock for all New Zealand over a period of 100 years lies in the range 7.45 - 8.55, the most likely value being of magnitude 8.0. According to seismologists, one earthquake of magnitude about 8 has occurred in New Zealand in the last hundred years, this being the South-west Wairarapa earthquake of January, 1855.

It is also of some scientific interest to note that the probable maximum shock for 1,000 years for all New Zealand is 9.1. As the greatest magnitude recorded anywhere in the world for shallow shocks(3) is 8.9 this figure of 9.1 seems to be a reasonable result and gives confidence to the overall soundness of the extreme value theory as applied to seismological phenomena.

Another valuable confirmation is to be found in the work of John M. Nordquist(4). Nordquist studied the data from Southern California, the area of which, about 100,000 square miles, is very close to that under consideration in this paper. However, his data and method of treatment were quite different. His records extended over the period 1934-1942 and included shocks of magnitude 2.5 or more. Because he had data for so many small shocks, he used periods of 10 days rather than 182 as used in this paper. His data have been recomputed in order to enable them to be plotted in figure 2. In this figure the solid line is the Gumbel line for the New Zealand shocks of focal depth less than 100 Kms, which makes the two sets of shocks quite comparable as earthquakes do not occur in California at a depth greater than 50 Kms. The broken line in figure 2 is the Gumbel line for Southern California. To avoid confusion, only the New Zealand points have been actually plotted. Considering the lengths of the records (5 years and 20 years respectively), the fact that the New Zealand data included only shocks of magnitude 4.5 or greater and the Californian data included shocks of magnitude 2.5 or greater and the fact that six-monthly periods were used in the calculation of the New Zealand data and 10 day periods in the Californian, the agreement is surprisingly close.

Figure 3 shows the Gumbel line (the solid line) for all New Zealand shocks of focal depth equal to or greater than 100 Kms. Again the points fit a straight line quite well, although at the low end, magnitudes 4.5 to 5.2, the points fall off. The broken line in figure 3 shows the Gumbel line for all New Zealand shocks of focal depth less than 100 Kms. There is no significant difference between the two lines.

As mentioned earlier in this paper, the magnitude of the 1,000 year maximum probable shock was calculated to be 9.1. As a test of plausibility and possible verification of this estimate, Nordquist's data for the world's largest earthquakes (taking again the maximum recorded magnitude in a 10 day period) have been recomputed and plotted, also in figure 2. It will be seen on this graph that the 100 year maximum probable shock for the whole world would have a magnitude of 8.93. The 1,000 year shock would have a magnitude of 9.58, so that it would appear that on a world basis the extreme value theory yields results in general agreement with the seismologists' views on the magnitude of what might be called the biggest possible earthquake.

The next point of interest is to see how the probable maximum shock in, say, 100 years, varies from one locality to another. To do this, a number of major cities and towns in the detection area were selected as the approximate centres of rectangles having sides 2 degrees of latitude and 3 degrees of longitude. These dimensions give rectangles of sides 138.3 and 148 miles or, in kilometres, 222 x 237. *These dimensions were chosen for two* reasons; first, the area had to be sufficiently large to provide shocks in most of the half-yearly periods and, second, the radius of damage for a shock of magnitude 8, is certainly of the order of 70 miles or 110 kilometres.

With such a relatively large area about each city or town, it is not surprising that the areas overlap. The results are shown in table I. In this table, the exact latitude and longitude are given for the town or city which names the locality, followed by the degrees of latitude and longitude which form the boundaries of the locality. It will thus be seen that in all cases, the town or city is approximately in the centre of the locality. The next three lines for each locality give the magnitude of the probable maximum shock to be expected in 1,000 years, 100 and 50 years respectively. These three values permit the plotting of the Gumbel line on Gumbel probability paper for any of the given localities. It will be noted that three entries are given for the 100 year period. The middle entry is the probable maximum, the two outer readings denoting the limits between which the probable maximum has a 2/3rds probability of occurrence. The final line shows the number of shocks of magnitude 6.5 or greater to be expected within each locality over a period of 100 years. The numbers in the brackets show the magnitudes of such shocks which actually occurred in the particular locality over the years 1848 to 1955, as recorded by Hayes. A map showing the positions of the epicentres of these shocks has been published by Eiby(5). In the case of Hamilton, none were predicted and none observed. It will be noted that the agreement between prediction and observation is very good, in every case.

Admittedly, one cannot be sure of the accuracy and completeness of the historical records, but in any event, the magnitude of the greatest shock (which should be subject to the least error) always lies in the 2/3 probability range for the magnitude of maximum probable 100 year shock.

Two exceptional cases must be noted. In the case of the Auckland locality, in only three of the 39 half-yearly periods were there any shocks of magnitude of 4.5 or more. These were of magnitudes 4.5, 4.9 and 5.3. Because of this paucity of shocks, it was not thought wise to include the Auckland locality in the table. For what it is worth, fitting the three points for the Auckland locality by least squares would give a probable maximum magnitude in 100 years of 6.46. A more reliable estimate for the Auckland region would be possible, if one were able to obtain magnitudes of shocks of, say, 3 or greater.

In the case of the Christchurch locality, all the shocks (except one) of magnitude 4.5 or more in the twenty year record, occur at a distance of 35 or more miles away from the city. Whether this is a statistical freak or whether there is some geological phenomenon thought capable of explaining this peculiarity is a question that the author is unable to judge. Again, information regarding the frequency of smaller shocks would be helpful in elucidating this question. However, there is no doubt that shocks of magnitude 7.3 to 8.1 can be expected within 70 miles of that city and that such shocks could produce more than superficial damage.

It is realised, of course, that knowledge of the magnitude of a shock with an epicentral depth of less than 100 Kms, is of less interest to the engineers and architects than a knowledge of the felt intensity at the site of the building.

It is hoped at a later date to see if extreme value theory can be applied to felt intensity data as successfully as appears to have been done in the case of instrumental magnitudes. As far as the author is aware, lack of data on ground accelerations unfortunately prevents this type of analysis from being applied to data from even one spot anywhere in the world.

Finally, it is worth noting that in a personal communication, Dr J.H. Darwin of the Applied Mathematics Division, D.S.I.R., has pointed out that in this application we may expect a straight line quite apart from the usual extreme value arguments. If shocks occur in unit time at rate a, and if the probability of a shock of magnitude greater than x is exp(-bx), (the frequency-magnitude law), then the probability that all shocks in a period t are less than x is:-

$$F(x) = \exp(-at\exp(-bx)).$$

In extreme value theory:-

$$F(x) = \exp(-\exp - p(x-q)).$$

Hence, it appears that the statistical theory of extreme values gives formulae that might well be expected from seismological evidence, principally the frequency-magnitude law and the fact that the time of occurrence of the largest earthquake in a six-monthly period tends to be randomly distributed in time.

Acknowledgements

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References

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3. Richter, C.F., "Elementary Seismology," Freeman and Co., p.350, 1958.

4. Nordquist, John M., "Theory of Largest Values Applied to Earthquake Magnitudes," Transactions American Geophysical Union, Vol.26, part 1, August, 1945.

5. Eiby, G.A., "Earthquakes," Frederick Muller Ltd., London, 1957, p.117.

TABLE I

HAMILTON 37°48'S 175°15'E

Lat. 37°S - 39°S
Long. 173°30'E - 176°30'E
1000 yrs 7.6
100 yrs 5.9 - 6.4 - 7.3
50 yrs 5.85
None

WANGANUI 39°52'S 175°2'E

Lat. 39°S - 41°S
Long. 173°30'E - 176°30'E
1000 yrs 8.7
100 yrs 6.8 - 7.3 - 8.2
50 yrs 6.7
5 (7, 7.3, 7.5)

NAPIER 39°38'S 176°54'E

Lat. 38°30'S 176°54'E
Long. 175°30'E - 178°30'E
1000 yrs 8.3
100 yrs 6.6 - 7.0 - 7.8
50 yrs 6.5
4 (7.2, 7.0, 7.7)

WESTPORT 41°52'S 171°54'E

Lat. 41°S - 43°S
Long. 170°30E - 173°30'E
1000 yrs 8.7
100 yrs 6.5 - 7.0 - 7.5
50 yrs 6.5
4 (6.7, 7.0, 7.0, 7.3)

TAUPO 38°45'S 175°55'E

Lat. 38°S - 40°S
Long. 174°30'E - 177°30'E
1000 yrs 8.6
100 yrs 6.6 - 7.1 - 8.0
50 yrs 6.5
4 (6.7, 7.0, 7.0, 7.3, 7.5)

NEW PLYMOUTH 39°1'S 174°2'E

Lat. 38°S - 40°S
Long. 172°30E - 175°30'E
1000 yrs 7.7
100 yrs 5.9 - 6.4 - 7.2
50 yrs 6.1
1 (7.0)

WELLINGTON 41°20'S 174°45'E

Lat. 40°S - 42°S
Long. 173°E - 176°E
1000 yrs 8.6
100 yrs 7.1 - 7.7 - 8.3
50 yrs 7.3
5 (6.7, 7, 7.3, 7.3, 7.5, 8.0)

TAURANGA 37°40'S 176°10'E

Lat. 36°30'S - 38°30'S
Long. 174°30'E - 177°30'E
1000 yrs 7.6
100 yrs 6.2 - 6.7 - 7.2
50 yrs 6.4
1 (6.7)

CHRISTCHURCH 43°32'S 172°40'E

Lat. 42°30'S - 44°30'S
Long. 171°E - 174°E
1000 yrs 8.9
100 yrs 6.6 - 7.3 - 8.1
50 yrs 6.8
3 (7.0, 7.0, 7.5)

GISBORNE 38°40'S 178°3'E

Lat. 37°30'S - 39°30'S
Long. 176°30'E - 179°30'E
1000 yrs 8.8
100 yrs 7.0 - 7.6 - 8.3
50 yrs 7.2
4 (6.7, 7.2, 7.3, 7.7)

ALL NEW ZEALAND FOCAL DEPTH LESS THAN 100 Kms.

1000 yrs 9.1
100 yrs 7.45 - 8.0 - 8.5
50 yrs 7.68

SOUTHERN CALIFORNIA

1000 yrs 9.26
100 yrs 7.45 - 8.05 - 8.6
50 yrs 7.68

ALL NEW ZEALAND FOCAL DEPTH GREATER THAN 100 Kms.

100 yrs 7.83
50 yrs 7.5

WORLD'S LARGEST EARTHQUAKES

1000 yrs 9.6
100 yrs 8.93
50 yrs 8.78

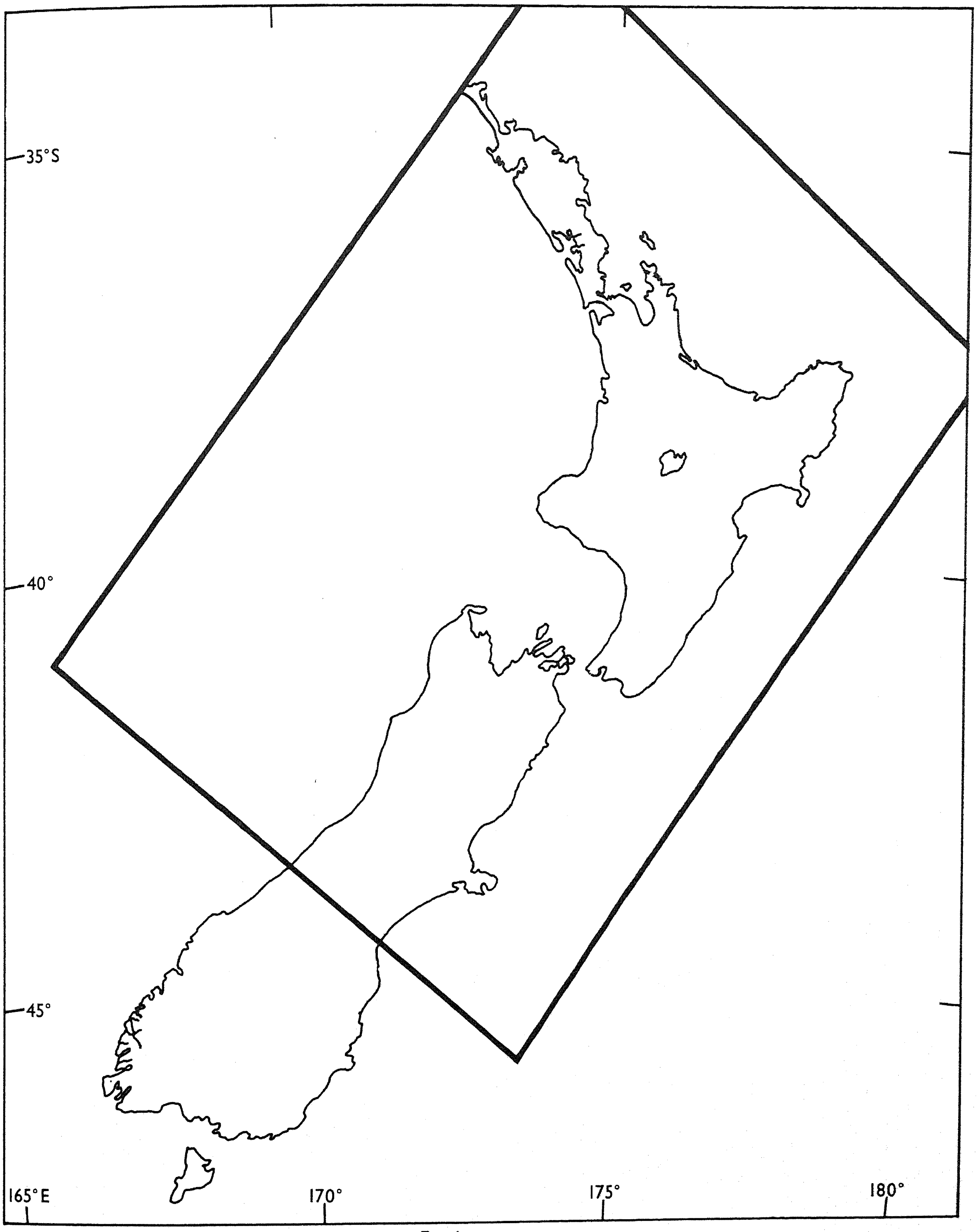


Fig 1

Fig 2

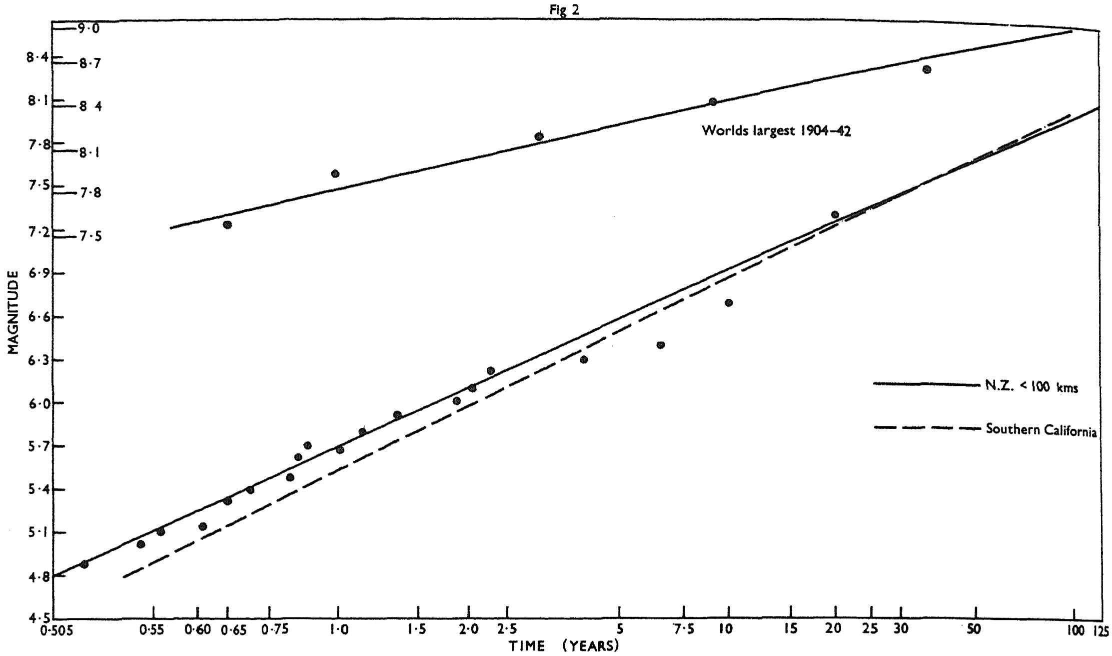
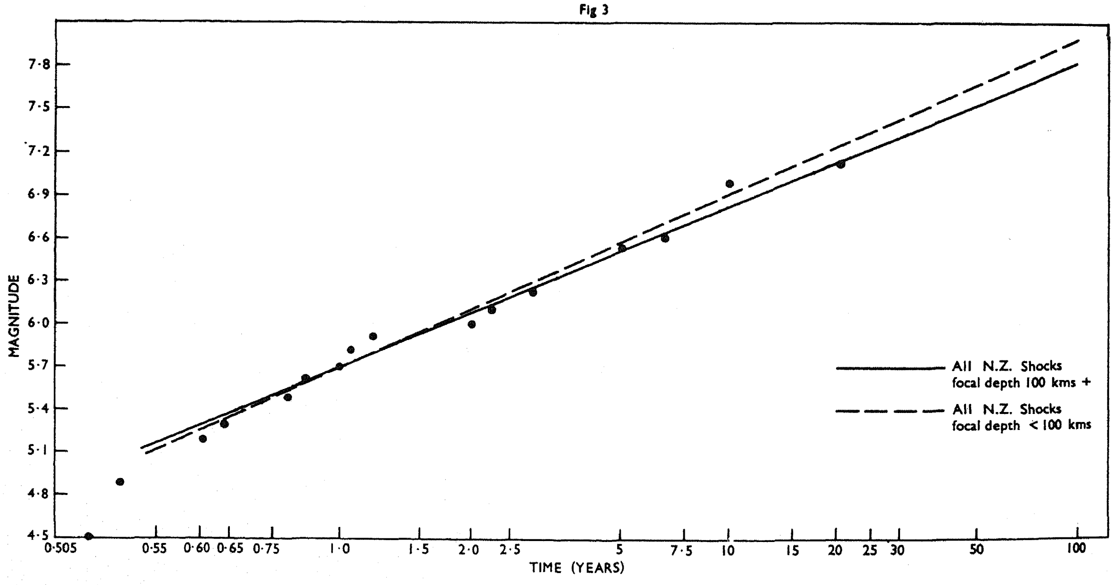

MAGNITUDE
TIME (YEARS)
Worlds largest 1904–42
N.Z. < 100 kms
Southern California
4·5
4·8
5·1
5·4
5·7
6·0
6·3
6·6
6·9
7·2
7·5
7·8
8·1
8·4
7·5
7·8
8·1
8 4
8·7
9·0
0·505
0·55
0·60
0·65
0·75
1·0
1·5
2·0
2·5
5
7·5
10
15
20
25
30
50
100
125

Fig 3


MAGNITUDE
TIME (YEARS)
All N.Z. Shocks focal depth 100 kms +
All N.Z. Shocks focal depth < 100 kms
4·5
4·8
5·1
5·4
5·7
6·0
6·3
6·6
6·9
7·2
7·5
7·8
0·505
0·55
0·60
0·65
0·75
1·0
1·5
2·0
2·5
5
7·5
10
15
20
25
30
50
100

EXTREME VALUE THEORY AND EARTHQUAKES

BY I. D. DICK

QUESTION BY: W.G. MILNE - CANADA

Do you propose to relate the results to seismic zoning?

AUTHOR'S REPLY: In my opinion it is difficult to know how this statistical analysis of earthquake magnitudes should be interpreted for purposes of seismic zoning. First there is the problem of translating the magnitude parameter into terms of ground acceleration. Secondly, it is not clear how one should regard a statistical type of result when one is concerned with the protection of human life. For example, are we to take the 100 year, the 200 year, or the 500 year maximum? Finally, it has only been possible to make statistical estimates for the limited number of regions listed in Table 1; one should note, however, that the least of the expected maximum magnitudes over a 100 year period - that for the regions around Hamilton and New Plymouth - is 6.4, which certainly represents a potentially destructive earthquake.