STATISTICAL PARAMETERS APPLIED TO SEISMIC REGIONALIZATION

By

W. G. Milne [1] and A. G. Davenport [2]

ABSTRACT

Previous studies on the seismicity of a small area of Western Canada include a plot of the epicentres of all earthquakes between 1951 and 1962; a strain-energy release study; and a computation of the formula relating magnitude and frequency of occurence of the earthquakes. The extreme value theorem of statistics is applied to the same data to calculate a) the maximum magnitude of an earthquake expected in an area with a certain return period, and b) the maximum ground motion expected at selected points within an area with a certain return period. The parameters of this computation are shown to be of use in evaluating the seismicity of an area.

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INTRODUCTION

Prior to making a study of the seismicity of the whole of Canada, a small area on the Pacific Coast is being used as a base for testing new methods. The seismicity of this area has already been studied to a certain degree (Milne, 1964), and the results of this study are outlined below. In this small area complete data are available for the year 1951 to 1962, although all the earthquakes are small.

The seismicity of an area can be expressed in several ways. The basic method is to simply mark all the epicentres on a map. To refine this slightly, one might indicate an epicentre by a circle the radius of which is proportional to the magnitude of the earthquake represented. Others have expressed seismicity in terms of strain-energy release in some selected unit area and time. This method seems to define the active tectonic zones within the area under study. It is possible to calculate the magnitude - frequency of occurrence relationship which indicates the number of earthquakes of a given magnitude per unit of time. Another manner of expressing the same feature is to use the extreme value theorem of statistics to calculate the largest earthquake with a return period of a certain number of years. Finally in this report, an attempt is made to calculate the largest ground amplitude expected at points within the area with a certain return period. These results are displayed on a contour map. In all but the latter method the attention of the reader is focused on the epicentre of the earthquake; but in the final case the point of observation of the event becomes the centre of attention.

It is anticipated that the seismicity of the whole of Canada can be expressed by similar methods, but in so doing considerable more attention can be paid to boundaries of tectonic features, and local superficial geology than has been possible in this paper.

RESULTS OF PREVIOUS STUDIES

The area selected for this study includes southern Vancouver Island, British Columbia, and some of the mainland. The earthquakes in this area have been listed in Dominion Observatory publications since 1951. Epicentres are within a short distance of a three-station seismograph network. Magnitudes have been calculated by Richter's method (1942) from maximum trace amplitudes recorded at the three stations.

Insert Figure 1

Figure 1 shows the distribution of all located earthquakes during the period. A rather randon scatter is apparent. No attempt has been made to indicate the magnitude of the individual earthquakes on this diagram, for the range of magnitudes is too small to present a pattern.

The strain-energy release diagram for the area is shown as Figure 2.

Insert Figure 2

The value of the strain-energy release ($E^{1/2}$) for each earthquake is calculated from the magnitude by the equation (Richter, 1958)

$$\log_{10} E = 11.4 + 1.5\, M$$

The area was divided into sections of 0.1° square and the value of $E^{1/2}$ was summed for each unit area for the period under study. The results of this study indicate some tendency to follow local geological features along the Strait of Georgia.

The relationship between magnitude and frequency of occurrence takes the form

$$\log_{10}N = a + b(8-M)$$

where N is the number of earthquakes of magnitude $M \pm 0.5$. For this area a, and b, are found by a least squares method to be -1.26 and +0.67 respectively. This means that in this limited area, one can expect to find the following earthquakes in a ten year period:

0.1	earthquakes	with	magnitude	between	7 and 8	(or 0.01	per	annum)
0.5	"	"	"	"	6 and 7	(or 0.05	"	")
3.0	"	"	"	"	5 and 6	(or 0.3	"	")
12	"	"	"	"	4 and 5	(or 1.2	"	")
58	"	"	"	"	3 and 4	(or 5.8	"	")
269	"	"	"	"	2 and 3	(or 27	"	")
1000	"	"	"	"	1 and 2	(or 100	"	")

It must be pointed out that this relationship was derived from data over limited area, and is based upon observations of relatively small earthquakes over about a ten year period.

MAXIMUM MAGNITUDE STUDIES

A relationship, quite similar to that of the previous paragraph, can be found relating magnitude, and return period by means of the extreme value theorem of statistics. The same data are used as in the last paragraph of the preceding section.

If "p" is the probability that in a given series of earthquakes the maximum magnitude will have a value less than M, it can be shown that:

$$M = U + \alpha \left[-\log_e \left(-\log_e p \right) \right]$$

If the values of M versus $\left[-\log_e \left(-\log_e p \right) \right]$ are plotted for several periods of observation, U can be called the mode (or the value of M which occurrs most often). α is the slope of the line so plotted. U and α are parameters in this case. If N is the number of periods during which observations are made, p then becomes $N / (N + 1)$, or in other words p is the return period of an earthquake of magnitude M. From this relationship it is possible to estimate the largest magnitude to be expected with a return period of 50, 100 etc. years.

For the discussion here, an individual period is considered to be 0.5 years. Values of 3.49 and 0.77 have been obtained for U and α respectively. Methods outlined by Gumbel (1943) have been followed. In table I the maximum magnitude expected for certain integral return periods are shown. Thus it can be seen that an earthquake of magnitude 7.05 has a return period of 50 years (= 100 periods).

TABLE I

Return Period	No. of Years	Magnitude
15	7.5	5.6
20	10	5.8
40	20	6.3
100	50	7.05

Again it must be noted that the interval during which observations were made is short, and the magnitudes are all small, so this may not be a true physical picture of this area. When a larger tectonic region is studied over a longer period of time, the stronger earthquakes will have more influence.

MAXIMUM GROUND AMPLITUDE STUDIES

The original version of the Richter magnitude scale (1935) provides a means of obtaining the magnitude (M_L) of earthquakes within a 600 kilometre radius of the recording station. It is stated that M_L is equal to the common logarithm of the maximum trace amplitude of the horizontal Wood Anderson seismograph minus the common logarithm of the standard earthquake. This latter term includes the distance factor and is published in the above paper as ($-\log A_0$). If the value of M_L is known, and if the distance is known so that ($-\log A_0$) can be obtained, the value of the logarithm of the maximum trace amplitude of the earthquake as recorded on a standard Wood Anderson seismograph can be obtained. The constants of the standard Wood Anderson seismograph are known, so that it is possible to obtain the ground amplitude in millimetres. Since most seismographs are upon bedrock, this ground amplitude calculated is that of the rock. The earthquakes in this discussion are small, and within 600 kms. of the seismograph; thus the frequency spectrum is not wide. The published work of authors who have studied the spectra of strong earthquakes (Housner and Jennings, 1964) indicate that the maximum amplitude has a frequency between 1 and 2 c.p.s. The same is evident for small earthquakes of this series. Thus the static magnification (2800) of the standard Wood Anderson torsional seismograph can be used to calculate the ground motion. Again it must be pointed out that this ground motion is the horizontal component of the motion on bedrock.

Points have been selected within the area under study at which the ground amplitude is to be determined. This process is carried out for each earthquake in this interval. Again a period of 0.5 years is selected, and for each point chosen, the maximum ground amplitude is found. Proceeding as in the previous section the extreme value theorem of statistics can be used to determine the maximum expected ground amplitude at each point with a return period of a selected number of years. The return period of 20 years (= 40 periods) has been chosen for this report. On Figure, the maximum horizontal ground motion on bedrock with a return period of 20 years is indicated at each of the 7 points selected. Contour lines have been drawn through points of equal ground amplitude.

Insert Figure 3

Of equal interest are the values of "U" and α for the seven points of Figure 3. These are listed in Table 2, and plotted on Figure 4.

Insert Table 2

"U" is the zero intercept as has been pointed out; or alternatively it is called the mode, that is, the value of logarithm of the amplitude which occurrs most frequently. "U" thus indicates the general level of seismic activity. "α" is the slope of the line obtained or the dispersion when amplitude is plotted versus $[-\log_e(-\log_e p)]$

It is used to estimate the probable maximum value of the amplitude at some future date. If for one site (such as Alberni), a low value of "U" but a relatively high value of "α" is obtained one would expect few small earthquakes with a very strong one from time to time. At another location, for example Victoria, a larger "U", with a small "α", would mean that there are many moderate sized earthquakes, with few large events. A relatively large "U" and "α", indicates the presence of a dangerous situation, whereas low values of both parameters point to the absence of any serious earthquake condition. Thus if a structure

Insert Figure 4

were designed for temporary use the "U" term should be the guide, for it establishes the background level of seismic activity. However, if the structure is to have a long life, the "α" term, which indicates the maximum event expected, must also be considered.

Values of these parameters would be related to the geological structure in nearby areas. A high mode would be found at sites where there are many small earthquakes in a much faulted geological structure. In these cases the strength of the earth's crust would be weak. Where a high value of "α" is found, the system of fault must be such that a large accumulation of strain energy arises which must be followed by a rather large earthquake.

The pattern of Figure 3 is what one might expect. A magnitude 6 earthquake northwest of Alberni influences this station greatly. The contours seem to follow the geological features on this map, but this need not be so when large earthquakes are experienced in the map area. When such a routine is carried out for all Canada, a map will be available showing the maximum expected horizontal ground motion on bedrock with frequencies within a narrow band.

The next step in such a process is to predict the amplitudes on any soil structure, given the amplitude on bedrock. This has been attempted by Kanai (1956) by the use of an admittance function.

SUMMARY AND CONCLUSIONS

The use of the statistical parameters, U and α is suggested as a practical measure of seismicity of an area. With these values one can define the background level of seismic activity at location, and can also estimate the duration of large events from this background. The value of the parameters in this case refers to a very special area, but a similar program has been prepared to cover all of Canada.

When the computation has been completed for a large area, an attempt will be made to relate the values of the parameters with local geological features.

ACKNOWLEDGEMENTS

The writers wish to acknowledge the support and assistance of Dr. A.E. Beck, Dr. R. F. Mereu, and Dr. R.J. Uffen of the University of Western Ontario, and Dr. John H. Hodgson of the Dominion Observatory. Financial assistance has been provided through a grant-in-aid of research made by the National Research Council of Canada to Dr. Uffen.

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TABLE II

Location	$\log_{10} U$	$\log_{10}$	$\log_{10} A$ for T= 20 years	Ground Ampl (mn) for T= 20 years
Alberni	.18	.70	2.75	.201
Victoria	.68	.51	2.55	.130
Vancouver	.45	.30	1.55	.012
(1)	.38	.59	2.47	.105
(2)	.30	.52	2.22	.059
(3)	.58	.43	2.16	.051
(4)	.58	.60	2.78	.215

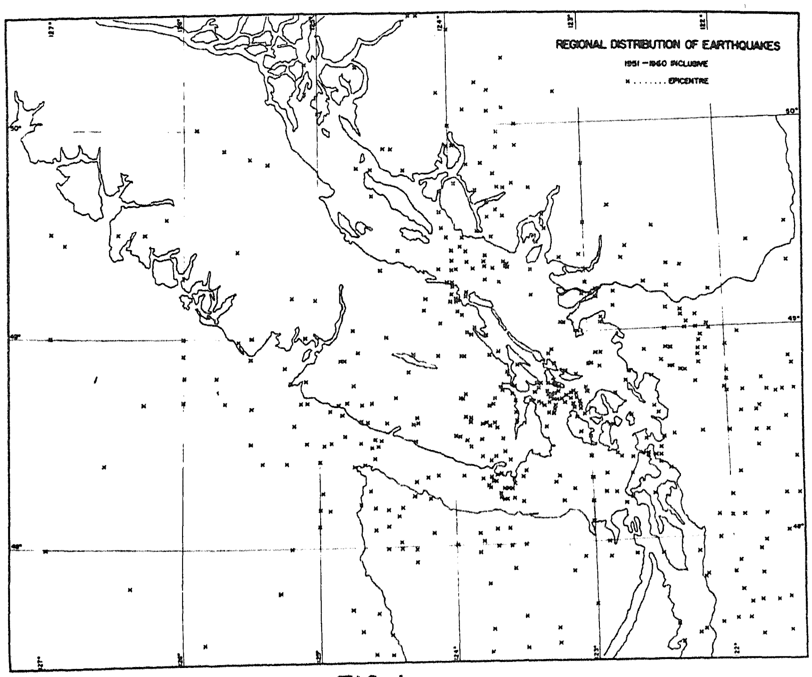

REGIONAL DISTRIBUTION OF EARTHQUAKES
1951 – 1960 INCLUSIVE
x . . . . EPICENTRE

FIG. I

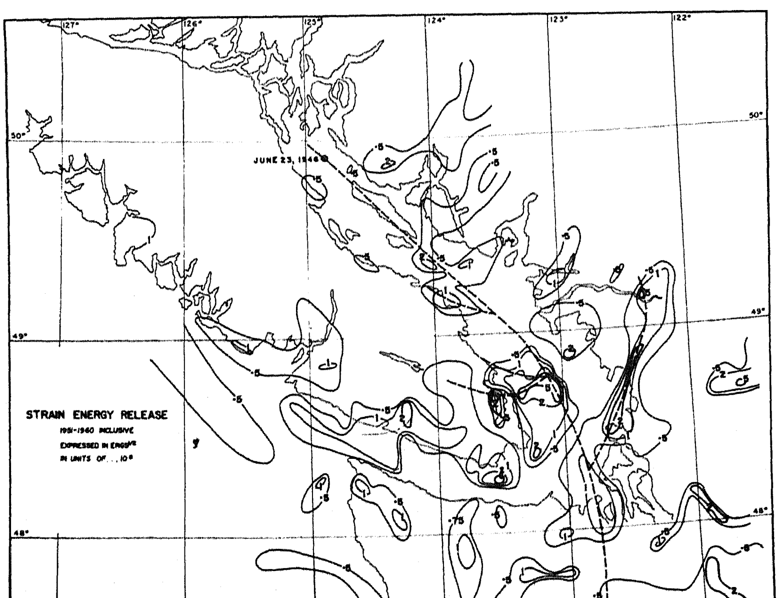
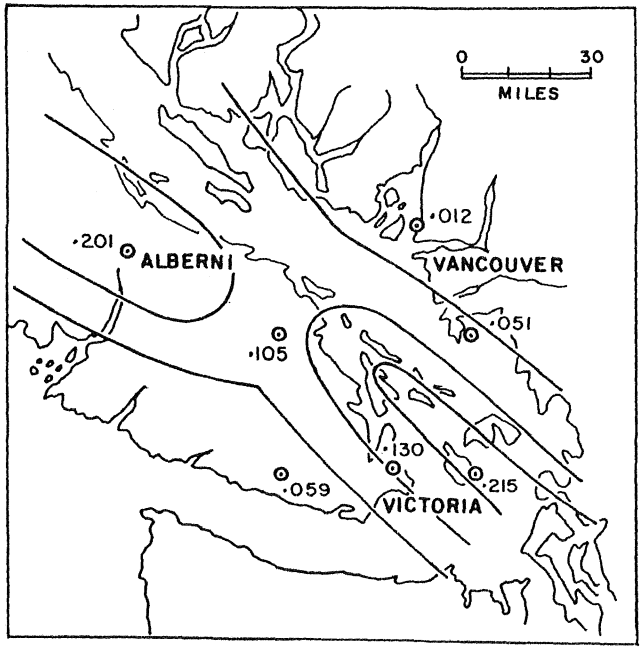
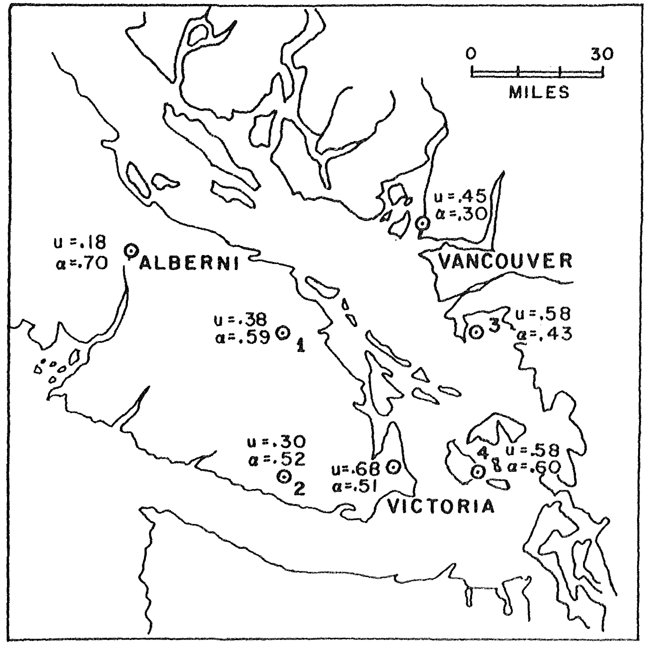

JUNE 23, 1946
STRAIN ENERGY RELEASE
1951-1960 INCLUSIVE
EXPRESSED IN $ERGS^{1/2}$
IN UNITS OF . . . 10

FIG. 2



MAXIMUM GROUND AMPLITUDE WITH 20 YR.
RETURN PERIOD

FIG. 3



STATISTICAL PARAMETERS

FIG. 4

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BY W.G. MILNE AND A.G. DAVENPORT

QUESTION BY: J.A. FISCHER - U.S.A.

Are all your present seismograph stations, and those proposed for the near future, on bedrock?

AUTHORS' REPLY: The seismographs operated in Canada are now on competent bedrock. In the area included in this paper the bedrock is granite or granidiorite. We have only a few more sites to finish in the Canadian network and bedrock is available at these sites.

QUESTION BY: R.D. ADAMS - NEW ZEALAND

I noticed the very fine detail of your figures. Could you tell us the width between the contours and if you consider this detail to be significant.

AUTHORS' REPLY: In the area covered by this paper the earthquakes are within the seismograph network, and they are all small. This paper is an attempt to define the seismicity of the area on the basis of these earthquakes. Since the method works in this case, we now propose to include the whole of Canada in a similar study using a longer period of time and larger earthquakes. The contours on the maps in the paper are thus significant only for the study of this select series of earthquakes. In the maps the width of the whole area is some 150 kilometers.

QUESTION BY: D.S. CARDER - U.S.A.

What are the limits of magnitude of earthquakes in S.W. Canada and adjoining N.W. United States.

AUTHORS' REPLY: The lower limit of magnitude is 2. This is chosen to eliminate many earthquakes recording on one station only, and to eliminate blasts. The largest earthquake of the series is one of magnitude 6 at the north west tip of the map. The next magnitude is 4.5 and the majority are smaller than this.