Probabilistic Assessment of Masonry Building Clusters

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SUMMARY:
A Masonry Building Cluster (MBC) is made from an assemblage of buildings and thus has a non-unitary origin: however, under certain conditions, Structural Units (SUs) can be identified and analysed independently.
Starting from a deterministic simplified procedure developed by the authors, a fully probabilistic method of analysis of a SU has been developed, which treats as uncertain all relevant variables, such as: general data, geometry, construction details, materials.
This allows to perform fully probabilistic Monte-Carlo-based analyses that allow to compute the failure probability of the MBC. The subsequent sensitivity analyses allow in turn to identify the effect of the different variables on the global response. The latter step proved to be very useful to single out the structural elements worth of a deeper investigation, thus reducing the uncertainty of the assessment outcome.

Keywords: Masonry building aggregate; FME method; Uncertainties; Sensitivity analysis; Reliability analysis

1. INTRODUCTION

Starting from a deterministic model developed by Monti and Vailati (2009), further developments resulted in a fully probabilistic procedure to assess Masonry Building Clusters (MBC). Such model follows the seismic Italian code (NTC-08) that prescribes to assess these buildings by means of nonlinear analyses. Furthermore, in the case of infinitely rigid floors, the following simplifications are allowed:

- the analysis can be performed floor by floor;
- rocking effects on masonry walls are neglected;
- torsional effects are neglected.

By following such indications, the deterministic procedure was implemented into an easy-to-use spreadsheet that allows to perform the assessment in a straightforward way.
The probabilistic procedure is an extension of the above procedure, which includes all uncertainties that may affect the seismic structural response. It permits to focus the diagnostics - and eventually to design strengthening measures - on those elements whose influence is more significant on the overall structural response.
In the next section, the relevant steps of the simplified procedure are summarized; the probabilistic procedure, composed by a deterministic analysis method and a stochastic input, is described in more detail in section 3.
Figure 1.1 shows a schematic view of both the deterministic and the probabilistic procedure for evaluating the deterministic and stochastic response, respectively.

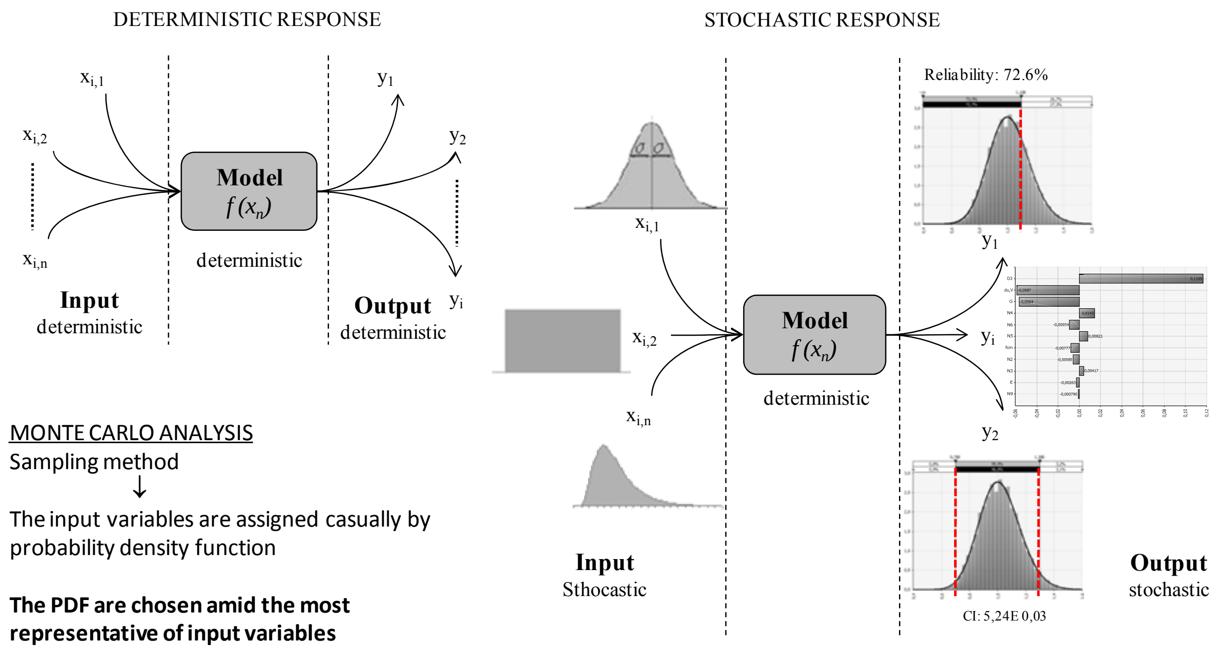

Figure 1.1. – Scheme of the deterministic (left) and of the probabilistic assessment procedure (right).

2. DETERMINISTIC PROCEDURE: SIMPLIFIED NONLINEAR ANALYSIS

The deterministic procedure can be briefly summarized in the following steps:

- definition of a constitutive bilinear law for each masonry wall in terms of three parameters (“yield” strength, “yield” and ultimate displacements);
- derivation of a constitutive law for each floor, by summing each wall contribution, when rigid floor condition is assumed;
- derivation of an equivalent bilinear constitutive law for each floor;
- computation of the dynamic response by means of a simplified modal analysis;
- computation of the interstory drift;
- comparison between capacity and demand for each interstory; if the ratio of the two is larger than one for all interstories, the SU is verified, otherwise it is not.

A detailed description of the simplified procedure can be found in (Monti and Vailati, 2009), while the extension to the case of flexible floors is discussed in (Vailati and Monti, 2011).

3. SOURCE OF UNCERTAINTIES: IDENTIFICATION OF VARIABLES AND THEIR MODELLING

A classification of the variables by type of uncertainty is shown in Table 3.1.

Table 3.1. Uncertainty nature of variables

Categories of variables	Associated parameter	Uncertainty
Geometry	Dimensions	Epistemic
Materials	Strength, modulus	Intrinsic, epistemic
Construction details	Connections	Epistemic

Having defined the uncertainty nature of each variable, a corresponding appropriate distribution model is assigned.
It is worth noticing that the type of model has an influence on the probability distribution of the output variables. Therefore, the most appropriate distributions should be identified by characterizing the

study parameters through statistical identification methods applied to the collected data and to the output control parameters.

The χ^2-test has been used in this study, which allows to ascertain if the observed frequencies of a measured event significantly differ from the theoretical frequencies provided by the considered probability distributions models.

If $\chi^2 = 0$, the observed frequencies exactly coincide with the theoretical ones. Instead, if $\chi^2 > 0$, the values of the frequencies differ from each other; as the estimator assumes high values, the greater the difference between observed and theoretical frequencies.

The relational scheme proposed in Figure 3.1 allows to associate the most suitable probability distribution to the characteristics of the collected data.

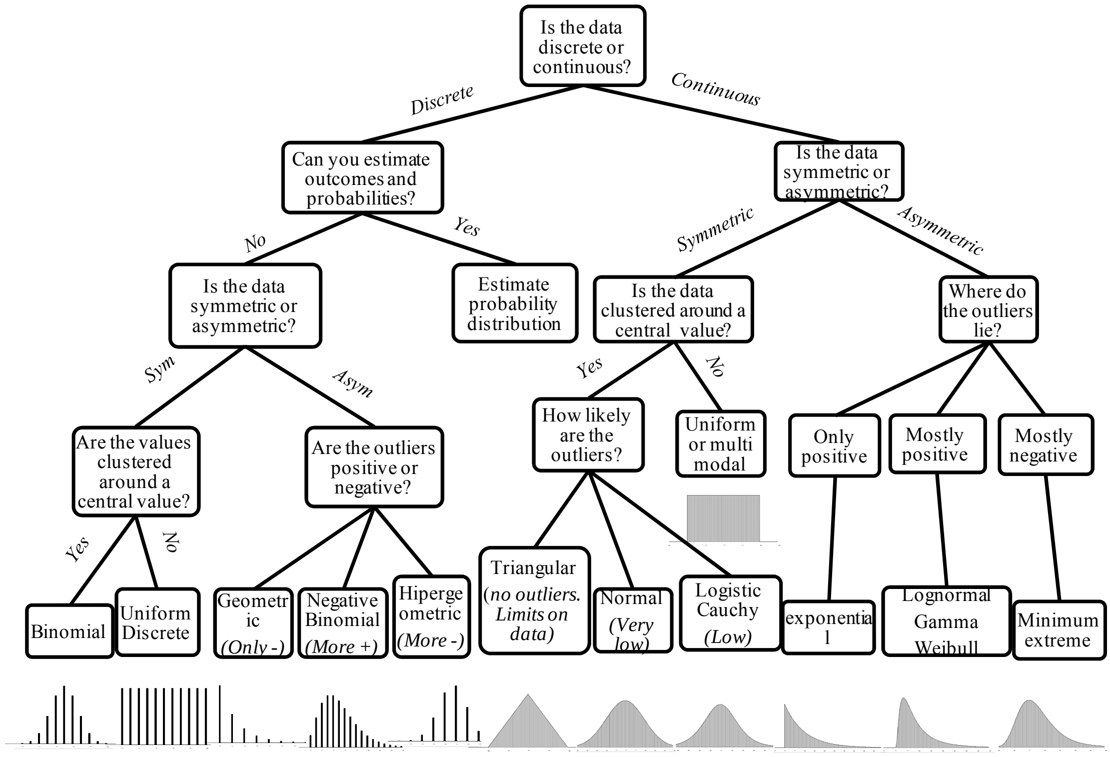

Figure 3.1. Relational scheme between probability distributions and characteristics of collected data

A better knowledge level for each variable can be attained by reducing its epistemic uncertainties. The activities of data acquisition and subsequent probabilistic description are based on Bayes inference, as follows. By denoting the null hypothesis and the observed empirical data with *"X"* and *"D"*, respectively, the Bayes' theorem can be stated as follows:

$$P(X|D) = \frac{P(X|D) \cdot P(X)}{P(D)} \tag{3.1}$$

The null hypothesis will have to be formulated before the observation D.

In Bayes' rule, as widely known:

- P(X) is the prior probability;
- P(D|X) is the likelihood function on which the statistical inference is based;
- P(D) is the marginal likelihood or "model evidence", i.e. the likelihood of observing D, without any prior information;
- P(X|D) is the posterior probability, given D has been observed.

The scale factor P(D|E)/P(E) can be considered as a measure of the effect that the observation of D has on the confidence level of the researcher in the null hypothesis. The latter is in turn represented by the

prior probability P(X); if it is unlikely that D is observed, unless X is not really true, the scale factor will be high. Consequently, the posterior probability (confidence) combines the prior beliefs of the researcher with those deriving from the observation of the empirical data.
In summary, the Bayesian approach aims at providing an increasingly reliable model by enhancing the information collected on the variables described as random.

4. RISK ANALYSIS

After having assigned the appropriate models to the selected random variables, the analysis can be performed using Monte Carlo simulation, according to the following steps:

- development of a parametric model, $y = f(x_1, x_2, ...x_q)$;
- assignment of the probability distributions to the uncertain variables (E, G, f_m, τ_0, ...);
- generation of random numbers ($x_{i1}, x_{i2}, ..., x_{in}$) for the *n* variables according to the distributions assigned to the previous step;
- evaluation of the output model and results storage as y_i;
- analysis of results (bar charts, summarizing statistics, confidence intervals).

5. SENSITIVITY ANALYSIS

Once the simulations on the input stochastic variables are performed, their effect on the overall response is evaluated. To this purpose, the distributions of both input and output data have to be used for some calculation steps, summarized in the following:

1. calculation of median and standard deviation of the input samples ($M_{ei,1}$, $\sigma_{t,1}$);
2. determination of a subset for each input collecting the only iterations which achieve a given objective;
3. calculation of median for each subset ($M_{ei,3}$);
4. comparison, for each input, between the difference of the median values computed at the steps 1 and 3 and the standard deviation at step 1.

If:

$$\Delta_t = \left| M_{e_{i,1}} - M_{e_{i,3}} \right| > \frac{\sigma_{t,1}}{2} \tag{5.1}$$

the input significantly affects the response. The steps 1-through 4 are repeated for each study variable.
At the end of the iterations, a value to each input parameter included in a subset is assigned; such value represents the variation produced on the control parameter when the input variable increases of $+\sigma$. As an example, the following conditions can be assumed:

- ρ is the control variable, which is the ratio between the displacement capacity and the demand one of the inter-storey k; $\rho = 0.6$ when the mean values are assigned to all the variables;
- τ is one of the examined input variables, which is the shear strength of masonry; the given strength distribution has standard deviation and median equal to 0.05 MPa and 0.28 MPa, respectively;
- the variation of τ over ρ is equal to 0.03

This means that if $\tau = M_e + \sigma$, then $\rho = 0.63$, i.e. if the input variable is increased by a standard deviation, the control parameter has a variation equal to the index provided by the sensitivity analyses.
In the considered case, τ weighs on the variable ρ by 5% every time it has a variation of $+\sigma$.
Obviously, each variable can positively or negatively affect the analysis outcome of which the control parameter is an estimator. Therefore, it is also possible that a $+\sigma$ variation yields to a negative variation of the control parameter.

6. RELIABILITY OF SEISMIC RESPONSE: SOME EXAMPLE CASES OF SUs

The reliability analysis presented in the previous paragraphs is now applied on three example cases of SUs to show the powerful and usefulness of reliability analysis of MBC.
The chosen cases are representative of a larger number of SUs, which were randomly generated by changing the parameters influencing the dynamic response. In this way, three buildings classes were identified, defined by: period, percentage of shear and flexural collapse, ratio between resistant and total area. Figure 6.1 depicts the geometric configuration of the SUs considered for performing the sensitivity analyses. All the random variables considered in such analyses are listed in Table 6.1. Conversely, the seismic hazard parameters, reported in Table 6.2, are assumed to be deterministic; other fixed parameters are the number of floors, respectively equal to 4, 3 and 2, and the number of walls, equal to 10 for the first two, 16 for the last one.

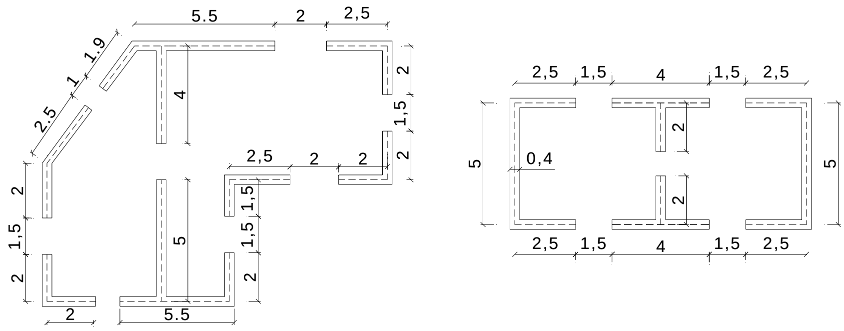

Figure 6.1. Example cases: at the left the SU with two floors, at the right the SUs with 3 and 4 floors and the same plan. X direction is horizontal, Y direction is vertical.

Table 6.1. Global and local uncertainties, distribution models, statistical quantities

Category	Variables	Distribution type	Correlation	Min	Max	Mean	σ
General data	du,v (%)	Uniform	no	0.003	0.005	0.004	0.057
	du,f (%)			0.005	0.007	0.006	0.057
	α			0.35	0.65	0.5	0.086
	G_k (kN/m^2)			3	5	4	0.578
	Q_k (kN/m^2)			1	2	1.5	0.289
Materials	f_m (kN/m^2)	Normal	no	1000	1800	1400	300
	τ (kN/m^2)		$0.025 \cdot f_m$			35	
	E (kN/m^2)		$700 \cdot f_m$			980000	
	G (kN/m^2)		0.4·E			392000	
	γ (kN/m^3)	uniform	no	17	21	19	1.154
Geometry	L (%)	Uniform	no	-5%	+5%	0	2.88
	t (%)			-20%	+20%	0.4-0.5	11.54
	H (%)			-10%	+10%	0	5.77
	ζ_T			0.75	1	0.875	0.072
	ζ_G			0.5	0.75	0.625	0.072
Structural details	η	discrete	no	0	1	0.5	0.5

The variables in Table 6.1 have the following meaning:

$d_{u,v}$ ultimate diagonal shear displacement;
$d_{u,f}$ ultimate flexural displacement;
α factor on modulus E to estimate damage effects;
G_k, Q_k dead and live load;
f_m, τ compressive and shear strength of masonry;
E, G longitudinal and transverse elastic modulus;
γ weight of masonry;
L, t percentage error on measure of length and wall thickness, assigned as mean value;
H percentage error on measure of interstory height assigned like mean value;
ζ_T null point moment at the last floor;
ζ_G null point moment at the generic floor;
η binary variable equal to 0 when web walls is connected to its flange, otherwise 1.

The Factor η modifies the ductility, strength and stiffness of the walls. The effectiveness of the web/flange connection was studied in a recent work (Vailati and Monti, 2010) and the ensuing equations were implemented in the procedure.

Table 6.2. Fixed parameters

Category	Variables	Value
Seismic hazard	a_g	1.357
	F_0	2.48
	T^*_C	0.27

Table 6.3 contains the three different parameters (i.e., percentage of shear and flexural collapse, period, resistant/total area ratio) considered for identifying three groups of SUs, and allows to rapidly estimate which variables have more influence on the structural response, depending on their class.

Table 6.3. Statistical quantities of PDF for period, % failure, areas ratio: in parenthesis the values for η= 1

SU	Dir	Flexure failure (%)	Shear failure (%)	Period (s)				Areas ratio (%)			
				Min	Max	Mean	Std. dev.	Min	Max	Mean	Std. dev.
1	X	43.8 (23)	56.2 (77)	0.21(0.16)	0.83 (0.57)	0.37 (0.28)	0.067 (0.047)	5.57 (5.55)	11.48 (11.26)	8.16 (8.16)	1.13 (1.13)
	Y			0.16 (0.13)	0.83 (0.46)	0.28 (0.23)	0.047 (0.038)	7.34 (7.31)	15.19 (14.84)	10.75 (10.75)	1.49 (1.43)
2	X	29.53 (7.25)	70.25 (92.75)	0.27 (0.23)	0.97 (0.77)	0.48 (0.39)	0.083 (0.066)	7.02 (6.96)	18.74 (18.52)	12.00 (11.96)	2.25 (2.23)
	Y			0.34 (0.29)	1.21 (0.96)	0.59 (0.50)	0.101 (0.084)	5.46 (5.42)	14.47 (14.41)	9.33 (9.33)	1.75 (1.74)
3	X	16.23 (1.15)	83.77 (98.85)	0.39 (0.34)	1.30 (1.18)	0.69 (0.56)	0.124 (0.100)	9.61 (9.37)	22.30 (22.20)	15.00 (15.00)	2.50 (2.50)
	Y			0.46 (0.42)	1.60 (1.46)	0.84 (0.70)	0.150 (0.124)	7.48 (7.28)	17.38 (17.27)	11.66 (11.67)	1.94 (1.94)

It is immediately observed that the web/flange connection modifies the structural behaviour by changing the stiffness. It is noted that this parameter can be understood also as an effect induced by a strengthening measure that aims at re-connecting wall corners.
In general, the procedure allows to evaluate any variable affecting the structural behaviour, both in terms of PDF and its statistical quantities (see table 6.3). Acceleration and displacement demands on each floor, or the capacity of a generic wall, as well as the modal response of the structure, are only a few of many parameters whose variability can be evaluated. In this case, according to Italian code NTC-08, the performance of a SU is assessed by means of the Capacity/Demand displacement ratio, hereafter called C/D ratio. Analysis results for the chosen parameters are given in Table 6.4.

Table 6.4. Collapse probability for each SU and floor: in parenthesis the values for η= 1

SU	Dir	Floor											
		PT			1			2			3		
		μ	Pr (%)	C/D	μ	Pr (%)	C/D	μ	Pr (%)	C/D	μ	Pr (%)	C/D
1	X	1.18 (1.74)	38.0 (8.5)	1.17 (1.74)	1.648 (3.16)	20.6 (0.5)	1.145 (3.2)						
	Y	2.67 (3.72)	0.7 (0.1)	2.68 (3.72)	3.22 (5.67)	0.2 (0.0)	3.22 (5.69)						
2	X	1.60 (2.13)	3.2 (0.7)	1.57 (2.10)	1.87 (1.874)	0.5 (0.5)	1.81 (2.67)	3.85 (5.22)	0.0 (0.0)	4.02 (5.28)			
	Y	0.93 (1.07)	65 (43.3)	0.95 (1.06)	1.56 (1.949)	2.8 (0.2)	1.61 (1.95)	3.138 (3.3)	0.0 (0.0)	3.13 (3.39)			
3	X	1.51 (1.74)	5.0 (1.2)	1.55 (1.76)	1.61 (1.987)	3.1 (0.2)	1.64 (2.02)	1.83 (2.24)	1.5 (0.1)	1.85 (2.27)	2.46 (3.32)	1.3(0.0)	2.78 (3.1)
	Y	0.90 (1.03)	69.5 (49.7)	0.92 (1.04)	1.37 (1.69)	10.6 (1.4)	1.41 (1.75)	1.37 (1.62)	12.3 (4.1)	1.42 (1.67)	2.72 (3.85)	0.2 (0.0)	2.47 (3.92)

The variables in Table 6.4 have the following meaning:

μ mean value of PDF;

Pr (%) failure probability;

C/D Capacity and Demand displacement ratio. The values given in Table 6.4 are obtained when all input variables assume the mean value.

The results show that the larger failure probability is found at the ground floor, especially in the taller buildings; at the top floors the probability decreases, until the last where the probability is almost zero.
We can observe another important effect given by web/flange connection (η= 1): in all cases the failure probability decreases, much more when the percentage of flexural failure is smaller (compare the results given in Table 6.4).
Assumably, this behaviour can depend on the fact that, in most walls, the failure mode changes from diagonal shear to flexural. This change increases the local strength and ductility, by increasing the interstory ultimate drift. Globally, the structure increases its displacement capacity, with an increment of the C/D ratio. This behaviour is more pronounced in the first SU and partially in the second. At the same time, the C/D ratio is inversely proportional to the failure probability, thus its decrease corresponds to an increase of C/D.
The next table and figures show the results of the sensitivity analyses. To simplify the presentation of results, only those of SU 1 and 3 are given. Note that the cells of column "variables" contains two variables when the web/flange connection produces a failure mode change.

Table 6.5. SU 1: Influence of input variables on global control parameter C/D: in parenthesis the values for η= 1

Floor	Direction	Variables	Category	Mean	σ	Δ C/D
PT	X	F_m	Materials	1400	300	0.248 (0.372)
		$d_{u,v}$ / **α**	General data	0.4	0.057	0.197 (0.304)
		α / $\mathbf{d_{u,v}}$	General data	0.5	0.086	0.173 (0.253)
	Y	F_m	Materials	1400	300	0.582 (0.793)
		α	General data	0.5	0.086	0.464 (0.650)
		$d_{u,v}$	General data	0.4	0.057	0.389 (0.540)
1°	X	F_m	Materials	1400	300	0.558 (0.687)
		α	General data	0.5	0.086	0.279 (0.553)
		τ / $\mathbf{d_{u,v}}$	Materials	35	-	0.168 (0.459)
	Y	F_m	Materials	1400	300	0.705 (1.205)
		α	General data	0.5	0.086	0.557 (0.991)
		$d_{u,v}$	General data	0.4	0.057	0.467 (0.823)

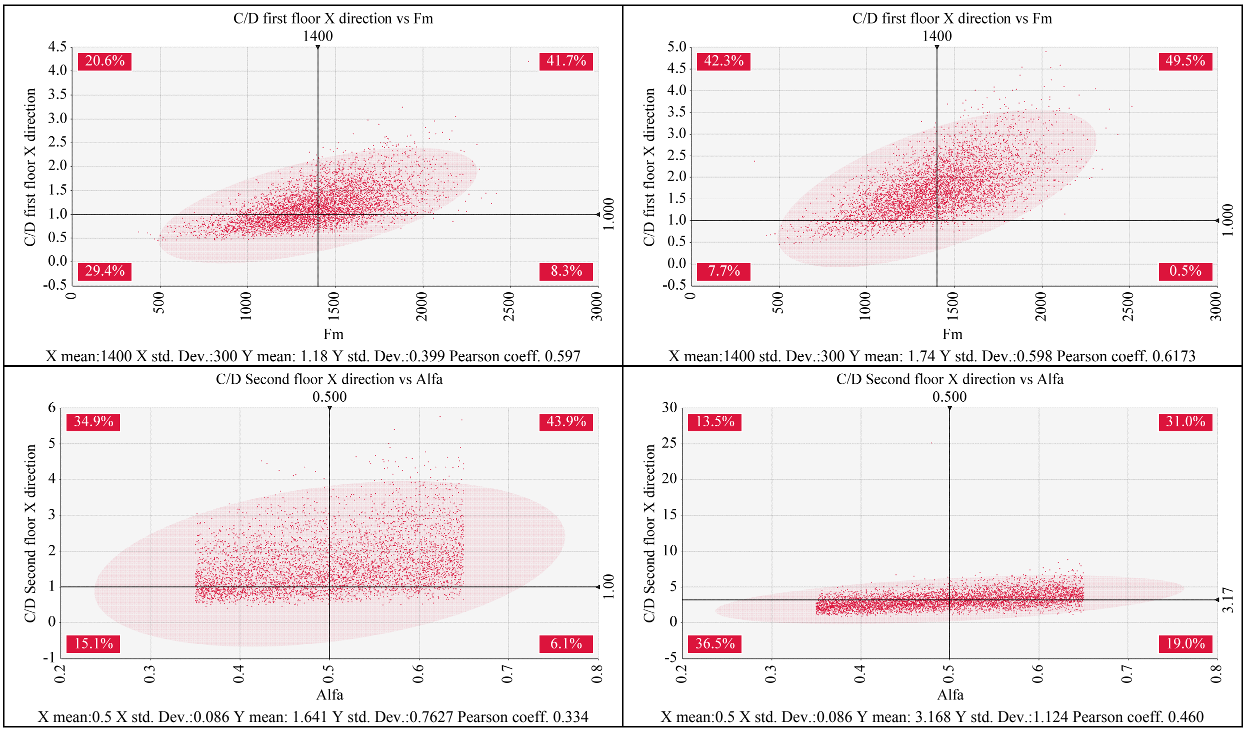

Figure 6.2. SU 1: scatter graphs of most influential variables on C/D ratio in X direction for each plane, by starting from first floor on the top line. Section I on the left (η= 0) and section T on the right (η= 1).

Table 6.6. SU 3: Influence of input variables on global control parameter C/D: in parenthesis the values for η= 1

Floor	Direction	Variables	Category	Mean	σ	Δ C/D
1°	X	$d_{u,v}$	General data	0.4	0.057	0.220 (0.252)
		F_m	Materials	1400	300	0.162 (0.187)
		α	General data	0.5	0.086	0.133 (0.162)
	Y	$d_{u,v}$	General data	0.4	0.057	0.620 (0.590)
		F_m	Materials	1400	300	0.460 (0.400)
		α	General data	0.5	0.086	0.380 (0.350)
2°	X	$d_{u,v}$	General data	0.4	0.057	0.232 (0.287)
		F_m	Materials	1400	300	0.175 (0.234)
		α	General data	0.5	0.086	0.142 (0.187)
	Y	$d_{u,v}$	General data	0.4	0.057	0.198 (0.242)
		F_m	Materials	1400	300	0.149 (0.183)
		α	General data	0.5	0.086	0.122 (0.148)
3°	X	$d_{u,v}$	General data	0.4	0.057	0.264 (0.326)
		F_m	Materials	1400	300	0.199 (0.266)
		tickness 4/3 floor	Geometry	0.4	0.057	0.164 (0.216)
	Y	$d_{u,v}$	General data	0.4	0.057	0.199 (0.235)
		F_m	Materials	1400	300	0.149 (0.176)
		tickness 4/3 floor	Geometry	0.4	0.057	0.125 (0.157)
4°	X	F_m	Materials	1400	300	0.514 (0.578)
		tickness 4/3 floor / **$d_{u,v}$**	Geometry	0.4	0.057	0.363 (0.479)
		$d_{u,v}$ / **tickness 4/3 floor**	General data	0.4	0.057	0.217 (0.458)
	Y	tickness 4,3 floor / **F_m**	Geometry	0.4	0.057	0.521 (0.609)
		$d_{u,v}$ / **thickness 4,3 floor**	General data	0.4	0.057	0.493 (0.559)
		F_m / **α**	Materials	1400	300	0.273 (0.417)

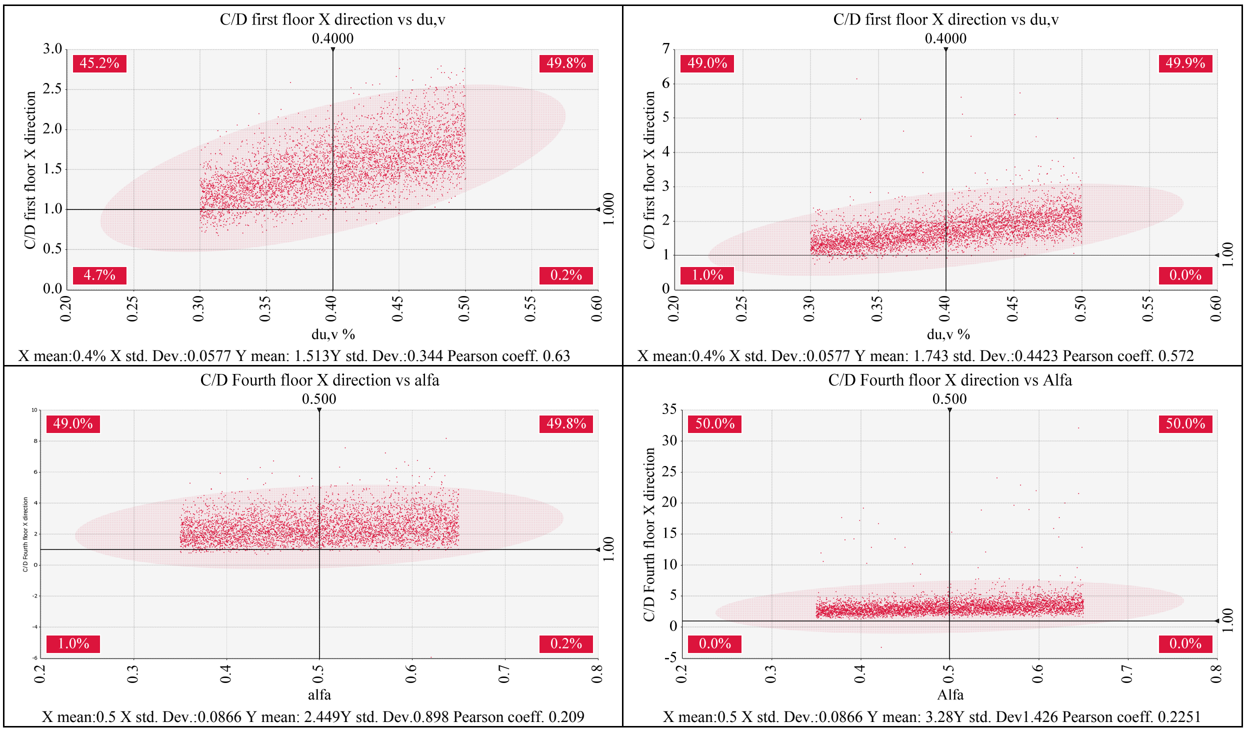

Figure 6.3. SU 1: scatter graphs of most influential variables on C/D ratio in X direction for first and fourth plane, by starting from first floor on the top line. Section I on the left (η= 0) and section T on the right (η= 1).

The four sectors of scatter graphs contain more details about the failure probability, in particular:

- the first left at the top gives the probability that the C/D ratio is larger than 1 when the sampled variable is smaller than the mean value;
- the first right at the top gives the probability that the C/D ratio is larger than 1 when the sampled variable is larger than the mean value;
- the first left at the bottom gives the probability that the C/D ratio is smaller than 1 when the sampled variable is smaller than the mean value;
- the first right at the bottom gives the probability that the C/D ratio is smaller than 1 when the sampled variable is larger than the mean value.

The results show that the main variables influencing the structural response are:

- d_{uv} ultimate diagonal shear displacement;
- f_m compressive strength of masonry;
- α factor on modulus E to estimate damage effects;

In a few cases, we observe also other variables, but with less frequence, like τ or t.
We observe that the variables ζ_T and ζ_G, respectively null point moment at the last floor and null point moment at the generic floor, have a marginal influence on the local and global response. This means that the variability of end constraints of wall have a secondary importance.

7. CONCLUSIONS

A fully probabilistic procedure has been presented for the seismic assessment of Masonry Building Clusters, which allows to compute their failure probability through a simplified nonlinear analysis, easily implemented into a worksheet. The method allows a straightforward identification of the structural elements most affecting the structural vulnerability, thus optimizing both diagnosis activities and strengthening measures.

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