Experimental analysis for identification of bridges structural damage using Operational Modal Analysis based methods

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SUMMARY
Recent structural collapses of bridges and damaging occurred on deck and dampers of suspension bridges, are driving the development of monitoring techniques for the structural integrity assessment of primary lifelines. Structural Health Monitoring combines a variety of methods for observed dynamic systems response detection of different structural systems. Among others techniques, Operational Modal Analysis, combined with efficient techniques for modal parameters identification, seems well suited for Vibration-Based Damage Detection (VBDD) of bridges under operating traffic loads. This paper addresses the issue, of the effectiveness of VBDD algorithms based on strain energy or modal flexibility matrices for the damage detection of bridge structures under operational loads. A reduced scale model of a three-span bridge was developed to evaluate the interaction effects between moving loads and structure and to investigate the algorithms sensitivity to different input levels and the damage prediction for different sensors density and damage size.

Keywords: Structural Health Monitoring, Bridges, Operational Modal Analysis

1. INTRODUCTION

Recent cases of structural collapse of bridges, such as I-35W Mississippi River bridge Minnesota, USA (2007), and the damage of the deck and some seismic dampers device of large suspension bridges such as the Vincent Thomas Bridge in the harbor area of Los Angeles CA (Amaddeo et al., 2008), have revealed the need to develop and implement technologies for monitoring the safety assessment of bridges and major lifelines. The civil infrastructures and bridges in particular, over time inevitably deteriorate. The most common causes of degradation of bridges (Mufti, 2001) are related primarily to corrosion of structural steel parts and rebars, or deterioration of the concrete and damage of constraint devices. The continuous evolutions of traffic loads induce an increase of structural stresses, and it requires an adjustment, expansion or replacement of the bridge. In this context, monitoring techniques of structural integrity through estimation of dynamic parameters are emerging as tools for planning and retrofitting of involved structures. The Structural Health Monitoring (SHM) combines a variety of methodologies for monitoring the performance and health status of various structures. Generally, the objectives of Structural Health Monitoring (Mufti, 2001) are: (1) Monitor the behavior of a structure with precision and efficiency, (2) Locate the damage and degradation, (3) Determine the health status and the condition of a structure in order to evaluate performance. The analysis techniques can be divided into three main classes (Xianfei, 2008): (1) Vibration-Based Damage Detection method (VBDD), (2) Statically-Based Damage Detection method (SBDD), (3) Direct inspection of structural elements. VBDD weigh changes in dynamic characteristics of a system, such as the natural frequencies, mode shapes and damping, as indicators of damage. Then, in order that damage can be properly identified by the VBDD methods, a reliable evaluation of the structural dynamic parameters is necessary (Farrar et al., 1997). The properties of a structure can be obtained analytically or through experimental analysis (EMA). Particularly, there are two approaches: in the first, called Experimental Modal Analysis (EMA), input and output are known, in the second, called Operational Modal Analysis (OMA) unknown input is given by environment excitation (wind, traffic), similar to a random signal, while the response is measurable. EMA is feasible with difficulty in the case of large structures (Brincker et al., 1990) (Brincker et al., 1991) (Caicedo et al., 2004). The main advantages of OMA are (Aktan et al., 2005): the test is quicker and cheaper since they are not required equipment for the excitation of the structure; the measurements are performed under actual operating conditions of the structure and the modal parameters obtained are representative of the dynamic behavior of the structure in its actual conditions of use; the test does not interfere with the operation of the structure ; so that, for example, it is not necessary to close a bridge to the traffic when it is tested. It is worth noting that the output-only techniques for the identification of the system are necessary for the development of a continuous monitoring based on natural vibrations in order to assess the health status of the structure.

This paper contains results of numerical and experimental studies on a reduced scale model of a three spans bridge, with the aim to investigate same questions regarding the potential use of VBDD output-only for structural health monitoring of bridges. The dynamic identification of the system was realized through the combined use of the Eigensystem Realization Algorithm (ERA) (Juang, 1994) and the Natural Excitation Technique (NExT) (James et al., 1993) for the determination of the impulse response functions. The reliability of these methodologies primarily depends on the trustworthiness of the dynamic characteristics of the system identified by OMA, and the ability to localize with sufficient accuracy the damage. Main objectives of this study are: (1) Assessing the interaction between moving loads and structure in the dynamic and damage identification (2) Evaluating the quality of the dynamic identification and their sensitivity at different input levels, (3) Assessing the ability of the used identification damage methods to correctly locate damage and to evaluate the sensitivity of damage prediction for different sensors density and damage size.

2. METHODS FOR DAMAGE IDENTIFICATION

2.1 Damage Index Method (DI)

The Damage Index Method was developed by (Stubbs et al., 1995). The indicator of damage β is based on the variation of the beam strain energy. The variations in the strain energy may be related to changes in its curvature. The indicator of damage β_{ij} at the node j of the beam and at the mode shape i is defined as

$$\beta_j = \sum_{i=1}^{N} \frac{\left(\varphi_{ij}^{''*}\right)^2 + \sum_{k=1}^{n}\left(\varphi_{ik}^{''*}\right)^2}{\left(\varphi_j^{''}\right)^2 + \sum_{k=1}^{n}\left(\varphi_j^{''}\right)^2} \times \frac{\sum_{k=1}^{n}\left(\varphi_j^{''}\right)^2}{\sum_{k=1}^{n}\left(\varphi_{ik}^{''*}\right)^2} \tag{2.1}$$

where $\varphi_j^{''}$ e $\varphi_j^{''*}$ are respectively the curvature at node j of the mode shape before and after damage, n is the number of nodes and N is the number of identified modal shapes (Farrar et al., 1994).

Assuming that the collection of the damage indices, β_j, represents a sample population of a normally distributed random variable, a normalized damage localization indicator is obtained as follows

$$Z_j = \frac{\beta_j - \mu}{\sigma} \geq 2 \tag{2.2}$$

where μ and σ represent the mean and standard deviation of the damage indices, respectively. For a value of Z_j equal to two it has got a significance level of 95% (Wang et al., 2000).

2.2 Positive Bending Inspection Load (PBIL)

This method developed by (Koo et al., 2009) is based on the explicit relationship between the modal deformation of the structure before and after the damage. Through the dynamic flexibility matrix is possible to evaluate the deformed shape of the structure induced by an arbitrary load vector as follows:

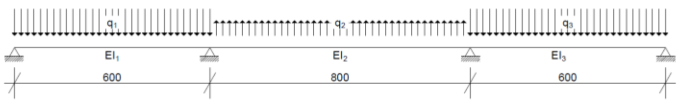
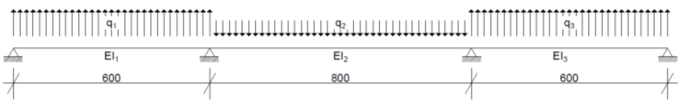
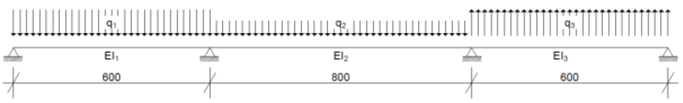
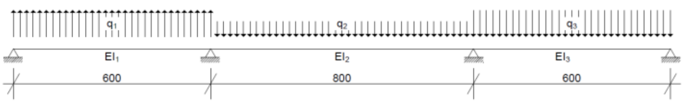
$$u = G_m f \tag{2.3}$$

where u is the vector containing the deformed shape, f is the load vector and G_m is the flexibility modal matrix that contains only the first m modes:

$$G_m = \Phi_m \Lambda_m^{-1} \Phi_m^T \tag{2.4}$$

where Λ is the diagonal matrix of modal frequencies and ϕ_m is the normalized mode shape matrix with respect to the mass. If you do not know the mass distribution, the normalization must be performed by the experimental tests of "added mass" or "mass perturbation method" (Brinker et al., 2002), (Bernal, 2002). To locate the damage in interest region, we must use a load or sets of loads that does not produce an inflection point in this region. It is possible to presume that damage, when is localized in the immediate vicinity of an inflection point, do not induce a significant additional strain on the structure since the bending moment in that section is negligible. In addition, in order to ensure easy recognition of the damage, the load vector will ensure a positive bending moment in the same portion of the structure (Koo et al., 2009).

Table 2.1: Positive Bending Inspection Loads for test model

Inspection region	Loads	Definition of PBILs
1st and 3rd span	$\frac{q_1}{q_2} = \frac{I_1}{I_2}\left(\frac{L_2}{L_1}\right)^3$; $q_1 = q_3$	
2nd span	$\frac{q_1}{q_2} = \frac{I_1}{I_2}\left(\frac{L_2}{L_1}\right)^3$; $q_1 = q_3$	
1st intermediate support	$\frac{q_1}{q_2} = \left(\frac{L_2}{L_1}\right)^2$; $q_1 = -q_3$	
2nd intermediate support	$\frac{q_3}{q_2} = \left(\frac{L_2}{L_1}\right)^2$; $q_1 = -q_3$	

After we calculated the difference between the flexibility matrix pre-and post-damage (G_m, G_m^*) as follows:

$$\Delta G_m = G_m^* - G_m \tag{2.5}$$

and we define all the load vectors in order to cover different areas of the structure to be inspected, following the references listed above, and we calculate for each of them the strain induced

$$\Delta u_i(x) = \Delta G_m f_i \tag{2.6}$$

The procedure of damage location occurs through the use of an approach of chord deformation $\Delta u_\Omega(x)$ (Damage-Induced Chord-wise deflection DI-CD), which can be defined as the additional deformation caused by damage and measured along the cord that connects two points x_a and x_b:

$$\Delta u_\Omega(x) = \Delta u(x) - \left[\frac{\Delta u(x_b) - \Delta u(x_a)}{x_b - x_a}(x - x_a) + \Delta u(x_a)\right] \tag{2.7}$$

where $\Omega = [x_a, x_b]$ is the region inspected.

The algorithm of location of the damage can be determined as follows:

$$\text{Damage at } x = x_d \iff f(x) = \begin{cases} \Delta u_\Omega(x) \text{ has a maximum at } x = x_d \\ \Delta u'_\Omega(x) \text{ changes abruptly at } x = x_d \end{cases} \tag{2.8}$$

The load vectors PBIL, as they have been defined, will ensure the inspected region Ω a value of positive bending moment. The damage can then be located in that region where the deformation reaches a maximum and at the same time its derivate abruptly changes. Deflections of the test models were estimated using Eq. (2.7) with the PBILs as shown in Table 2.1.

3. NUMERICAL AND EXPERIMENTAL STUDY

3.1 Characterization of model

The experimental validation was carried out through two scale models with different mass density. Tested models are composed of a continuous steel beam with rectangular cross section. To experimentally simulate the traffic loads, issue that it was treated by several authors including (Yun et al., 2004), it was realized a system constituted by moving arm with three pendulums connected in series reproducing a multiple-input/multiple-output system (MIMO). The arm connected to the shake table simulates a random moving load through three wheels that are dragged on the deck.

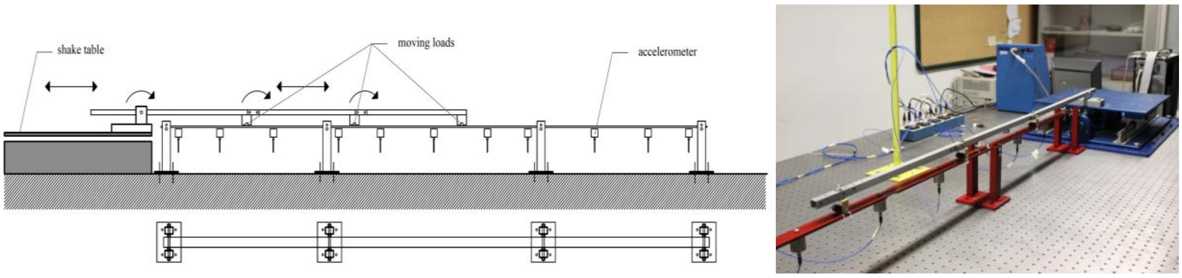


Figure 3.1: Scale model and accelerometers layout

The total length of the model is equal to 2 m, while the cross section is of 30x5mm. In Table 3.1 and Table 3.2 the geometrical dimensions of model and adopted scale factors are listed.

Table 3.1: Geometrical dimensions of prototype and model

	Prototype (m)	Model (m)
L_{tot} (m)	140	2
$L_{central\ span}$ (m)	56	0.8
$L_{lateral\ span}$ (m)	42	0.6

Table 3.2: Scale factors for 1st and 2nd test model

Physical quantities	*Scale factor 1st model*	*Scale factor 2nd model*
Length	$S_L = 70$	$S_L = 70$
Young's modulus	$S_E = 1$	$S_E = 1$
Mass	$S_M = 150\ S_L^2$	$S_M = 50\ S_L^2$
Moment of inertia	$S_I = 150\ S_L^4$	$S_I = 50\ S_L^4$
Force	$S_F = 150\ S_L^2$	$S_F = 50\ S_L^2$
Time	$S_t = \sqrt{S_L}$	$S_t = \sqrt{S_L}$
Frequency	$S_f = 1/\sqrt{S_L}$	$S_f = 1/\sqrt{S_L}$

3.2.1 Equipment

The chain of excitation-acquisition is composed by: Quanser Shake Table II, four PCB 393B04 and seven PCB 626B04 accelerometers, one unit LMS Scadas mobile M05 with 16 acquisition channels (24bit) and simultaneous sampling, LMS Test.Xpress v9.0A software.

3.2 Quality assessment of modal identification for different input levels

To assess the sensitivity of the modal identification technique were performed experimental tests on the model with three different levels of input intensity. The acceleration measurements were repeated 10 times for each level of input, and then they were measured average RMS of output acceleration values for each test (see Table 3.3). Figure 3.2 also shows the second mode shape identified and standard deviation values for the first and the second test. In the third test, due to low acceleration values, were not correctly identified the first two modes.

Table 3.3: Results of dynamic identification, frequencies and their deviation for each level of input

Test N.1 $20 \leq a_{RMS} \leq 40$ *(mg)*		*Test N.2* $5 \leq a_{RMS} \leq 12$ *(mg)*		*Test N.2* $0.5 \leq a_{RMS} \leq 2$ *(mg)*	
F(Hz)	Deviation	F(Hz)	Deviation	F(Hz)	Deviation
20,2431	0,03476	20,2510	0,01385	/	/
23,8768	0,01035	23,9206	0,00545	/	/
44,4853	0,01485	44,5379	0,02560	45,6422	2.13

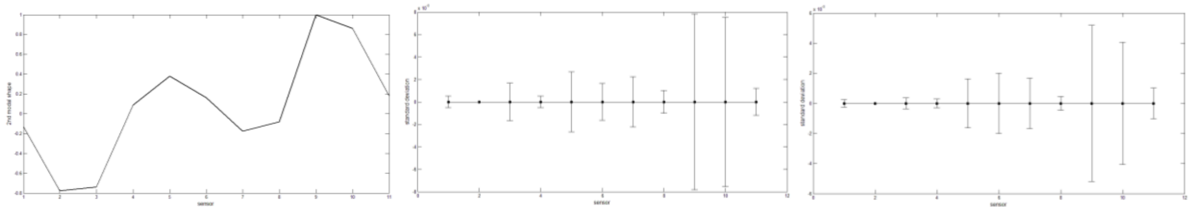


Figure 3.2: 2nd identified mode shape and standard deviation for the first and the second test

Figure 3.2 shows that the levels of standard deviation in the portion of the structure not subject to moving loads are less than $2x10^{-3}$ while they grow up to $8x10^{-3}$ in the section directly loaded, influencing negatively the identification of a possible small damage located just in that span.

3.3 Damage identification through numerical simulation

To assess the ability of the DI and PBIL methods to correctly locate the damage numerical simulations were conducted by varying sensors density placed in the structure.

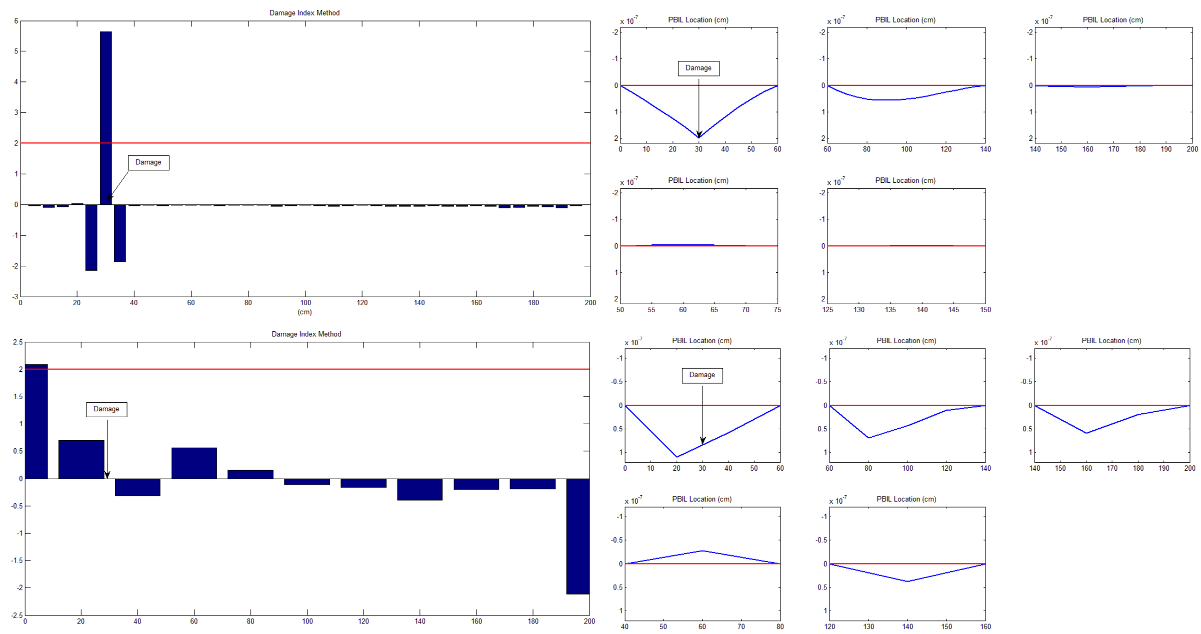


Figure 3.3: Numerical simulation of a single damage, DI and PBIL: 41 sensors (up); 11 sensors (down)

Two different damage scenarios were considered. In the first case a single mid span damage was introduced (see Table 3.5, Test n.2). In the second case damage was introduced at one support and mid span (see Table 3.6, Test n.4). The search of the damage was carried out by simulating the presence of 41 and 11 sensors. Through the Figure 3.3 and Figure 3.4 it is possible to observe how the damage with 41 sensors is properly located by both methods. With 11 sensors, the Damage Index is unable to locate the damage, while the PBIL locates the damage but also gives false positives.

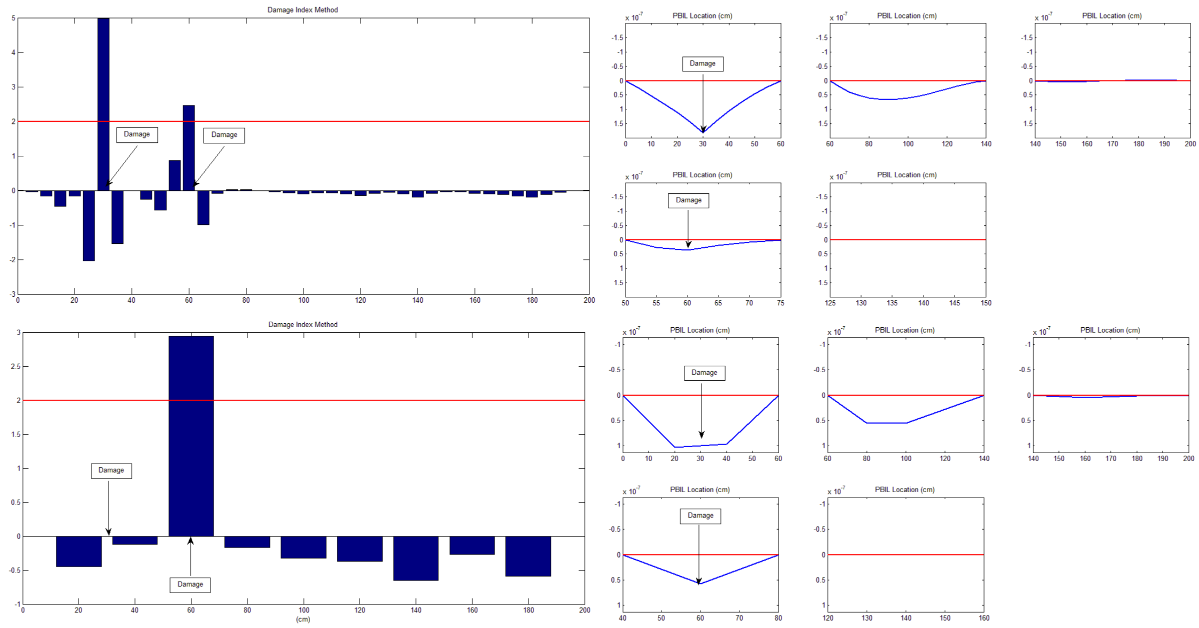


Figure 3.4: Simulation of two damage, DI and PBIL: (up) with 41 sensors; (down) with 11 sensors

3.4 Damage identification through experimental tests

The experimental validation was performed using four different input scenarios that differ in intensity, location, severity and presence of multiple damages. Eleven accelerometers were evenly placed on the lower surface of one side of the girder as shown in Figure 3.1. The different damages were imposed at two locations by cutting the deck sections (see Table 3.4).

Table 3.4: Damage severity on test section

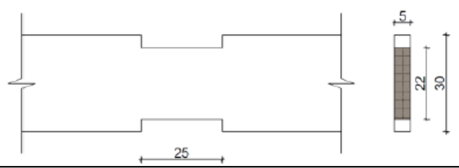
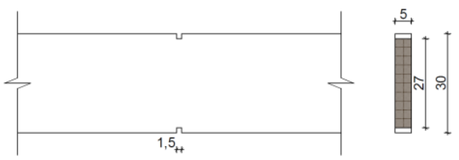
Damage	Damage severity (mm)	Moment of Inertia reduction	Area reduction
I		27%	27%
II		10%	10%

Table 3.5 and 3.6 show the different test combinations that were performed (1 to 4). Random vibration inputs were induced by shake table during 5 minutes and the acceleration responses were measured with 1000 Hz sampling rate. Then the signal has been filtered with a FIR filter and down-sampled to

200 Hz. The acceleration measurements were repeated 10 times for the intact case and 10 times for each damage case. During the execution of the tests the temperature was maintained nearly steady in order to make temperature effects negligible.
Finally, Table 3.7 and 3.8 show the results of the identification process. For each estimated parameters it was evaluated the relative standard deviation (RSD). Table 3.9 and 3.10 show the comparison between the parameters identified for the test N.1 and 3 before and after the damage; we can see how a local stiffness reduction of 27% causing limited natural frequencies variations (max 1.5%). This demonstrates how a good dynamic characterization of the undamaged model is of primary importance to be able to detect small variations of the dynamic properties of the system. To assess the significance level between the modal frequencies identified pre and post-damage it was used the statistical Student's t-test (DeCoursey, 2003). The results confirmed that the variations between the natural frequencies in the two conditions are statistically significant for all natural frequencies except the sixth, with a significance level less than 0.01, that it has 99% of the probability that there aren't casual differences between the two samples.

Table 3.5: Damage location – Single damage

Test N.	*Test model*	*Damage*	a_{RMS} *(mg)*	*Damage location*
1	1st	I	20÷50	
2	2nd	II	5÷10	

Table 3.6: Damage location – Multiple damage

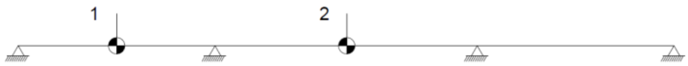
Test N.	*Test model*	*Damage*	a_{RMS} *(mg)*	*Damage location*
3	1st	I	20÷50	
4	2nd	II	5÷10	

Table 3.7: Modal parameters identified with NExT for the 1st undamaged model

Frequency identified (Hz)	Frequency RSD (%)	Damping (%)	Damping RSD (%)	Frequency FEM (Hz)	Δ(identified-FEM) (%)
21.349	0.069	1.488	1.97	20.827	2.45
32.812	0.230	5.575	3.76	33.794	-2.99
40.204	0.211	2.091	8.32	40.987	-1.95
85.121	0.206	4.170	7.51	78.070	8.28
126.051	0.021	0.401	5.59	122.010	3.21
135.211	0.015	0.266	7.45	132.810	1.78
176.510	0.024	0.455	4.80	167.991	4.83

Table 3.8: Modal parameters identified with NExT for the 2st undamaged model

Frequency identified (Hz)	Frequency RSD (%)	Damping (%)	Damping RSD (%)	Frequency FEM (Hz)	Δ(identified-FEM) (%)
12.558	0.2654	1.108	15.481	11.978	4.62
20.127	0.122	0.964	7.776	19.479	3.22
23.790	0.042	0.378	6.233	23.468	1.35
44.345	0.135	0.914	7.828	44.648	-0.68

Table 3.9: Comparison of modal parameters before and after the damage and statistic comparison with Student's t-distribution at for test N.1

Frequency Undamaged (Hz)	*Standard Deviation*	*Frequency Damaged (Hz)*	*Standard Deviation*	*Δ (%)*	*Significance level p*
21.349	0.014731	21.401	0.0238	0.240	<0.01
32.812	0.07547	32.495	0.0597	-0.979	<0.01
40.204	0.08483	39.928	0.0570	-0.692	<0.01
85.121	0.175349	83.586	0.0904	-1.836	<0.01
126.051	0.026471	125.813	0.0150	-0.189	<0.01
135.211	0.020282	135.207	0.0365	-0.004	/
176.510	0.042362	174.299	0.0310	-1.268	<0.01

Table 3.10:Comparison of modal parameters before and after the damage and statistic comparison with Student's t-distribution at for test N.3

Frequency Undamaged (Hz)	*Standard Deviation*	*Frequency Damaged (Hz)*	*Standard Deviation*	*Δ (%)*	*Significance level p*
21.349	0.014731	21.401	0.0022	-1.623	<0.01
32.812	0.07547	32.495	0.0523	-1.948	<0.01
40.204	0.08483	39.928	0.0465	-2.354	<0.01
85.121	0.175349	83.586	0.2058	-1.882	<0.01
126.051	0.026471	125.813	0.0651	-2.864	<0.01
135.211	0.020282	135.207	0.0047	-1.180	<0.01
176.510	0.042362	174.299	0.0366	-1.776	<0.01

Figure 3.5-3.8 show instead the results of the damage identification with DI and PBIL for the four conducted tests. It was found that all damage locations were reasonably identified for all the cases by PBIL, while the DI, as provided by the numerical simulations, was unable to locate the damage. However the estimated results contain false alarms at several locations.

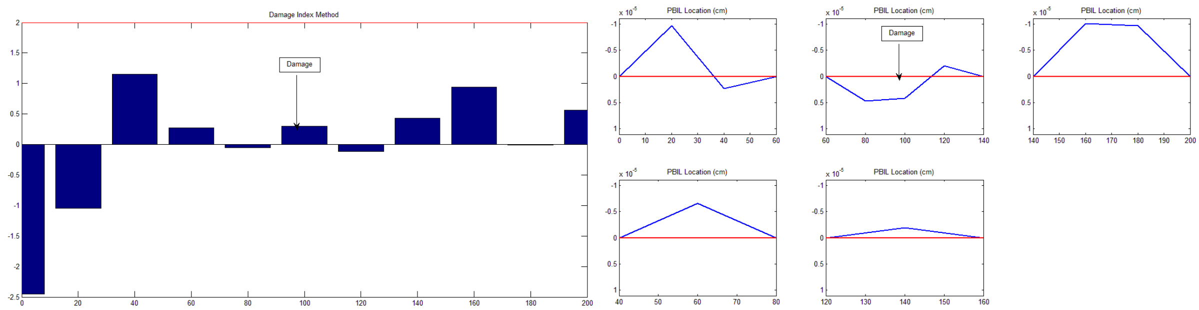


Figure 3.5: Test N.1 – Single damage, DI and PBIL

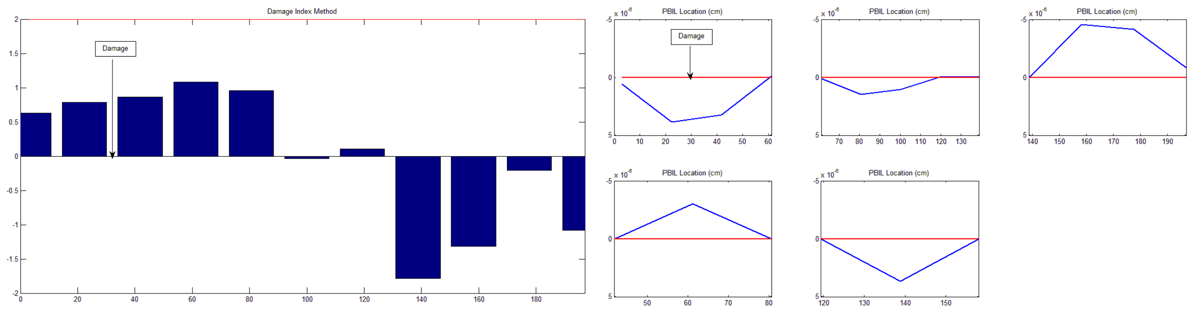


Figure 3.6: : Test N.2 – Single damage, DI and PBIL

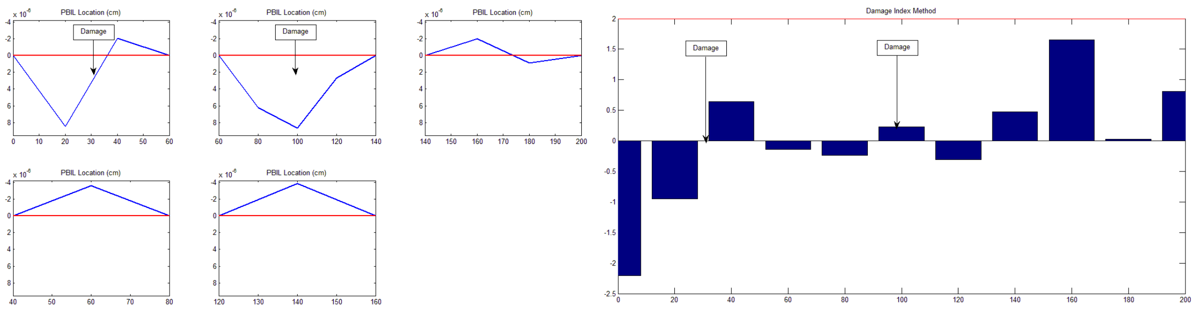


Figure 3.7: Test N.3 – Multiple damage, DI and PBIL

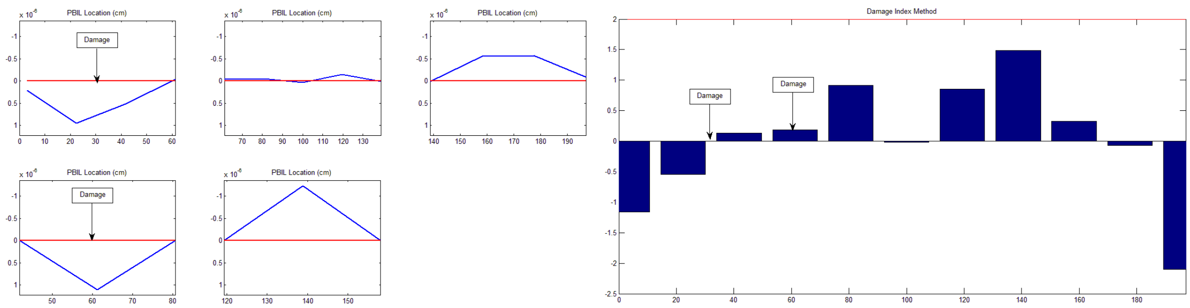


Figure 3.8: Test N.4 – Multiple damage, DI and PBIL

4. CONCLUSIONS

In this paper, results of numerical and experimental studies on a reduced scale model of a three spans bridge were presented using output-only methods and simulated traffic loads. At first, the modal parameters of the experimental model were estimated from different input levels to evaluate the sensitivity of the dynamic identification. The presence of moving loads increases the values of the standard deviation in the area directly loaded, negatively influencing the identification of a possible small damage located just in that span. Also it can be seen that if the input levels is too low it is impossible to identify a sufficient number of mode shapes. Then to assess the ability of the DI and PBIL methods to correctly locate the damage numerical simulations were conducted by varying sensors density. Finally, the experimental validation was performed using four different scenarios that differ in input intensity, location, severity and presence of multiple damages. Both methods used for the damage identification (Damage Index Method and Positive Bending Load) were able to correctly locate the damages, but one can see how their efficacy depends strongly on the sensors density. PBIL seems to have less sensitivity to the spacing of sensors, however the estimated results contain false alarms at several locations.

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