DYNAMIC RESPONSE OF COHESIVE SOILS FOR EARTHQUAKE CONSIDERATIONS

by

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ABSTRACT

Energy storage and dissipation characteristics of cohesive soil are important in earthquake phenomena. These are studied in steady state vibratory uniaxial compression. The soil response is represented in terms of viscoelastic parameters and shows definite nonlinear (underlinear) behavior, even at small values of dynamic strain. Phase angles (dissipation measure) between applied strains and resulting stresses are small and require special instrumentation. Nonlinear response gives calculated compression moduli and propagation velocities which decrease with dynamic stress or strain level. For the limited range considered, applied static stress level has little effect on the dynamic response.

INTRODUCTION

The energy storage and energy dissipation characteristics of soil are important considerations in the design and analysis of soil-structure systems subject to transient loadings such as those of a nature prevalent in earthquake phenomena. In addition to the size and mass of footing, configuration and mass distribution of the superstructure, and magnitude as well as nature of the applied excitation, the response of the soil-foundation phase of the soil-structure system is dependent on the dynamic stress-strain-time properties of the soil supporting the structure. Consideration must also be given to the possible influence of the static stress level on dynamic soil response. Dynamic soil properties also influence other important considerations of earthquake engineering, including wave propagation and attenuation of transient excitations. This paper deals with various aspects of the dynamic stress-strain-time response of cohesive soil important in earthquake phenomena.

Ideal elasticity and ideal flow are two special types of mechanical behavior. In general, when a material is deformed by means of external forces, part of the work is stored elastically and part is dissipated. For simplicity, such combined elastic and flow behavior will herein be called viscoelastic behavior. Since cohesive soils are neither ideally elastic nor ideally fluid in nature, a rheologic approach with a phenomenological point of view seems a quite rational way to study such materials. The research reported in this paper represents a phase of a dynamic viscoelastic investigation to obtain a clearer understanding of the actual linear and nonlinear response of a cohesive soil under steady state dynamic loading conditions. The advantage of using steady state techniques to study dynamic soil properties are illustrated in uniaxial compression. The present study is a continuation of the study of dynamic soil properties reported by Kondner.[1,2,3,4]

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THEORETICAL CONSIDERATIONS

Various aspects of the stress-strain-time response of cohesive soils reported in the literature indicate a nonlinear behavior. In addition, the dynamic loading of cohesive soil-solid systems is nonlinear. For a harmonic excitation, the response in nonlinear vibration may be periodic but not harmonic; hence, the assumption of a harmonic wave form inherently leads to the concept of linearity. Depending upon the degree of nonlinearity, it is possible to use a harmonic approximation for the response, particularly when only amplitude data are being considered on an individual point by point basis.

The method of testing utilizes cylindrical specimens compressed uniaxially under steady state sinusoidally imposed deformations and, hence, sinusoidally varying strains at a specified frequency of vibration. Figure 1 indicates a sinusoidally varying strain and the resulting periodic stress which may be approximated by a harmonic function (sine wave) generally out of phase (phase angle δ) with the strain, plotted as functions of time for a particular frequency of oscillation, ω.

Steady state vibratory deformation of a soil specimen generates an inertial force with amplitude $m\ddot{x}$ and a soil resisting force. Consideration of the soil as a viscoelastic material allows the soil resistance to be divided into two component forces - a dissipation or damping force of amplitude R_1 and a restoration or storage force of amplitude R_2. Since the response is considered to be harmonic, the various forces involved can be represented in phase diagram form as indicated in Fig. 2. This development utilizes the inertial term $m\ddot{x}$ as a force whose direction is opposite to that of the acceleration vector. The restoring force vector $R_2(x)$ is opposite to that of the displacement and the dissipation force vector $R_1(\dot{x})$ is in the opposite direction of the velocity while the amplitude of the forcing function is shown as leading the displacement by the phase angle δ.

Resolving the force F_D into components parallel and perpendicular to the inertial force and applying the equilibrium conditions at an instant of time gives

$$m\ddot{x} - R_2(x) + F_D \cos \delta = 0 \qquad (1)$$

and

$$R_1(\dot{x}) - F_D \sin \delta = 0 \qquad (2)$$

Equations (1) and (2)give the respective amplitudes of the restoring force as

$$R_2(x) = m\ddot{x} + F_D \cos \delta \qquad (3)$$

and the dissipation force as

$$R_1(\dot{x}) = F_D \sin \delta \qquad (4)$$

Dividing Eqs. (3) and (4), respectively, by the cross-sectional area of the soil specimen gives a restoring stress amplitude, σ_2, as

$$\sigma_2 = \sigma_I + \sigma_D \cos \delta \qquad (5)$$

and a dissipation stress amplitude, σ_1, as

$$\sigma_1 = \sigma_D \sin \delta \tag{6}$$

By proper choice of the length of specimen and frequency of oscillation, it is possible to make the inertial stress, σ_1, negligible. This requirement has been expressed by Ferry[5] as $L/\lambda \leq 0.1$, where L is the length of the specimen and λ is the wave length of the propagated wave. For negligible inertial stress, the restoring stress amplitude becomes

$$\sigma_2 = \sigma_D \cos \delta \tag{7}$$

The dissipation and restoring stresses of Eqs. (6) and (7) are functions of the applied strain amplitudes ε_D and the frequency of oscillation. By changing frequency in the test program, it is possible to vary the time of loading or strain rate amplitude since frequency has dimensions of the inverse of time (T^{-1}). Strain rate amplitude is proportional to the product of strain amplitude and frequency of oscillation and can be written as

$$\dot{\varepsilon}_D = \omega\varepsilon_D. \tag{8}$$

Stress, strain and strain rate can also be represented diagrammatically by the three rotating vectors given in Fig. 3. The stress vector may be separated into two components, one in phase and the other 90 degrees out of phase with the strain vector. The component of the stress in phase with the strain divided by the strain is called the storage modulus, E', and can be written

$$E' = \frac{\sigma_D \cos \delta}{\varepsilon_D} = \frac{\sigma_D}{\varepsilon_D} \cos \delta \tag{9}$$

The storage modulus is proportional to the energy stored and completely recovered for a single cycle of deformation. The component of the stress 90 degrees out of phase with the strain divided by the strain is called the loss modulus, E'', and can be written

$$E'' = \frac{\sigma_D \sin \delta}{\varepsilon_D} = \frac{\sigma_D}{\varepsilon_D} \sin \delta \tag{10}$$

The loss modulus is proportional to the energy dissipated for a single cycle of deformation.

The storage and loss moduli can be added vectorially in the complex plane to give a complex modulus, E*, written as

$$E^* = E' + iE'' \tag{11}$$

where $i^2 = -1$. If E' and E'' are real, the absolute value of the complex modulus can be written

$$|E^*| = |E' + E''i| = \sqrt{(E')^2 + (E'')^2} \tag{12}$$

Substitution of Eqs. (9) and (10) into Eq. (12) gives

$$|E^*| = \frac{\sigma_D}{\varepsilon_D}\left[\cos^2 \delta + \sin^2 \delta\right] = \frac{\sigma_D}{\varepsilon_D} \tag{13}$$

Thus, the absolute magnitude of the complex modulus is equal to the ratio of the stress amplitude to the strain amplitude; that is, ratio of peak stress to peak strain. This parameter is frequently used in viscoelasticity.

Note that Eqs. (9) and (10) are simply special cases of Eqs. (7) and (6), respectively, when σ_2 is assumed proportional to ε_D (proportionality coefficient E') and σ_1 is assumed proportional to ε_D (proportionality coefficient E'').

An important aspect of transient loading is the propagation of waves. The analysis of the propagation of waves in viscoelastic materials is complicated by the fact that the moduli are time dependent. For the case of a linear material, Leaderman[6] has shown that the velocity of propagation of a dilatation wave, C_E, is

$$C_E = \left[\frac{2}{(1 + \cos \delta)} \frac{|E^*|}{\rho}\right]^{\frac{1}{2}} \tag{14}$$

where ρ = mass density of the material.

There is a similar equation for the propagation of a distortion (or rotation) wave, where $|E^*|$ and δ are replaced by a complex shear modulus $|G^*|$ and a shear phase angle δ, respectively. The attenuation factor in compression, α_E, can be expressed as

$$\alpha_E = \frac{\omega}{C_E} \tan \left(\frac{\delta}{2}\right) \tag{15}$$

Physically, this factor is the reciprocal of the distance for the amplitude of displacement to decrease by a factor "e" (equal to 2.71828 ...).

For the special case of an elastic material, the phase angle δ vanishes; hence, cos δ and tan (δ/2) become unity and zero, respectively. Substitution of these values into Eqs. (14) and (15) gives

$$C_E = \left[\frac{|E^*|}{\rho}\right]^{\frac{1}{2}} = \sqrt{\frac{E'}{\rho}} \tag{16}$$

and

$$\alpha_E = 0 \, . \tag{17}$$

SOIL STUDIED

The soil investigated is a remolded clay sold commercially under the name Jordan Buff by the United Clay Mines Corporation, Trenton, New Jersey, U.S.A. The particle size distribution of the clay is given in Fig. 5 and its characteristics are liquid limit 46%, plastic limit 30%, shrinkage limit 20%, plasticity index 16% and specific gravity 2.74.

The soil specimens were prepared from a dry, powdered form by mixing with distilled water to a predetermined moisture content and passing the soil-water mixture through a "Vac-Aire" extruder. This equipment has been described in detail by Matlock, Fenske and Dawson.[7] After extrusion, the specimens were covered with five or six coats of a special flexible wax and stored in a humidity box until they were ready to be tested. The degree of saturation obtained using the "Vac-Aire" extruder ranged from 89 to 98 per cent with an average value of approximately 94 per cent. The majority of dynamic tests were performed on specimens with a length of 8.20 cm and a diameter of 3.65 cm, giving a length-to-diameter ratio of 2.25. Specimens tested at various intervals over a period in excess of one year indicated that thixotropic effects during this period are either non-existent or negligible.

EXPERIMENTAL PROCEDURE

A schematic diagram of the experimental apparatus is given in Fig. 5. In general descriptive terms, the vibratory uniaxial compression tests are conducted in the following manner. As indicated in Fig. 5, a cylindrical soil specimen is placed between the upper and lower platens and subjected to a specified static stress level with a spring system attached to the lower platen and supported by hooks fastened to the structural framework. The magnitude of the static stress level can be varied as desired by the proper selection of an appropriate group of springs and poper adjustment of the spring support hooks. The creep deformation due to the static load is compensated with a manually operated screw mechanism such that the lower platen is maintained about its "load-free" equilibrium position.

After most of the transient creep due to the static load is complete, a small sinusoidally varying deformation is induced at the bottom of the specimen through oscillation of the lower platen. This lower platen is fixed to the moving coil of an electromagnetic vibration exciter. After a constant frequency-constant amplitude dynamic oscillation has been exerted for a sufficiently long period to minimize transient dynamic creep effects, a record is made of the time-dependent variation of the induced oscillatory deformation and the resulting force necessary to maintain the deformation. The oscillatory deformation is measured by means of an LVDT (Linear Variable Differential Transformer) while the resulting force is measured with a specially constructed dynamometer unit consisting of semiconductor strain gages (gage factor of 127 + 2%) mounted on a thin-walled stainless steel tube. Both signals are amplified and recorded on a dual channel oscillograph. Additional creep deformation of the specimen is continually compensated with the screw mechanism by continual positioning of the LVDT about its null position, as indicated by the oscillograph trace. Throughout the process, the total creep deformation is obtained with an indicator dial which is mounted above the upper platen.

Maintaining the frequency constant, the amplitude of oscillation is increased and the same process repeated. Amplitude of oscillation is monotonically increased in this manner until the specimen fails or the output capacity of the power supply equipment is reached. A monotonically increasing amplitude was used to minimize any effects which the material might exhibit due to "strain-hardening." Various aspects of the apparatus and experimental procedure are given by Kondner and Krizek.[8]

Thus, for each amplitude of dynamic deformation, the amplitude of the resulting dynamic force is obtained as well as the phase angle between the dynamic force and deformation waves. In addition, the total length of the specimen about which the vibratory motion is taking place is obtained. This allows cross-sectional area corrections on a constant volume basis with changes in axial length.

EXPERIMENTAL RESULTS

The deformation of a specimen under a dynamic loading imposed by vibratory unconfined compression is very similar to that obtained from a conventional unconfined compression test. Typical results of such a vibratory compression test are shown in Fig. 6 where the data are plotted in the form of dynamic stress amplitude, σ_D, (based on a constant-volume area correction) versus dynamic strain amplitude, ε_D, (based on the deformed length). It can readily be seen that the relationship between these two quantities is nonlinear, even for the small values of strain considered. Since the curve of Fig. 6 was obtained using only amplitude response on a point by point basis (measured stress amplitude for each imposed strain amplitude), the assumption of harmonic wave forms used in the theoretical developments has no apparent effect.

For the tests reported herein, the use of the relatively slow-speed oscillograph made it difficult to obtain accurate determinations of the phase angle. However, the response indicated that these phase angles are quite small in magnitude (on the order of a few degrees). For a phase angle variation from three to six degrees, the value of sin δ varies from 0.0523 to 0.1045 (a variation of approximately 100 per cent) while the value of cos δ only varies from 0.9986 to 0.9945 (a variation of less than 0.5 per cent). The values of cos δ and sin δ are the coefficients in Eqs. (7) or (9) and Eqs. (6) or (10), respectively. Thus, for small phase angles, fairly large per cent variations in phase angle would have little effect on the restoring stress or storage modulus, but would have considerable effect on the dissipation stress or loss modulus.

Because of the difficulty of accurately measuring the small phase angles, only the restoration or energy storage will be considered, with cos δ approximated as one. This corresponds to a phase angle of zero. Thus, the error induced by this assumption is quite small with respect to energy storage, but extremely large with respect to dissipation. Although the results given in the present paper do not consider directly the energy dissipation response, a detailed study of such response is currently being conducted with a more accurate recording system.

Assuming for practical purposes that the phase angle is zero, the dynamic response given in Fig. 6 may be considered that of a nonlinear elastic material with underlinear or "soft" restoration and represents the energy storage characteristics of the cohesive soil. In addition, this condition means that the storage modulus of Eq. (9) equals the complex modulus of Eq. (13). However, the storage modulus or complex modulus is not a constant but varies inversely as a function of either stress or strain level, as indicated by the nonlinearity in Fig. 6 and Eqs. (9) or (13). The propagation velocity of a dilatation wave given by Eq. (14) simply degenerates to the elastic case of Eq. (16). However, the propagation is not constant, but varies because E^* or E' varies with either stress or strain level.

Figure 7 is a plot of storage modulus versus dynamic strain level ε_D for the results given in Fig. 6. The modulus decreases as the dynamic strain level increases. This is simply an expression of the underlinear or "soft" type of nonlinear behavior given in Fig. 6.

Figure 8 is a plot of the propagation velocity calculated with Eq. (16) using the results of Figs. 6 or 7. It must be emphasized that E' is not constant but a function of either stress or strain level, as indicated by the nonlinear response of Fig. 6. Hence, as the excitation level increases with a corresponding increase in stress and strain, the propagation velocity decreases from approximately 15,000 in per sec to nearly 3,400 in per sec.

The preparation of a large number of soil specimens with identical moisture contents is a very formidable task. Small changes in moisture content may radically influence the response of a cohesive soil to a significant extent. This can be taken into account conveniently by dividing the dynamic stress amplitude by the conventional unconfined compressive strength, q, of the specimen to give a dimensionless dynamic stress-strength parameter, σ_D/q, Krizek and Kondner[9].

To determine the effect of static stress level on the dynamic response, a series of tests were conducted at different values of applied static stress. The results given in Fig. 9 indicate that if any effects exist they are small and of the same order as the experimental error. Although the results are presented in terms of σ_D/q, the general trends of Fig. 9 are similar to those given in Fig. 6 and similar interpretation can be made.

Extensive studies of moisture content, specimen geometry, frequency of oscillation and strain rate amplitude were conducted. In general, the effects of these variables were small or of the same order as the experimental error. Details of these aspects are reported by Kondner, Krizek and Haas[10].

CONCLUSIONS

The energy storage and energy dissipation characteristics of cohesive soils are important in the design and analysis of soil-structure systems subject to earthquake loadings. These can be studied using steady state testing techniques. This is illustrated for a particular clay tested in uniaxial vibratory compression. The soil response can be expressed in terms of viscoelastic parameters, such as storage and loss moduli, or in terms of restoration and dissipation stress amplitudes. The energy storage or elastic portion of the clay response is definitely nonlinear, even at small values of strain. Experimental results indicate that the phase angles (a measure of dissipation) between sinusoidally applied strains and the resulting stresses are small (a few degrees) and require special instrumentation to obtain accurate measurement. Use of the underlinear or soft restoration response of the cohesive soil gives calculated values of compression moduli and velocities of propagation of compression waves which decrease with either dynamic stress or strain level. For the limited range considered, the applied static stress level has little effect on the dynamic response.

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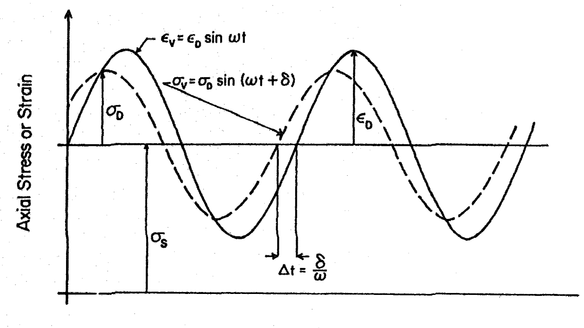

Figure I. Vibratory Stress-Strain-Time

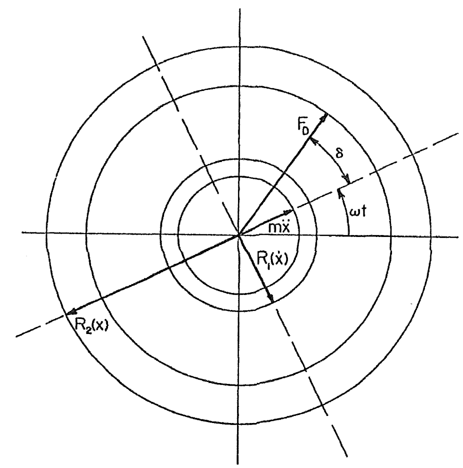

Figure 2. Phase Diagram: Force Parameters

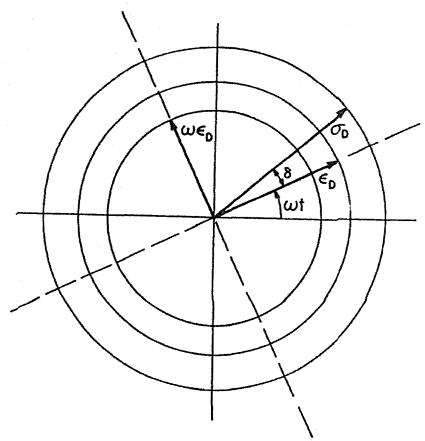

Figure 3. Phase Diagram: Stress, Strain and Strain Rate

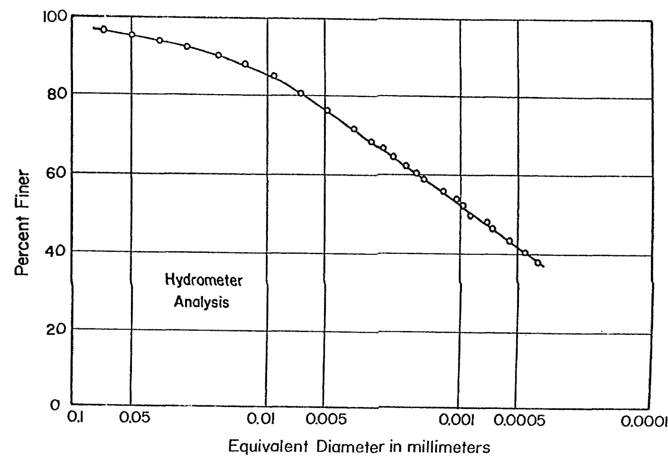

Figure 4. Grain Size Distribution

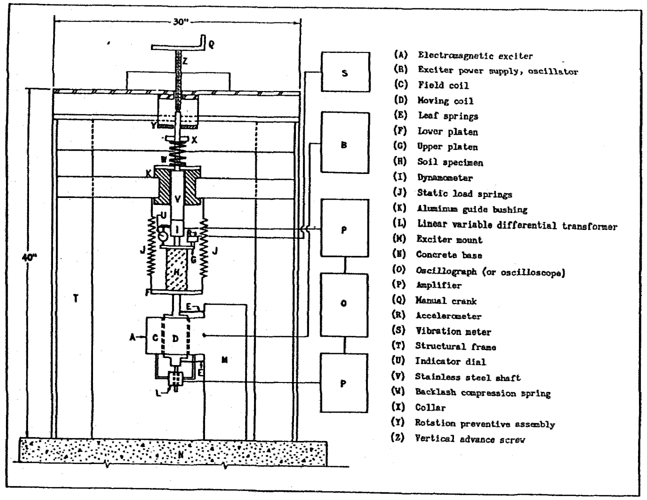

Figure 5. Schematic Diagram: Vibratory Unconfined Compression Apparatus

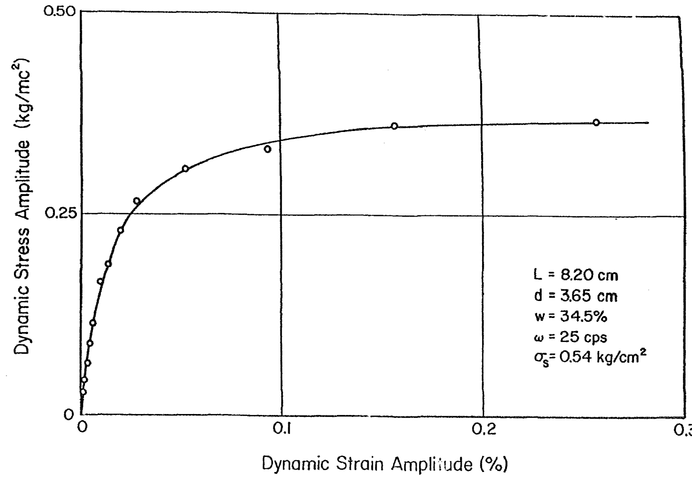

Figure 6. Typical Plot of Dynamic Stress Amplitude versus Dynamic Strain Amplitude

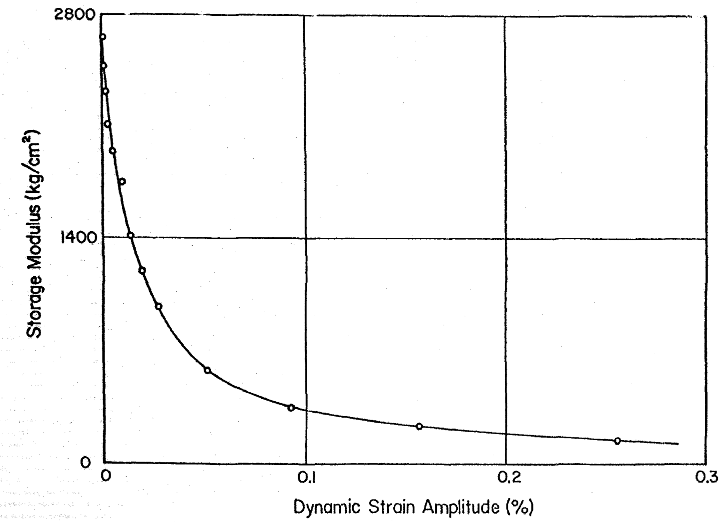

Figure 7. Storage Modulus versus Dynamic Strain Amplitude

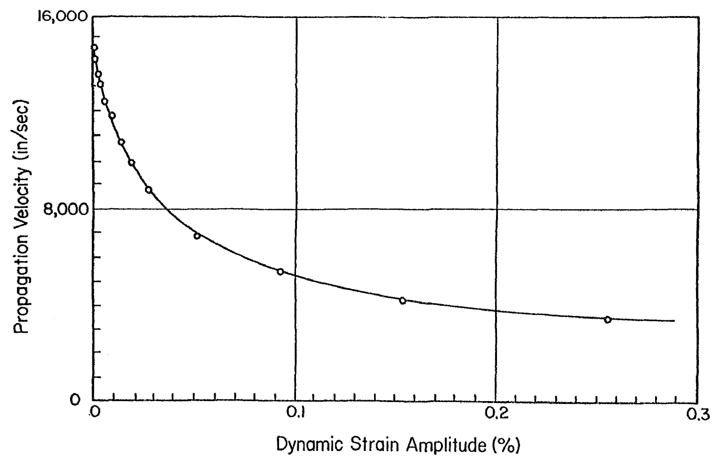

Figure 8. Wave Propagation Velocity versus Dynamic Strain Amplitude

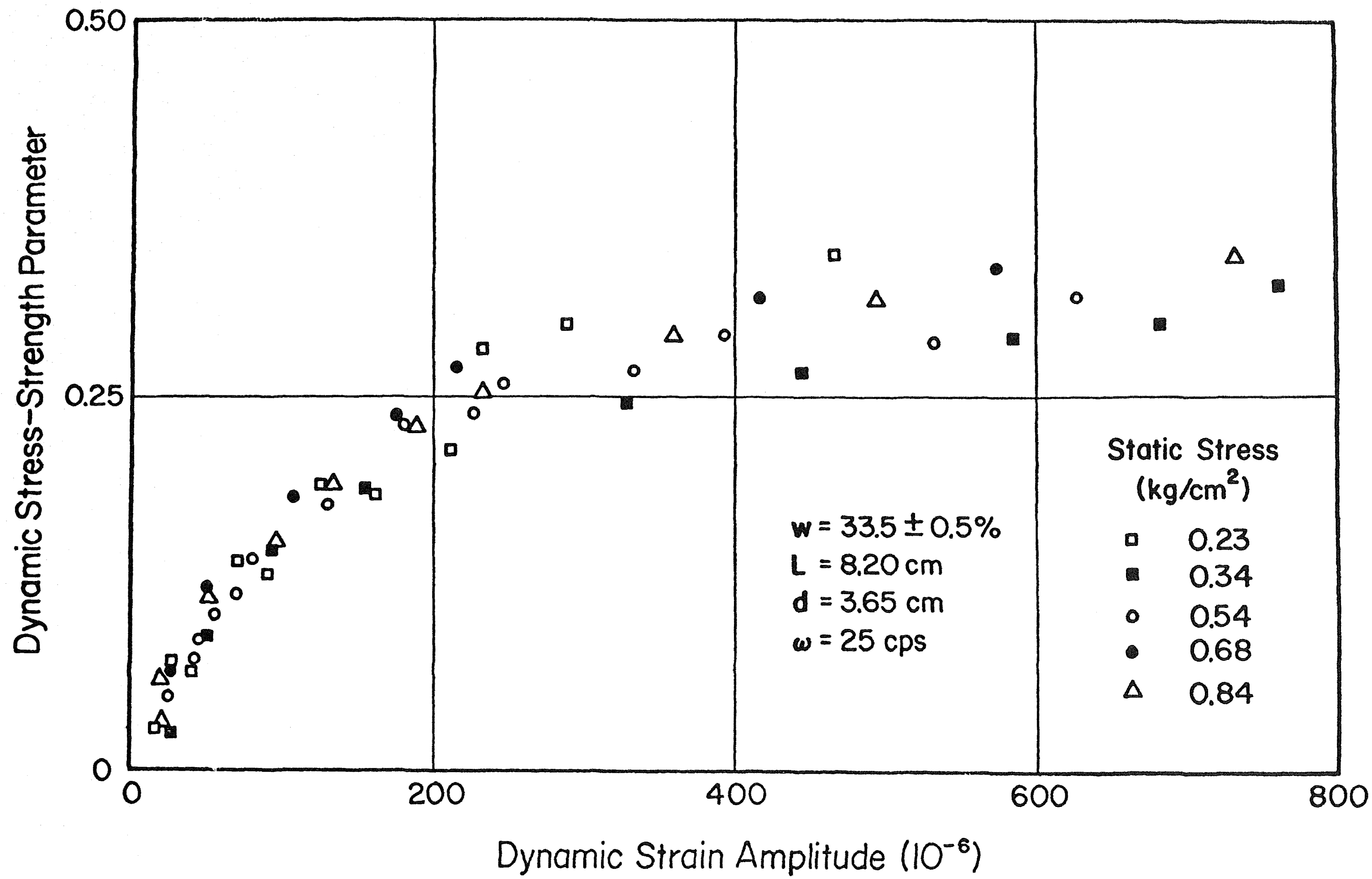

Figure 9. Dynamic Response: Different Static Stress Level