FOUNDATION SUPERSTRUCTURE INTERACTION UNDER EARTHQUAKE MOTION

by

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ABSTRACT

A method of analysis is presented for determining the dynamic response of structures under earthquake motion taking into account the flexibility of the foundation. The structure is replaced by a lumped mass mathematical model which is attached to the moving rock layer by a flexible member having the same force-displacement relationship as the foundation. Several example problems are discussed which show the effect of the soil stiffness upon the response of the structure.

INTRODUCTION

Although much time and effort has been devoted to the study of earthquakes, very little is known about them and most of the information which is available is of a qualitative nature. The quantitative information which is available is comprised of records of ground surface motion at specific points due to specific earthquakes. Unfortunately, the motion of the ground surface at any point is strongly influenced by the nature of its immediate surroundings. It has been shown (1) that the use of the record of the foundation movement of a particular structure at one location, under a certain earthquake, is of questionable value for the design of another structure to resist that same earthquake at another location. For design purposes, it is necessary that the foundation movement of the particular structure be known under the design earthquake.

The purpose of this paper is to develop a mathematical model which includes the effect of the foundation properties upon the dynamic response of the structure to a particular earthquake. The earthquake is assumed to occur as an acceleration of a rock layer at some distance below the structure. If the force

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deformation properties of the soil between the rock layer and the foundation are known, the response of the structure can be determined. The behavior of the foundation is influenced not only by the super-structure but also by the properties of the soil on which it is supported. The mathematical model can include both linear and non-linear soil properties; however, it must be realized that considerable research work would be needed to obtain the force-deformation relationships for the foundation for real structures.

MATHEMATICAL MODEL AND EQUATIONS OF MOTION

The case considered will be that of a N-story building, as shown in Fig. 1a, assumed to be capable of displacement in a single plane. The mass of the structure is assumed to be concentrated at the N floor levels, as shown in the mathematical model of Fig. 1b, while the mass of the foundation is assumed to be concentrated at the base of the structure, thus resulting in $n = N + 1$ concentrated masses. The foundation mass may include not only the mass of the physical foundation, but also the mass of a certain amount of soil that moves along with it. Further research is necessary, however, to determine realistic values of the mass of the foundation to be used. The foundation is assumed to be connected to a point on the rock layer directly below it by a fictitious member 0-1, as shown in Fig. 1b, which has the horizontal force deformation characteristics of the soil. In addition, the foundation is assumed to be free to rotate about its center of gravity. This movement is resisted by a rotational spring having the moment-rotation characteristics of the foundation.

The excitation of the model can consist of either an acceleration applied to the rock layer or a set of time dependent forces applied at the mass points. If the applied forces at the mass points are zero and an acceleration is imposed on the rock layer, the model will simulate the response of the structure to an earthquake. If, on the other hand, the acceleration of the rock layer is zero, while forces are applied at the mass points, the model will simulate the response of the structure to a blast load.

Figure 1c shows the model in the deformed state. The absolute horizontal displacement of mass M_i is designated x_i while the displacement relative to the point directly below the structure, on the rock layer, is designated x_i'. Rotation of the base, designated by α, is measured about its center of gravity and is considered positive in a clockwise direction.

The equations of motion, considering horizontal displacements of the concentrated masses, neglecting damping in the structure, are

$$F_i - R_i = M_i \ddot{x}_i \qquad (i = 1, 2, \ldots, n) \qquad (1)$$

where M_i is the magnitude of the concentrated mass at point i, $\ddot{x}_i$ is the absolute horizontal acceleration of mass M_i, R_i is the internal resisting force in the structure at point i, and F_i is the external force acting on mass M_i. The internal resisting forces can be written as

$$R_1 = \sum_{j=2}^{n} \left[b_{1j} \left(x_j' - x_1' - h_{j1} \tan \alpha\right) \right] + K \qquad (2)$$

and

$$R_i = \sum_{j=2}^{n} \left[b_{ij} \, (x'_j - x'_1 - h_{j1} \tan \alpha) \right] \qquad (i = 2, 3, \ldots, n) \tag{3}$$

where b_{ij} is a stiffness influence coefficient for the structure, h_{j1} is the distance between masses M_j and M_1, and K is the total horizontal resisting force in the soil. (The form of the expression for K will depend upon the properties of the soil. In general, it will be some function of x'_1 and $\dot{x}'_1$.) If it is assumed that the rotation of the structure is small; i.e., $\tan \alpha = \alpha$, and if it is also noted that

$$\ddot{x}_i = \ddot{x}_o + \ddot{x}'_i \tag{4}$$

the previous equations of motion for horizontal movement of the concentrated masses can be rewritten as

$$F_1 - \sum_{j=2}^{n} \left[b_{ij} \, (x'_j - x'_1 - h_{j1} \, \alpha) \right] - K = M_1 \, (\ddot{x}_o + \ddot{x}'_1) \tag{5}$$

and

$$F_i - \sum_{j=2}^{n} \left[b_{ij} \, (x'_j - x'_1 - h_{j1} \, \alpha) \right] = M_i \, (\ddot{x}_o + \ddot{x}'_i) \qquad (i = 2, 3, \ldots, n) \tag{6}$$

The additional equation of motion which is required is that governing the rotation of the structure about its foundation, which, assuming a small rotation angle, is

$$I\ddot{\alpha} + R_\alpha = \sum_{i=2}^{n} \left[F_i - M_i \, (\ddot{x}_i - \ddot{\alpha} \, h_{i1}) \right] \tag{7}$$

where I is the moment of inertia of the entire structure about its base and R_α is the resisting moment due to the soil. (The form of the expression for R_α will, in general, be some function of the properties of the soil and the geometry of the foundation.) Upon rearranging, Eq. 7 becomes

$$I_1 \, \ddot{\alpha} + R_\alpha = \sum_{i=2}^{n} \left[F_i - M_i \, (\ddot{x}_o + \ddot{x}'_i) \right] h_{i1} \tag{8}$$

where I_1 is the moment of inertia of the foundation about its center of gravity

NUMERICAL SOLUTION OF EQUATIONS OF MOTION

Since, in general, the quantities K and R_α will be non-linear functions of velocity and displacement while, at the same time the acceleration of the rock layer, $\ddot{x}_o$, will be an irregular function of time, it is not possible to formulate a general exact solution for the equation of motion of the mathematical model. An approximate solution can be conveniently obtained, however, by means of a single step forward numerical integration procedure presented by Fleming and Romualdi (2). In this method, both the deflection-velocity and velocity-acceleration relationships are assumed to vary linearly over a small time interval, Δt, thus resulting in the relationships

$$\ddot{x}_i(2) = \frac{2}{(\Delta t)}\left[\dot{x}_i(2) - \dot{x}_i(1)\right] - \ddot{x}_i(1) \tag{9}$$

and

$$\dot{x}_i(2) = \frac{2}{(\Delta t)}\left[x_i(2) - x_i(1)\right] - \dot{x}_i(1) \tag{10}$$

where $x_i(1)$, $\dot{x}_1(1)$, and $\ddot{x}_1(1)$ are the displacement, velocity and acceleration of mass M_i at a time t_1 while $x_i(2)$, $\dot{x}_i(2)$ and $\ddot{x}_i(2)$ are the same quantities at a time $t_2 = t_1 + \Delta t$. Substituting Eqs. 9 and 10 into Eqs. 5, 6 and 8 and simplifying leads to the matrix equation

$$\left[A\right]\left\{x'(2)\right\} = \left\{c\right\} \tag{11}$$

where the matrices $[A]$ and $\{c\}$ are defined in Tables I and II, respectively, while the matrix $\{x'(2)\}$ is defined as

$$\{x'(2)\} = \left\{\begin{matrix} x'_1(2) \\ x'_2(2) \\ \cdot \\ \cdot \\ \cdot \\ x'_n(2) \\ \alpha \end{matrix}\right\} \text{(order } n+1\text{)} \tag{12}$$

At any time step, the complete deformed shape of the structure can be obtained, if $\{c\}$ is known, from

$$\{x'(2)\} = [A]^{-1}\{c\} \tag{13}$$

From an examination of Table II, it can be seen that the soil resisting force and moment, $K(2)$ and $R_\alpha(2)$, at time t_2 are required to define the matrix

$\{c\}$. Since the quantities are, in general, a function of x_1' (2), $\dot{x}_1'$ (2), α(2) and $\dot{\alpha}$ (2), their values at time t_2 cannot be defined before Eq. 11 is solved. A solution can be found by means of an iterative procedure. As a first approximation K(2) and R_α(2) can be approximated by K(1) and R_α(1), after which a first approximation for $\{x'(2)\}$ can be found from Eq. 13. These values can then be used to obtain a better approximation for K(2) and R_α(2) after which a second approximation for $\{x'(2)\}$ can be found. This process can be repeated as many times as necessary until two succeeding values of $\{x'(2)\}$ agree within a previously determined acceptable limit. Once $\{x'(2)\}$ is known, $\{\dot{x}'(2)\}$ and $\{\ddot{x}'(2)\}$ can be found from Eqs. 9 and 10. By now replacing $\{x'(1)\}$, $\{\dot{x}'(1)\}$, and $\{\ddot{x}'(1)\}$ by $\{x'(2)\}$, $\{\dot{x}'(2)\}$ and $\{\ddot{x}'(2)\}$, the process can be repeated for the next time step. This procedure has been programmed for the IBM 709 digital computer.

EFFECT OF SOIL PROPERTIES UPON DYNAMIC RESPONSE

In order to be able to design structures to resist earthquake motions, it is necessary that the effect of the foundation properties, upon the response of the structure, be known. To completely understand the problem of foundation-superstructure interaction will require a very exhaustive research program; however, a few preliminary conclusions can be reached by considering several example problems. The specific problem which will be considered is a two story shear building, described originally by Housner (3), whose mathematical model is shown in Fig. 2a and for which the rock layer is subjected to the acceleration, $\ddot{x}_o$, described in Fig. 2b. For the purpose of the example problem, the foundation mass is taken to be equal to the concentrated masses at the floor levels although, as mentioned previously, further research is necessary to obtain realistic values for this quantity. The fundamental period of the structure is 0.67 seconds, while the period of the rock acceleration used was taken to be 0.16 seconds. This value was chosen since it is approximately equal to that occurring in the El Centro, California, Earthquake of May 18, 1940 (4).

For the preliminary studies, the soil force-deformation relationship was assumed to be linear elastic. It is realized that this assumption is not completely realistic; however, it will simplify the initial phase of the investigation. Since the method of analysis described can handle any form for the force-deformation relationship, the effect of the non-linearity of the soil can be studied at a later time. The displacement, x_i', of the foundation mass and superstructure masses with respect to the rock layer are shown in Figs. 3 to 5 for soil stiffness of 1,000, 100,000 and 1,000,000 pounds per inch, and in Fig. 6 for a rigid soil. These stiffnesses range from a very flexible to a very stiff foundation compared to the stiffness of the structure.

Several interesting, although not surprising, facts can be seen upon inspecting the results of the example problem. For the case of a very flexible foundation, as shown in Fig. 3, the displacement with respect to the rock layer is essentially a rigid body movement of the total structure. Since there is little relative movement between the floor levels, the stresses induced in the structure will be very small. This would indicate that, assuming a foundation failure does not occur, the most desirable condition when considering the stresses induced in the superstructure is a flexible foundation compared to the stiffness of the structure. On the other extreme, it can be seen from Fig. 6

that for the case of a completely rigid foundation, the foundation does not move relative to the rock layer; however, there are rather large relative movements between the foundation and the floor levels, thus inducing appreciable stresses in the columns. Essentially the same response is shown in Fig. 5 for a foundation stiffness of 1,000,000 pounds per inch; therefore, for the particular structure being studied, this corresponds to approximately a rigid foundation.

A very interesting condition can be observed by considering the response curves shown in Fig. 4 which correspond to a soil stiffness of 100,000 pounds per inch. For this case, the displacement of the foundation with respect to the rock layer became very large while the displacements of the floor levels are approximately the same as in Figs. 5 and 6. Due to these large foundation displacements, the relative displacements between the foundation and the floor levels are very large, thus resulting in high stresses in the columns. This condition is not surprising when the natural period of the total system, including both the foundation and the superstructure, of 0.14 seconds and the period of the rock acceleration of 0.16 seconds are compared. This represents a condition very close to resonance. Of course, it must be realized that damping in both the foundation and the superstructure have been neglected in these calculations. If damping were included the displacements of the system would be greatly reduced. Before damping can be intelligently taken into account, however, it is necessary that considerable research be performed in order to arrive at a realistic value for soil damping. It must be realized that such values may be functions of the displacement as well as the velocity in addition to the configuration of the foundation-structure system. Once a form for the foundation damping expression is known, it can be easily included in the calculation.

CONCLUSIONS

Several conclusions and opinions can be arrived at from the study which has been performed to date. It can be seen that for the mathematical model used, the stiffness of the soil is very important. For identical structures and rock accelerations, the displacements occurring in the structure will be completely different depending upon the relative stiffnesses of the foundation and the structure. Although specific cases are not demonstrated here, it can also be shown that for identical foundation stiffnesses and rock acceleration different foundation displacements will be obtained if the type or stiffness of the structure is changed. It is, therefore, the opinion of the authors that any method of analysis for the response of a structure to an earthquake can lead to completely erroneous results if the foundation superstructure interaction is not considered. It is not sufficient to merely apply the measured foundation acceleration for one structure to the base of another in order to design it to withstand the same earthquake.

It is the further opinion of the authors that better information is necessary concerning the intensity of actual earthquakes. Rather than measuring the base acceleration of a particular structure, as is now done, it is suggested as a possible alternative that the acceleration of the rock layer below the structure might serve as a better standard. This would eliminate the effect of the structure and soil properties upon the measured acceleration. With this information and the mathematical model prescribed, a more rational approach to earthquake design can be made.

The preliminary results presented in this paper in no way exhaust the possibilities for further study in the field of earthquake engineering. In fact, they open up a completely new avenue of approach which should be explored.

ACKNOWLEDGMENTS

Acknowledgment is given to the Northwestern University Computing Center and to the National Science Foundation whose funds helped in the establishment of the center.

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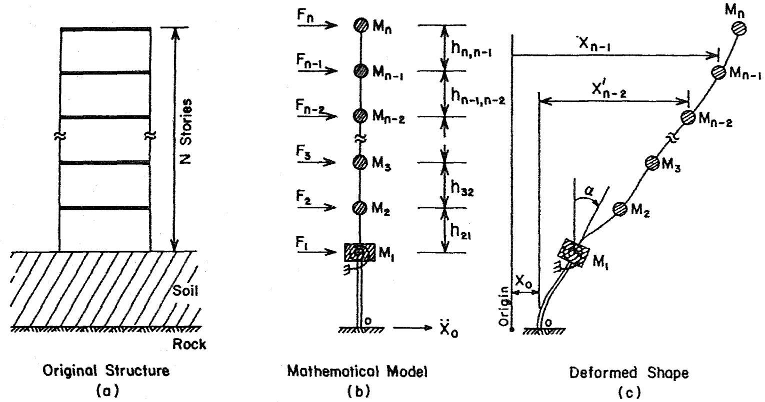


FIGURE 1

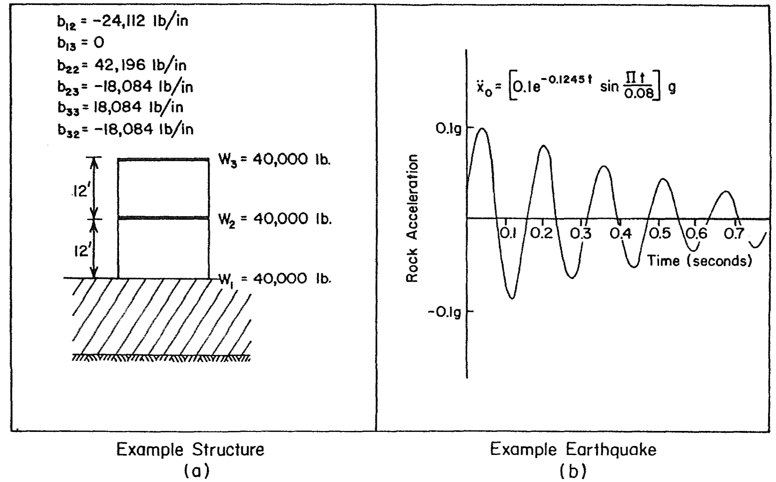
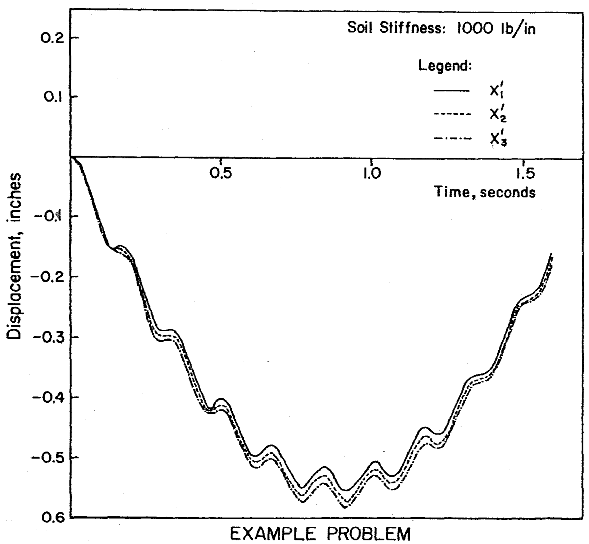
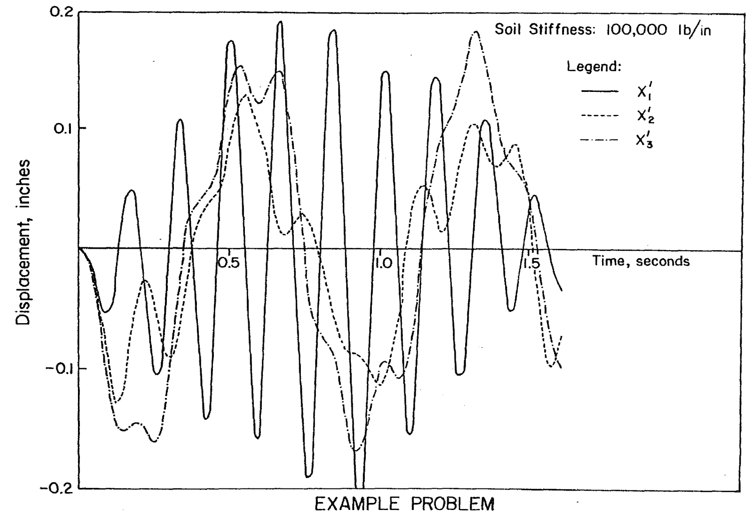


FIGURE 2



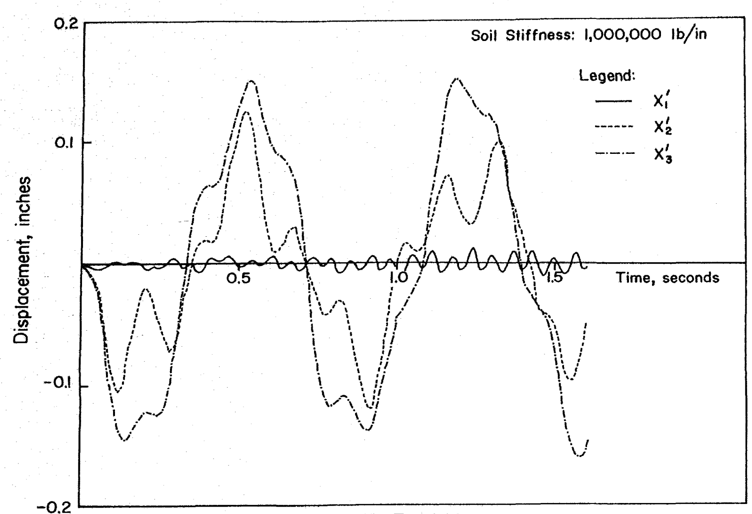
EXAMPLE PROBLEM

FIGURE 3



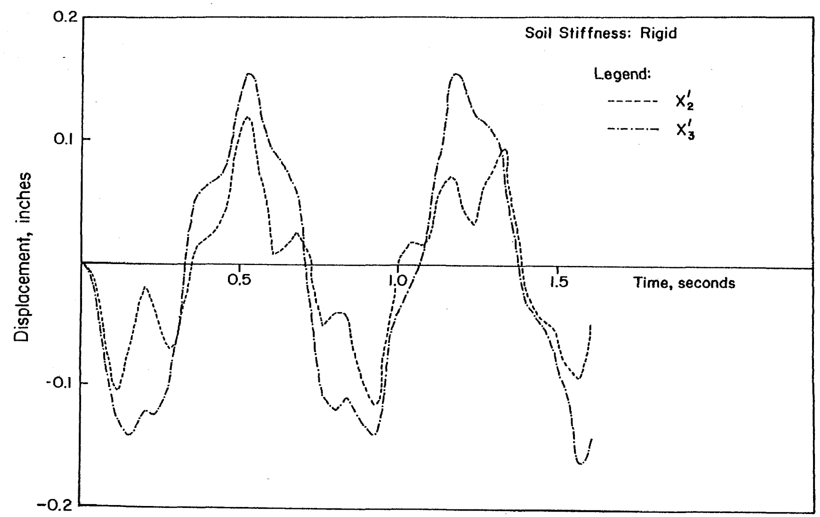
EXAMPLE PROBLEM

FIGURE 4



EXAMPLE PROBLEM

FIGURE 5



EXAMPLE PROBLEM

FIGURE 6

$$[A] = \begin{bmatrix} \frac{4M_1}{(\Delta t)^2} - \sum_{j=2}^{n} b_{1j} & b_{12} & b_{13} \text{ ------- } & b_{1n} & -\sum_{j=2}^{n} b_{1j} h_{j1} \\ -\sum_{j=2}^{n} b_{2j} & \frac{4M_2}{(\Delta t)^2} + b_{22} & b_{23} \text{ ------- } & b_{2n} & -\sum_{j=2}^{n} b_{2j} h_{j1} \\ -\sum_{j=2}^{n} b_{3j} & b_{32} & \frac{4M_3}{(\Delta t)^2} + b_{33} \text{ ------- } & b_{3n} & -\sum_{j=2}^{n} b_{3j} h_{j1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -\sum_{j=2}^{n} b_{nj} & b_{n2} & b_{n3} \text{ --- } & \frac{4M_n}{(\Delta t)^2} + b_{nn} & -\sum_{j=2}^{n} b_{nj} h_{j1} \\ 0 & \frac{4M_2 h_{21}}{(\Delta t)^2} & \frac{4M_3 h_{31}}{(\Delta t)^2} \text{ --- } & \frac{4M_n h_{n1}}{(\Delta t)^2} & \frac{4I_o}{(\Delta t)^2} \end{bmatrix}$$

DEFINITION OF MATRIX [A]

TABLE I

$$\{c\} = \left\{ \begin{matrix} F_1(2) - M_1 \ddot{x}_o(2) + \frac{4M_1}{(\Delta t)^2} x_1'(1) + \frac{4M_1}{(\Delta t)} \dot{x}_1'(1) + M_1 \ddot{x}_1'(1) - K(2) \\ F_2(2) - M_2 \ddot{x}_o(2) + \frac{4M_2}{(\Delta t)^2} x_2'(1) + \frac{4M_2}{(\Delta t)} \dot{x}_2'(1) + M_2 \ddot{x}_2'(1) \\ F_3(2) - M_3 \ddot{x}_o(2) + \frac{4M_3}{(\Delta t)^2} x_3'(1) + \frac{4M_3}{(\Delta t)} \dot{x}_3'(1) + M_3 \ddot{x}_3'(1) \\ \vdots \\ F_n(2) - M_n \ddot{x}_o(2) + \frac{4M_n}{(\Delta t)^2} x_n'(1) + \frac{4M_n}{(\Delta t)} \dot{x}_n'(1) + M_n \ddot{x}_n'(1) \\ \left[\sum_{i=2}^{n} \left[F_i(2) h_{i1} - M_i h_{i1} \ddot{x}_o(2) \right] + \sum_{i=2}^{n} \left[\frac{4M_i}{(\Delta t)^2} x_i'(1) + \frac{4M_i}{(\Delta t)^2} \dot{x}_i'(1) + M_i \ddot{x}'(1) \right] + \frac{4I_o}{(\Delta t)^2} \alpha(1) + \frac{4I_o}{(\Delta t)} \dot{\alpha}(1) + I_o \ddot{\alpha}(1) - R_\alpha(2) \right] \end{matrix} \right\}$$

DEFINITION OF MATRIX {c}

TABLE II