Non-stationary Seismic Soil-Structure-Soil Interaction

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SUMMARY:
The amplitude and frequency non-stationarities present in the ground motions modify structural responses to a great extent. The present paper illustrates a method to account for these non-stationarities while analyzing soil-structure-soil interaction effects in a tank-soil model. First, an equivalent linear two-degree-of-freedom model of a circular liquid storage tank has been developed with a combined mass-spring-dashpot system subjected to a horizontal seismic excitation applied at the base. Only translational vibration in the horizontal direction is considered. A linear soil interaction is considered in this case. The equilibrium equations have been formulated and solved using wavelet analytic technique to obtain the shear and moment at the base of the tank. The wavelet based technique captures non-stationary effects of ground motions. Only impulsive liquid motion is considered. Subsequently, a three dimensional finite element model of the soil-foundation system has been developed in DIANA. The base shear and the overturning base moment obtained from previous analytical solution are distributed appropriately on the surface of the soil domain. The soil behaviour has been assumed to be nonlinear and represented by Mohr-Coulomb constitutive law. The model has thereafter been numerically solved to obtain the soil responses. A few comparisons between displacement, strain and stress responses of the soil medium due to different ground motions have been presented to depict the effects of ground motion non-stationarities.

Keywords: wavelet, seismic, soil-structure interaction, Mohr-Coulomb, DIANA

1. INTRODUCTION

The seismic ground motion process is likely to be highly non-stationary in nature due to local concentrations of frequency components that may be attributed to the dispersive phenomenon associated with propagating waves. The amplitude and frequency non-stationarities present in such ground motions may considerably affect the responses of a structure (Basu and Gupta (1998), Yeh and Wen (1990)) and in turn may even influence the underlying soil responses to a great extent. As liquid storage tanks are of prime importance in oil, gas and petrochemical industries, a reasonable prediction of structural and soil responses is of paramount importance. The researchers in the past have studied seismic responses of liquid storage tanks quite extensively (Housner (1963), Haroun (1983), Toki and Miura (1983), Peek (1988), Veletsos and Tang (1990), Tam (2008), Livaoglu (2008)). Some of the studies were carried out for fixed base tanks and in some cases the soil interaction has been duly considered. However, in none of these studies, the non-stationary effects of ground motions have been accounted for. Recently, the usefulness of application of wavelets in capturing the non-stationarities contained in a random ground motion process has opened up a clear avenue to access this area of engineering applications. In a number of studies (Chatterjee and Basu (2001, 2004, 2006)) the applicability of wavelet analytic technique has been shown in characterizing ground motions (using unique time-frequency localization property of wavelets) as well as solving coupled dynamic equations of the system in wavelet domain to obtain the desired responses in terms of expected largest peak displacements or accelerations. Though these studies considered dynamic soil-structure interaction effects including non-stationary effects, the reaction response of soil domain due to soil-structure interaction effects have not been studied much. However, it may be essential to see the responses of the soil as a result of soil-structure interaction due to seismic activity underground. In this paper, an attempt has been made to include non-stationary effects in soil-structure-soil interaction analysis. A particular tank has been imagined to be erected at four different sites with four different (horizontal) seismic ground motions (thus, four different types of non-stationarities). The equations of motion have been formulated for linear soil-structure interaction part and solved with wavelet based technique as explained in Chatterjee and Basu (2001) to obtain the expected peak largest values of base shears and overturning base moments. Next, a finite element model of the underlying soil domain has been constructed in DIANA using nonlinear soil parameters and those base shears and moments have been applied in the model suitably at the soil surface and the corresponding soil responses were obtained. The results were obtained for four pairs of shear and moment values and plotted to see the difference in the responses in soil due to various non-stationarities. It may be mentioned here that DIANA is a finite element based software package which offers a wide range of applications across the industry including structural and geotechnical engineering, tunnelling and oil and gas engineering. It also offers choices in terms of material constitutive models, various analysis procedures including nonlinear analysis, dynamic analysis, and hybrid frequency time domain analysis and advanced modelling features.

2. MODEL

A right circular cylindrical tank of height H (this is also the height of the incompressible liquid in the tank), radius R and wall thickness h has been considered. The liquid in the tank is assumed to vibrate only in impulsive mode. The tank is rigidly clamped to a thick and rigid circular basemat having same radius (R) as that of the tank. The tank-foundation system is supported on the surface of a homogeneous elastic half space. The whole system is subjected to a free-field horizontal seismic ground motions, $\ddot{x}_g(t)$. The tank–liquid system is represented by a two-degree-of-freedom (2-DOF) mass–spring–dashpot model with impulsive mass of the liquid, m_i, a linear spring with stiffness, K_i and a linear viscous damper with coefficient, C_i. The foundation system of the tank is of mass, m_f with a complex valued linear translational impedance function, K_x due to the surrounding soil. Only translational vibration is considered. The basemat is assumed to be thick enough to arrest any rocking. The model is shown in Figure 1.

3. GROUND MOTION NON-STATIONARITIES

The seismic response of linear as well as non-linear structures is affected considerably by the temporal and spatial non-stationarities present in the ground motion. The temporal and spatial non-stationarities may be defined as changes in the amplitude and frequency content respectively of the ground motion with time which arise from the evolving nature of the seismic waves arriving at a site. This evolving frequency content of the ground motion must be modeled accurately since it can be critical to the response of degrading structures as such structures have resonant frequencies which tend to decay with time as the structure responds to excitation.

4. THEORETICAL FORMULATION

The absolute lateral displacement of the foundation is denoted by x_f(t) and the lateral displacement of the liquid in the tank relative to the foundation is denoted by x_i(t). The equation of motion of the tank–fluid–foundation system may be written as

$$m_i\left(\ddot{x}_i(t)+\ddot{x}_f(t)\right)+m_f\ddot{x}_f(t)+K_x(x_f(t)-x_g(t))=0 \qquad (4.1)$$

where,

$$K_x=\frac{8GR}{2-\nu}(\alpha_x+ia_0\beta_x) \qquad (4.2)$$

In the above expression, G is the shear modulus of the soil, where $G = \rho V_s^2$ with V_s being the shear wave velocity of the soil and ρ, its mass density and ν is the Poisson's ratio of the soil.

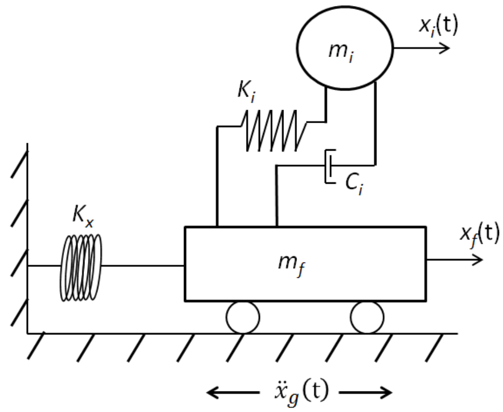

Figure 1. Model of tank-liquid-foundation-soil system

The term, a_0 denotes the dimensionless frequency parameter and may be expressed as $a_0 = \frac{\omega R}{V_s}$. The over dots represent differentiation with respect to time. On considering the equilibrium of only the tank-fluid system, the equation of motion becomes

$$m_i\ddot{x}_i(t) + C_i\dot{x}_i(t) + K_i x_i(t) = -m_i\ddot{x}_i(t) \tag{4.3}$$

It may be noted here that in the above equation, $C_i = 2\zeta m_i\omega_i$ and $K_i = m_i\omega_i^2$. The term, ω_i denotes the lateral natural frequency of the tank having the following expression (Veletsos and Tang (1990))

$$\omega_i = \frac{C}{H}\sqrt{\frac{E_t}{\rho_t}} \tag{4.4}$$

where, E_t and ρ_t are elastic modulus and mass density of the tank respectively and ζ represents the damping of the tank material. The value of C, a dimensionless coefficient which depends on ratios $\frac{H}{R}, \frac{h}{R}, \frac{\rho_l}{\rho_t}$ and the Poisson's ratio of the tank material, may be chosen from Table II of Veletsos and Tang (1990)). The term ρ_l denotes the mass density of the tank liquid. The ground motion may be assumed to have zero mean, non-stationary Gaussian characteristics. The ground acceleration, ground displacement, foundation displacement and impulsive liquid displacement may be expressed in terms of wavelet coefficients respectively as

$$\ddot{x}_g(t) = \sum_i\sum_j \frac{K''\Delta b}{a_j} W_\psi \ddot{x}_g\,(a_j, b_i)\psi_{a_j,b_i}(t) \tag{4.5}$$

$$x_g(t) = \sum_i\sum_j \frac{K''\Delta b}{a_j} W_\psi x_g\,(a_j, b_i)\psi_{a_j,b_i}(t) \tag{4.6}$$

$$x_i(t) = \sum_i\sum_j \frac{K''\Delta b}{a_j} W_\psi x_i\,(a_j, b_i)\psi_{a_j,b_i}(t) \tag{4.7}$$

$$x_f(t) = \sum_i\sum_j \frac{K''\Delta b}{a_j} W_\psi x_f\,(a_j, b_i)\psi_{a_j,b_i}(t) \tag{4.8}$$

In the above expressions, the parameter, b centers the basis function at $t = b$ and its neighbourhood by windowing over a certain temporal stretch depending on the parameter a $(a_j = \sigma^j, b_j = (j-1)\Delta b)$. The term, $K^{"}$ may be defined as

$$K^{"} = \frac{1}{2\pi C_\psi}(\sigma - \frac{1}{\sigma}) \tag{4.9}$$

The term, C_ψ is admissibility criteria as per Basu and Gupta (1998) for the mother wavelet, $\psi(t)$ that will be defined later in the paper. The wavelet coefficients of ground displacement may be written in terms of wavelet coefficients of ground acceleration (Chatterjee and Basu, 2001) as follows.

$$\sum_i \sum_j \frac{K^{"}\Delta b}{a_j} W_\psi x_g\left(a_j, b_i\right)\psi_{a_j,b_i}(t) = (-\frac{1}{\omega^2})\sum_i \sum_j \frac{K^{"}\Delta b}{a_j} W_\psi \ddot{x}_g\left(a_j, b_i\right)\psi_{a_j,b_i}(t) \tag{4.10}$$

The expressions for base shear, Q(t) and base moment, M(t) developed at the tank base due to soil-structure interaction effects may be written as

$$Q(t) = m_i\{\ddot{x}_i(t) + \ddot{x}_f(t)\} \tag{4.11}$$

$$M(t) = m_i\{\ddot{x}_i(t) + \ddot{x}_f(t)\}h_i \tag{4.12}$$

In the above expression for tank base moment the term, h_i denotes the height above the tank base at which the impulsive mass should be located to give correct moments at a section immediately above the tank base. The base shear and base moment may also be expressed in terms of wavelet coefficients as

$$Q(t) = \sum_i \sum_j \frac{K^{"}\Delta b}{a_j} W_\psi Q\left(a_j, b_i\right)\psi_{a_j,b_i}(t) \tag{4.13}$$

$$M(t) = \sum_i \sum_j \frac{K^{"}\Delta b}{a_j} W_\psi M\left(a_j, b_i\right)\psi_{a_j,b_i}(t) \tag{4.14}$$

5. WAVELET DOMAIN SOLUTION

The wavelet domain solution of the equations may be obtained with the wavelet based formulation of Basu and Gupta (1998). The Littlewood-Paley (LP) wavelet basis function has been chosen for the analysis in the present study. This mother wavelet has the following expression.

$$\psi(t) = \frac{1}{\pi\sqrt{\sigma-1}}\frac{\sin\sigma\pi t - \sin\pi t}{t} \tag{5.1}$$

The Fourier transform of the LP basis function yields

$$\psi(\omega) = \frac{1}{\sqrt{2(p-1)\pi}}; \ \pi \le |\omega| \le p\pi$$

$$= 0 \qquad ; \text{otherwise} \tag{5.2}$$

The value for σ in the above equation is $2^{0.25}$. On substituting Equations (4.5) – (4.10) in Equations (4.1) and (4.2) one may get

$$(m_i + m_f) \sum_i \sum_j \frac{1}{a_j} W_\psi x_f\,(a_j, b_i) \ddot{\psi}_{a_j,b_i}(t) + K_x \sum_i \sum_j \frac{1}{a_j} W_\psi x_f\,(a_j, b_i) \psi_{a_j,b_i}(t) +$$
$$m_i \sum_i \sum_j \frac{1}{a_j} W_\psi x_i\,(a_j, b_i) \ddot{\psi}_{a_j,b_i}(t) = K_x \sum_i \sum_j \frac{1}{a_j} W_\psi x_g\,(a_j, b_i) \psi_{a_j,b_i}(t) \tag{5.3}$$

and

$$\sum_i \sum_j \frac{1}{a_j} W_\psi x_i\,(a_j, b_i) \ddot{\psi}_{a_j,b_i}(t) + \omega_i^2 \sum_i \sum_j \frac{1}{a_j} W_\psi x_i\,(a_j, b_i) \psi_{a_j,b_i}(t) +$$
$$2\zeta\omega_i \sum_i \sum_j \frac{1}{a_j} W_\psi x_i\,(a_j, b_i) \dot{\psi}_{a_j,b_i}(t) = -\sum_i \sum_j \frac{1}{a_j} W_\psi x_f\,(a_j, b_i) \ddot{\psi}_{a_j,b_i}(t) \tag{5.4}$$

Further, on taking Fourier transform of both sides of the above equations and subsequently simplifying, one may obtain the following equations.

$$\sum_i W_\psi x_f\,(a_j, b_i) \hat{\psi}_{(a_j,b_i)}(\omega) \left[\frac{K_x}{m_f} - (1+\gamma)\omega^2\right] + \sum_i W_\psi x_i\,(a_j, b_i) \hat{\psi}_{(a_j,b_i)}(\omega)[-\gamma\omega^2] =$$
$$\sum_i W_\psi \ddot{x_g}\,(a_j, b_i) \hat{\psi}_{(a_j,b_i)}(\omega) \left[-\frac{K_x}{\omega^2 m_f}\right] \tag{5.5}$$

$$\sum_i W_\psi x_i\,(a_j, b_i) \hat{\psi}_{(a_j,b_i)}(\omega)(-\omega^2) + \omega_i^2 \sum_i W_\psi x_i\,(a_j, b_i) \hat{\psi}_{(a_j,b_i)}(\omega) +$$
$$2\zeta\omega_i \sum_i W_\psi x_i\,(a_j, b_i) \hat{\psi}_{(a_j,b_i)}(\omega) = \omega^2 \sum_i W_\psi x_f\,(a_j, b_i) \hat{\psi}_{(a_j,b_i)}(\omega) \tag{5.6}$$

where, γ represents the ratio of impulsive mass of the liquid, m_i to the foundation mass, m_f. The above equations (5.5) and (5.6) may be solved to obtain the following expressions for impulsive displacement and foundation displacement responses in terms of functionals of wavelet coefficients.

$$\sum_i W_\psi x_i\,(a_j, b_i) \hat{\psi}_{(a_j,b_i)}(\omega) = \tau_i(\omega) \sum_i W_\psi \ddot{x_g}\,(a_j, b_i) \hat{\psi}_{(a_j,b_i)}(\omega) \tag{5.7}$$

$$\sum_i W_\psi x_f\,(a_j, b_i) \hat{\psi}_{(a_j,b_i)}(\omega) = \tau_f(\omega) \sum_i W_\psi \ddot{x_g}\,(a_j, b_i) \hat{\psi}_{(a_j,b_i)}(\omega) \tag{5.8}$$

In the above equations (5.7) and (5.8), $\tau_i(\omega)$ and $\tau_f(\omega)$ are the frequency dependent complex valued transfer functions relating the wavelet coefficients of horizontal displacements of impulsive tank-liquid and supporting foundation respectively to the wavelet coefficients of horizontal seismic accelerations. These transfer functions have the following expressions.

$$\tau_i(\omega) = \frac{K_x/m_f}{\left[\frac{K_x}{m_f} - (1+\gamma)\omega^2\right]\left[(\omega_i^2 - \omega^2) + i(2\zeta\omega_i\omega)\right] - \gamma\omega^4} \tag{4.9}$$

$$\tau_f(\omega) = \frac{\frac{K_x}{\omega^2 m_f}\left[(\omega_i^2 - \omega^2) + i(2\zeta\omega_i\omega)\right]}{\left[\frac{K_x}{m_f} - (1+\gamma)\omega^2\right]\left[(\omega_i^2 - \omega^2) + i(2\zeta\omega_i\omega)\right] - \gamma\omega^4} \tag{4.10}$$

The expressions relating the wavelet coefficients of base shear and base moment to the wavelet coefficients of the ground acceleration have been derived from Equations (4.13) and (4.14) and may be written respectively as

$$\sum_i W_\psi Q\,(a_j, b_i) \hat{\psi}_{(a_j,b_i)}(\omega) = \tau_Q(\omega) \sum_i W_\psi \ddot{x_g}\,(a_j, b_i) \hat{\psi}_{(a_j,b_i)}(\omega) \tag{4.11}$$

$$\sum_i W_\psi M\,(a_j, b_i) \hat{\psi}_{(a_j,b_i)}(\omega) = \tau_M(\omega) \sum_i W_\psi \ddot{x_g}\,(a_j, b_i) \hat{\psi}_{(a_j,b_i)}(\omega) \tag{4.12}$$

The transfer functions, $\tau_Q(\omega)$ and $\tau_M(\omega)$ in Equations (5.11) and (5.12) finally may be expressed as follows (after dividing these functions respectively by $m_i g$ and $m_i g H$ to obtain non-dimensional coefficients of base shear and base moment).

$$\tau_Q(\omega) = -\frac{1}{g}\left[\frac{\left\{\frac{K_x}{m_f}\right\}\{\omega_i^2 - i2\zeta\omega_i\omega\}}{\left\{\frac{K_x}{m_f} - (1+\gamma)\omega^2\right\}\{(\omega_i^2-\omega^2)+i(2\zeta\omega_i\omega)\} - \gamma\omega^4}\right] \quad (5.13)$$

$$\tau_M(\omega) = -\frac{h_i}{gH}\left[\frac{\left\{\frac{K_x}{m_f}\right\}\{\omega_i^2 - i2\zeta\omega_i\omega\}}{\left\{\frac{K_x}{m_f} - (1+\gamma)\omega^2\right\}\{(\omega_i^2-\omega^2)+i(2\zeta\omega_i\omega)\} - \gamma\omega^4}\right] \quad (5.14)$$

On computing the expectation of the square of the amplitude of Equations (5.11) and (5.12), integrating over ω, using orthogonality and wavelet analysis relationship as explained in Basu and Gupta (1998), one may obtain the contribution to the mean square shear and moment response for the frequency band corresponding to the dilation factor, a_j. Subsequently, using time-localization property of wavelets, assuming point wise relation taking advantage of narrow energy bands, instantaneous power spectral density function of base shear and overturning base moment may be computed (Chatterjee and Basu 2001) as follows.

$$S_{x_i}(\omega) = \sum_j \frac{E\left[\left|X_i^j(\omega)\right|^2\right]}{\Delta b} = \sum_j \frac{K''\Delta b}{a_j} E\left[\left|W_\psi \ddot{x}_g(a_j, b_i)\right|^2\right] |\tau(\omega)|^2 \left|\hat{\psi}_{a_j,b_i}(\omega)\right|^2 \quad (5.15)$$

where, $\tau(\omega)$ in the above equation represents either $\tau_Q(\omega)$ or $\tau_M(\omega)$. The moments of instantaneous PSDF, the rate of instantaneous zero crossings and the band-width parameter have been evaluated (Basu and Gupta (1998)) to obtain the expected value of the largest peak shear and overturning moment at tank base. The ground motion has also characterized by calculating wavelet coefficients from single accelerogram at each of sites at Dumbarton bridge (Loma Prieta, USA, 1989), Miyagi (Japan, 1978) and El Centro (USA, 1940) (see Chatterjee and Basu, 2001). The height, radius, wall thickness, elastic modulus, mass density, Poisson's ratio and damping ratio of the tank are assumed as 12 m, 4 m, 4 mm, 2.1 X 10^8 kN/m^2, 7850 kg/m^3, 0.2 and 5% respectively. The values of linear elastic soil properties viz. elastic modulus, mass density and Poisson's ratio have been assumed as 3.5 X 10^5 kN/m^2, 1500 kg/m^3 and 0.33 respectively. The values of shear and overturning moment obtained at tank base from the wavelet based analysis have been given in Table 1. The wavelet based pseudo-spectral acceleration spectra (PSA) of the tank have been computed for all three seismic motions at 5% damping and shown in Figure 2. It may be seen from this figure that Miyagi earthquake generated dominant PSA responses for the widest range in time period (0.06 s to 5.0 s) and indeed the strongest of the three motions as is also evident from the values of the base shear and moment in Table 5.1. The Loma Prieta seismicity has milder effect on PSA responses and is critical for a shorter range of time period (0.1 s to 1.0 s).The El Centro ground motion resulted in the mildest peak PSA responses for similar range like Loma Prieta one but without any sharp peaks.

Table 5.1. Expected largest peak value of responses at tank base

Responses	Loma Prieta (1989)	Miyagi (1978)	El Centro (1940)
Base Shear (kN)	1207.60	3902.90	446.34
Base Moment (kN-m)	661.73	2138.80	244.75

6. NUMERICAL MODEL

A three dimensional (3D) soil-footing model has been developed in DIANA and a 3D nonlinear static analysis of this model has been carried out separately for each of the three seismic motions considered. The finite element (FE) model is shown in Figure 3. The circular tank of radius 4 m is placed at the center of the soil surface. The soil domain is assumed to extend 7 times the tank radius on both sides along two mutually perpendicular directions. The vertical extent is assumed to be 5 times the tank radius. In the finite element model, the circular concrete footing (assumed to be 1 m thick) and the soil have been meshed by 2D plate elements and six-node iso-parametric solid wedge elements respectively. The base shear obtained from the analysis has been distributed equally among all the nodes of the footing along horizontal direction (+X) and the overturning base moment has been equally distributed about Y axis at the nodes at the center of the footing (at x = 28 m). The self-weight of the soil and the tank has been considered during the analysis.

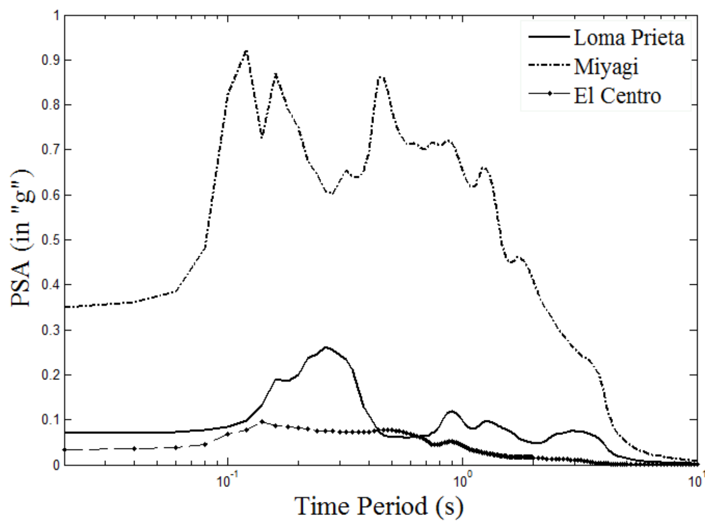

Figure 2. Wavelet based PSA spectra of tank for various ground motions

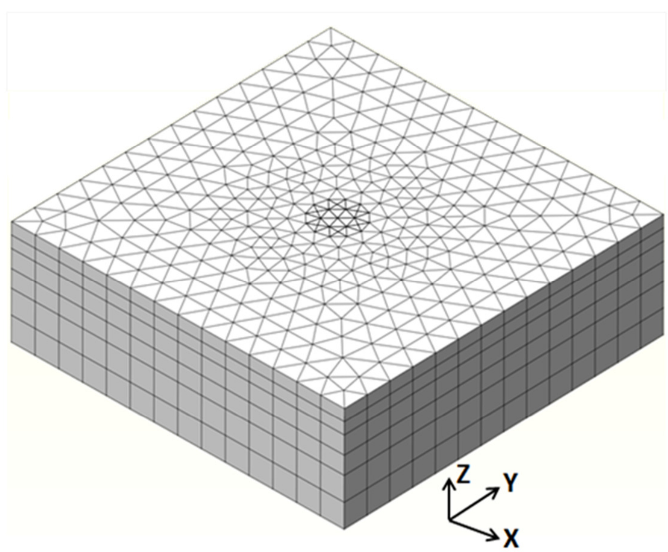

Figure 3. 3D FE model of the soil in DIANA

The soil has been represented by Mohr-Coulomb parameters (cohesion = 5 kN/m^2, friction angle = 36^0 and dilatancy angle = 6^0) and K_0 = 0.5 is also assumed in addition to the values stated above for linear elastic analysis done before. The elastic modulus, Poisson's ratio and the density of the footing material are 3 X 10^7 kN/m^2, 0.2 and 2400 kg/m^3 respectively. Figure 4 represents normalized horizontal displacements of the soil at a depth of 7 on *XZ* plane through the footing center as obtained from finite element analysis in DIANA for 3 different seismic motions. Figures 5 and 6 show variation of normalized normal stresses (S_{xx}) and the maximum horizontal principal strain (free from shear strain) inside soil domain at 7 m below the surface on *XZ* plane through center of tank-footing system. These results include the non-stationarity effects of the ground motions as these effects have been captured while obtaining the values of the shear and moment responses from wavelet based analysis. It may be mentioned here that in the plots shown, the normalized distance along X axis has been obtained dividing the distance by the tank diameter and the values of the displacements and stresses have also been normalized dividing the respective values by the corresponding maximum value obtained considering all three types of earthquakes. The soil near the boundary is deformed more due to the more powerful Miyagi earthquake and the nature is anti-symmetric in case of milder earthquakes like Loma Prieta and El Centro (Figure 4). As the base shear is applied in +X direction and the moment has been applied clockwise about Y axis, so the stresses seem to increase immediately on the right side of the centerline of the tank-soil model (around x = 3.5 m) as evident from Figure 5. The maximum horizontal principal strain (Figure 6) seems to follow a symmetrical trend about the centerline of the tank.

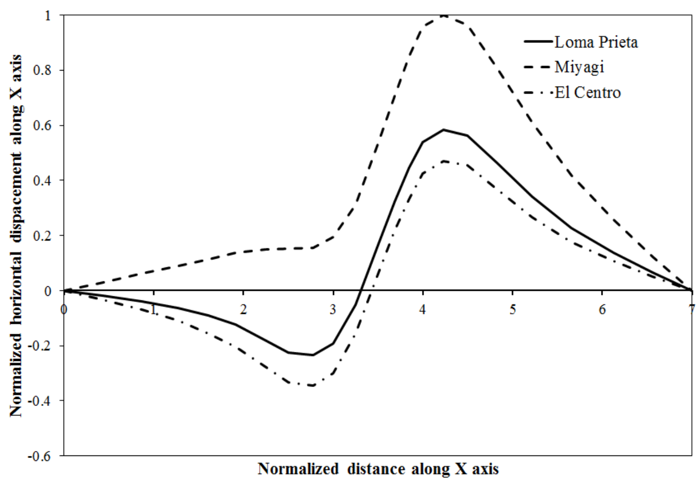

Figure 4. Horizontal (X) soil displacements at 7 m depth (from DIANA analysis)

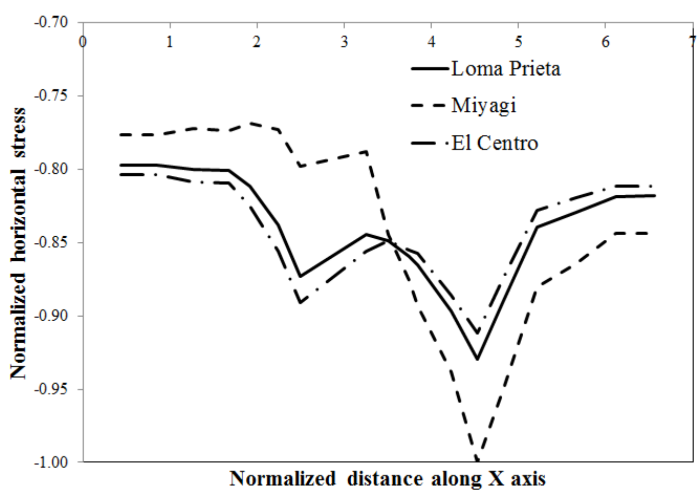

Figure 5. Horizontal stresses in soil at 7 m depth (from DIANA analysis)

7. CONCLUSIONS

In this paper, a method for soil-structure-soil interaction analysis subjected to earthquake excitations has been presented with reference to a tank-foundation-soil model. The first part on coupled dynamic linear soil-structure interaction analysis has been developed based on wavelet dependent technique to capture the non-stationary effects of ground motions. The values of the responses obtained from this part have been suitably applied to the finite element model of the soil-foundation system developed in DIANA to perform nonlinear static analysis and obtain corresponding responses in the soil domain thereby accounting for ground motion non-stationarities.

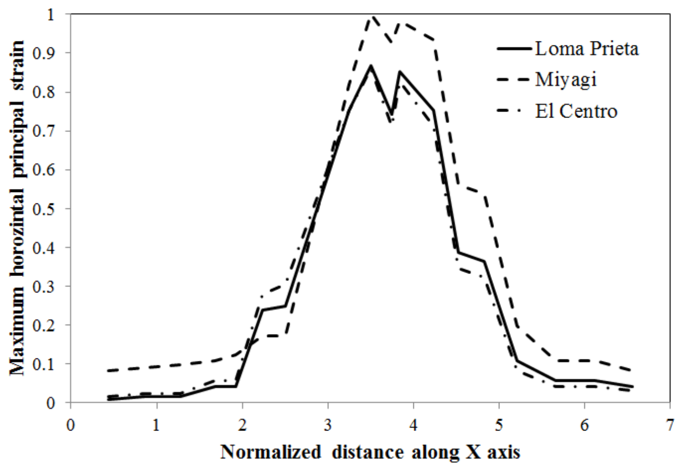

Figure 6. Variation of maximum horizontal principal strain (E_{xx}) in soil at 7 m depth (from DIANA analysis)

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