Evaluation of the frequency effects on the shear wave velocity of saturated sands

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SUMMARY:

The shear wave velocity of soils is one of the main parameters in geotechnical earthquake engineering. However, the frequency range in which this parameter is obtained in laboratory does not match with the frequency content of the vibratory motion during the operation of civil structures. To study the vibration influence on the shear wave velocity, it is necessary to develop different tests, which should be carried out in a unique specimen under the same stress condition in order to avoid influence of external conditions. A device capable of performing sequentially bender element, resonant-column, non-resonance, and cyclic torsional shear tests was implemented to study this phenomenon in three types of sands. Differences between results over a range of 0.1 to 10 kHz could be explained by the influence of the frequency excitation. The shear wave velocity increases as the input frequency increases and this effect is more important in coarse saturated sands.

Keywords: Bender Element test, resonant-column test, shear modulus, vibration frequency

1. INTRODUCTION

A number of factors can affect the soil stiffness at very low strain. Comprehensive studies conducted on the subject were useful to establish the most influencing factors. A start point was done by Hardin and Black (1968) who proposed a functional relation including factor such as effective stresses and stress history, phase relationships, soil structure, grain characteristics, temperature and amplitude and frequency of vibration. From this wide range of influencing factors, Seed and Idriss (1970) selected the most important identified by the studies available at time and the vibration frequency was not considered in this selection. In a most recently classification (Barros, 1997; quoted by Santos, 1999) the frequency effect was recognized as less important in clays.

The effect of the vibration frequency on the shear wave velocity has been studied in recent years, mainly due the advent of non-resonance techniques in resonant-column devices (Lai *et al.*, 2001; Rix and Meng, 2005; Khan *et al.*, 2008b). In sands, this effect is of particular interest because the high soil permeability increases the frequency dependency of the shear wave velocity (Qiu and Fox, 2008). According to the Biot's Theory (Biot 1956a; 1956b), coarse saturated sands are most susceptible to this effect. To study in detail de frequency effect on the shear wave velocity.

There is not a unique laboratory technique to evaluate this effect over a wide frequency range. Results from tests carried out in different apparatuses cannot be compared because the soil stiffness is very sensitive to small changes in stress or strain conditions. Therefore, it is necessary to implement various kinds of tests in a single triaxial chamber and run each of them sequentially. This paper presents a new combined resonant-column, bender element and cyclic torsional equipment. Results show that the shear wave velocity increases as the input frequency increases and this effect is more important when the soil permeability is relatively high.

2. THEORETICAL BACKGROUND

2.1. Influence of the vibration frequency on the shear wave velocity

Bray et al. (2003) summarized the studies on frequency effects. They refer that pioneers studies (Hardin and Drnevich, 1972) do not exhibit evidence of frequency effects. Isenhower and Stokoe (1981) compared resonant-column against torsional shear data tests, attributing the discrepancies to the differences on frequencies involved in these tests. Additional evidence (Kramer et al., 1992; Shibuya et al., 1995; Tatsuoka et al., 1995; Lo Presti et al., 1997; Vucetic et al., 1998) suggested that the input frequency affects shear modulus and damping ratio especially on cohesive soils. Shibuya *et al.* (1995) explain that the damping ratio in cohesive soils: (i) increases at frequencies below 0.1 Hz, due to creep effects of soil skeleton; (ii) remains more or less constant between 0.1 and 10 Hz due the dominant hysteretic damping; and (iii) increases for frequencies greater than 10 Hz because the pore fluid viscosity.

Gookin et al. (1999) using a cyclic triaxial apparatus with broad frequency and strain ranges, found that the initial shear modulus increase and the damping ratio decreases as the input excitation frequency increase. The effect of the vibration frequency on the initial shear modulus has been studied in recent years, mainly due the advent of non-resonance techniques in resonant-column devices. In geomaterials, the method was firstly applied by Lai et al. (2001), who found that the shear modulus maintained approximately a same value, with a slight trend to increase with frequency.

Khan et al. (2008b) carried out a series of tests on two types of sands and on a sand-bentonite-mud mixture. Their results showed frequency effects on the last material, evidencing a trend to increase with frequency. The cause of this behaviour may be attributed to the dispersive nature of wave propagation on soils (Sachse and Pao, 1978) which implies that velocity and damping ratio are frequency dependent, as showed in preceding observations (Lai et al., 2001; Rix and Meng, 2005; Khan et al., 2008b).

2.2. Shear wave velocity in saturated sands

The influence of the vibration frequency was first studied by Biot (1956a; 1956b). This theory deals with the propagation of elastic waves in a liquid-saturated porous solid. The development of his theory starts with the relations between stresses and strains in the solid particles and the pressure and dilatation in the liquid. Biot assumed that the shear modulus is constant along the frequency while the shear wave velocity can be modified according to the frequency.

The concepts of apparent masses and dynamic coupling between water and solid were introduced by Biot to explain the slight variation of the shear wave velocity as the wave frequency increases. The frequency dependence of the shear wave velocity is explained as follows: at low vibration frequency the solid particles and the water move together due to the viscosity of the water. This region is upper bounded by the critical or characteristic frequency, f_c that is defined as the frequency in which the so called viscous skin depth of the flow matches with the radius of the pores. As the frequency increases, there is a relative motion between solid and water since the viscous forces are less important in comparison with the inertial forces. The Biot's theory is valid until frequencies with wavelengths equivalent to the diameter of the pores.

In the theoretical treatment described in detail by Biot (1956a), the force acting on the solid per unit volume and the force acting on the water per unit volume are obtained by two ways: first, by evaluation of the kinetic energy of the solid-water system. Second, these forces are expressed as stress gradients. Stoll (1977) derived a frequency equation of the form:

$$\begin{vmatrix} \rho_{\text{sat}} - \frac{G_0}{\left[V_{\text{s}}^*(\omega)\right]^2} & \rho_w \\ \rho_w & \frac{\alpha}{n} \rho_w - \frac{i \cdot g}{\omega \cdot k} \rho_w \cdot F\{a(\omega \rho_\omega / \eta)^{0.5}\} \end{vmatrix} = 0$$
(2.1)

where:

ρ_{sat}	:	saturated soil mass density
$\rho_{\rm w}$:	water mass density
G ₀	:	initial shear modulus
$V_{s}^{*}(\omega)$:	frequency dependant complex shear wave velocity
ω	:	angular frequency of the shear wave
α	:	tortuosity factor, $\alpha \ge 1$
n	:	porosity
g	:	acceleration of gravity
k	:	soil permeability
a	:	pore-size diameter
η	:	water viscosity

 $F\{a(\omega \rho_{\omega}/\eta)^{0.5}\}$ is a complex dimensionless factor that takes into account the gradual vanishment of the viscous effect when the frequency increases (Biot, 1956b), which in turn depends of a viscodynamic operator defined by Youn et al. (2008). From the last equation, the shear wave velocity can be explicitly determined (Yamamoto, 1983):

$$V_{\rm S}(\omega) = \sqrt{\frac{G_0}{\rho_{\rm sat}}} \operatorname{Re}\left\{ \left(1 + \frac{\rho_{\rm w}}{\rho_{\rm sat}} \frac{1}{\frac{{\rm i}\cdot g}{\omega \cdot k} \operatorname{F}\left\{a(\omega \, \rho_{\omega}/\eta)^{0.5}\right\} \frac{\alpha}{n}}\right)^{-0.5} \right\}$$
(2.2)

At very low frequency, V_s only depends on the shear modulus and the total mass density because the solid particles and the fluid move together:

$$V_{S0} = \lim_{\omega \to 0} (V_S) = \sqrt{\frac{G_0}{\rho_{sat}}}$$
(2.3)

On the other hand, at very high frequency, the shear wave velocity is expressed by:

$$V_{S\infty} = \lim_{\omega \to \infty} (V_S) = \sqrt{\frac{G_0}{(1-n)\rho_s + n\left(1-\frac{1}{\alpha}\right)\rho_w}} = V_{S0}\sqrt{\frac{1}{1-\frac{n}{\alpha}\rho_{wt}}}$$
(2.4)

The characteristic frequency f_c is computed by (Sheng and Zhou, 1988):

$$f_c = \frac{n \cdot g}{2\pi \cdot k \cdot \alpha}$$
(2.5)

The ratio $V_{S\infty}/V_{S0}$ for a given soil depends mainly on the tortuosity factor. For α =1 (i.e. tube-shaped pores) this ratio has the maximum value. Figure 1a shows an example of how V_S/V_{S0} varies with frequency for a given soil and different values of α (Soil properties: Gs=2.63; n=42.7%; a=0.012 mm; k=10⁻⁴ m/s. Water viscosity: η =0.001 Pa s. The figure illustrates that for particulate materials: $2<\alpha<3$ (Stoll, 1979), the shear wave velocity at high frequency is 4% to 6% greater than the shear wave velocity at very low frequency. Figure 1b shows the same curves but the frequency is normalized by their respective characteristic frequency and the vertical axis represents $(V_S-V_{S0})/(V_{S\infty}-V_{S0})$. This figure shows an additional effect of the tortuosity factor: the shear wave velocity increases faster after the characteristic frequency when α increases (Camacho-Tauta, 2011).



Figure 1. Influence of the tortuosity factor on V_s : a) $V_s/V_{s0} vs$ frequency; b) $(V_s - V_{s0})/(V_{s\infty} - V_{s0}) vs f/f_c$.

The hydraulic conductivity is another important factor that plays an important role in this theory. The more relevant effect, according with Eqn. 2.5, is in the value of the characteristic frequency. Figure 2a plots V_S/V_{S0} versus frequency for three different values of k (Soil properties: Gs=2.63; n=42.7%; a=0.012 mm; α =2). For high permeability, the shear wave velocity reaches its highest value at lower frequencies. In addition, Figure 2b reveals that the increasing rate of the shear wave velocity is lower for low permeability (Camacho-Tauta, 2011).



Figure 2. Influence of the hydraulic conductivity on V_S : a) V_S/V_{S0} vs frequency; b) $(V_S - V_{S0})/(V_{S\infty} - V_{S0})$ vs f/f_c.

Qiu and Fox (2008) proposed an "effective soil density" ρ_{eff} , as the proportional factor between the initial shear modulus (assumed to be independent of the frequency) and the shear wave velocity (frequency dependant):

$$G_0 = \rho_{\text{eff}} [V_{\text{S}}(\omega)]^2 \tag{2.6}$$

Combining Eqns. 2.2 and 2.6 yields the effective density:

$$\rho_{\text{eff}} = \rho_{\text{sat}} \cdot \mathbf{Re} \left\{ 1 + \frac{\rho_{\text{w}}}{\rho_{\text{sat}}} \frac{1}{\frac{1}{\omega \cdot k} \cdot F\{a(\omega \rho_{\omega}/\eta)^{0.5}\} \cdot \frac{\alpha}{n}} \right\}$$
(2.7)

Effective density has two extreme values: at very low frequency and at very high frequency:

$$\rho_{\text{eff}} = \rho_{\text{sat}} \quad \text{for} \quad \omega \to 0$$

$$\rho_{\text{eff}} = \rho_{\text{sat}} - \rho_{\text{w}} \frac{n}{\alpha} \quad \text{for} \quad \omega \to \infty$$
(2.8)

Qiu and Fox (2008) found that the importance of the effective soil density depends of the vibration frequency and the permeability of the soil. Moreover, they proposed that when ρ_{eff}/ρ_{sat} is greater than 95%, no correction by soil density is necessary in the computation of the shear modulus.

3. EQUIPMENT AND METHODS

A device capable of performing sequentially bender element (BE), resonant-column (RC), nonresonance (NR) and cyclic torsional shear (CTS) tests was designed and implemented in a triaxial stress system. In this device, it is possible to apply vibrations to the soil specimen ranging from 0.01 to 20 kHz. Figure 3 shows a schematic description of the apparatus with the main components.

The bender element subsystem is composed by: Bender-extender elements (B/E-E) manufactured by the University of Western Australia (UWA) (Ismail and Hourani, 2003) arranged in a T-shaped style. That is, each end platen is equipped with two bender elements orthogonally oriented encapsulated into a metallic encasement. The input signal is supplied by a function generator (Rigol, DG1022, 200 MHz). The current amplifier of the BE transmitter and the conditioner of the BE receiver were designed and manufactured at the UMNG. Input and output signals are acquired by a digital oscilloscope (Tektronix TDS210, 100 MHz)

The resonant-column subsystem is composed by four drive coils to produce the rotational vibration on the specimen. The input signal is generated by a function generator (Hameg HM8150, 12.5MHz) and the current amplified by an amplifier (STP-890, 700 W). The vibration is measured by an accelerometer (Dytran, 3055B2) and the signal amplified by a signal conditioner (Dytran, 4119B). The Cyclic torsional subsystem is composed by a servomotor (Yaskawa, Sigma-5 200W, 1.96Nm). The servomotor applies the torque and measures the resulting rotation with a precision of 10^{-5} Rad.

Axial and volumetric deformations, as well as cell, back and pore pressure, and axial force are measured by electronic sensors and all the information is acquired by a data acquisition card (NI, CompactDAQ 9174). The information is automatically processed by a software in LabView.

3.1. Non-resonance test (NR)

The resonant-column test is the most common laboratory testing method used for measuring the smallstrain dynamic properties of soils. A cylindrical soil specimen is subjected to torsional harmonic loading. The amplitude T_0 and frequency ω of the torque are controlled and a motion transducer measures the resulting rotation level θ_0 . The transfer function $H(\omega)$ of the dynamic system is represented by (Santos, 1999):

$$H(\omega) = \frac{\theta_0(\omega)}{T_0(\omega)} e^{i\phi(\omega)} = \left\{ \frac{J\omega^2}{\left(\frac{\omega h}{V_S^*}\right) \tan\left(\frac{\omega h}{V_S^*}\right)} - J_A \omega^2 + J_A \omega_A^2 (1 + i2\xi_A) \right\}^{-1}$$
(3.1)

where:

- ϕ : Phase lag between input and output signals
- J : rotational mass inertia of the soil specimen
- J_A : rotational mass inertia of the active end
- h : height of the specimen
- ξ : damping ratio of the soil
- ξ_A : damping ratio of the apparatus

ω_A : angular resonant frequency of the apparatus

The frequency dependent complex shear wave velocity $V_{S}^{*}(\omega)$ is defined by:

$$V_{\rm S}^*(\omega) = V_{\rm S}(\omega) \sqrt{\left(1 + i2\xi(\omega)\right)}$$
(3.2)

If a harmonic torque with known constant frequency and amplitude is applied to the system, and the resulting rotation and phase lag are measured, the only unknown variable is the complex shear wave velocity, since the apparatus constants are previously determined by calibration. As explained by Lai et al. (2001), the Newton-Raphson method applied for complex values of variables is useful to solve the transcendental equation. V_s and ξ are computed by Eqns. 3.3 and 3.4, respectively. The procedure can be repeated for different input frequencies in order to obtain the variation of the shear wave velocity.

$$V_{\rm S} = \mathbf{Re}\{V_{\rm S}^*\} \tag{3.3}$$

$$\xi = \frac{\mathrm{Im}\{G^*\}}{2\mathrm{Re}\{G^*\}} \tag{3.4}$$

The bar over the functions means average for multiple tests.



Figure 3. Schematic description of the combined BE-RC-CTS apparatus.

3.2. Resonant-column sine sweep test (RCSS)

The transfer function of the input towards the output isolates the inherent dynamic properties V_s and ξ , of a mechanical structure (Richardson, 1999). If experimental data supplies the transfer function (left side of Eqn. 3.1), then a numerical procedure like curve fitting of the complex data provides the shear wave velocity, which is the unknown parameter of the right side of Eqn. 3.1. The transfer function data can be obtained by means of experimental modal analysis techniques (Ramsey, 1975; Ewins, 1984). This alternative is suitable for small-strains, in which the system behaves linearly and consists in applying a sine sweep with frequency content in a bandwidth around resonance. The transfer function, between torque T(t) and rotation $\theta(t)$ signals is computed from a number of time series by:

$$H(\omega) = \frac{\mathcal{F}\{\Theta(t)\}\cdot\mathcal{F}\{T(t)\}^*}{\mathcal{F}\{T(t)\}\cdot\mathcal{F}\{T(t)\}^*} = \frac{\overline{G_{\Theta T}}}{\overline{G_{TT}}}$$
(3.5)

where:

$G_{\theta T}$:	Cross power spectrum between $\theta(t)$ and $T(t)$
G _{TT}	:	Power spectrum of T(t)
F{ }	:	Fourier transform
*	:	Complex conjugate function

3.3. Bender Element Test (BE)

The bender element makes use of a couple of small piezoceramic elements at two locations of the soil specimen for generation or detection of a shear wave test (Shirley and Hampton, 1978 and Shirley 1978). The degree of perturbation associated to the vibratory movement of the particles is appropriately small to avoid permanent deformation of the material. Therefore, it is assumed that the material behaves under elastic range, but a level of vibration sufficiently large to be detected by the sensor. A detailed description of the bender elements and their application in measuring shear wave velocity was presented by Dyvik and Madshus (1985). By analysis of the transmitted and received movements of the sensors, it is possible to derive the time of propagation of the signal tt, required to travel a distance L_{tt} . Thus, the shear wave velocity is computed by:

$$V_{\rm S} = \frac{L_{\rm tt}}{\rm tt} \tag{3.6}$$

3.4. Cyclic torsional shear (CTS)

In the cyclic torsional test, the cylindrical specimen is subjected to a torque at the upper end while the bottom end is constrained. Under this condition, the shear modulus is expressed by (Santos, 1999):

$$G = \frac{T_0 h}{I_P \theta_0}$$
(3.6)

Where I_P is the polar moment of inertia of the circular section. The shear wave velocity is:

$$V_{\rm S} = \sqrt{\frac{G}{\rho}} \tag{3.7}$$

4. RESULTS AND ANALYSIS

Three grain sizes of sands of the same origin were tested under 400 kPa of effective confinement. Table 4.1 shows the main properties of the materials. The testing programme included: BE test using single pulse sine signal with the frequency that produce the maximum output in the receiver, RCSS test at low strain and resonant frequency, NR test around resonance and CTS for 0.1 Hz. Table 4.2

shows summarizes the main results of each test. Results are compared against the Biot's model, whose parameters are listed in Table 4.1. The hydraulic conductivity can be estimated by the relationship proposed by (Chapuis, 2004):

$$k = 0.02462 \left[(D_{10})^2 \frac{e^3}{1+e} \right]^{0.7825}$$
(4.2)

where D_{10} is the effective diameter. The tortuosity factor is estimated by $\alpha = n^{-0.5}$ (Qiu and Fox, 2008). The pore-size diameter is approximately 15% of D_{10} (Stoll, 1979). A comparison of normalized result is presented in Figure 4. As can be seen in the Figure, differences between results are not completely explained by the input frequency.

Parameter	Symbol	Units	Sand 1	Sand 2	Sand 3
Properties of the solid particles					
 Mass density 	ρ_s	kg/m ³	2650	2650	2650
- Effective diameter	D ₁₀	mm	1.00	0.50	0.18
- Pore-size diameter	а	mm	0.15	0.07	0.03
Properties of water					
 Mass density 	$\rho_{\rm w}$	kg/m ³	1000	1000	1000
- Viscosity	η	Pa·s	0.001	0.001	0.001
Properties of the soil mass					
- Void ratio	е	-	0.96	0.88	0.79
- Saturated mass density	ρ_{sat}	kg/m ³	1841	1878	1922
- Permeability	k	m/s	1.33×10^{-2}	3.76×10^{-3}	$6.14 \text{x} 10^{-4}$
- Tortuosity factor	α	-	1.428	1.462	1.505

Table 4.1. Parameters of the Biot's model to the shear wave velocity of the saturated sand.

TEST	Sand 1		Sand 2		Sand 3	
	f	Vs	f	Vs	f	Vs
	[Hz]	[m/s]	[Hz]	[m/s]	[Hz]	[m/s]
BETD	18700	317	19320	300	19530	301
RCSS	185	300	181	289	175	287
NR	173	299	170	289	168	288
CTS	0.1	276	0.1	275	0.1	280



Figure 3. Schematic description of the combined BE-RC-CTS apparatus.

4. CONCLUSIONS

A new combined resonant-column/bender element/cyclic torsional shear system was developed including the basic data acquisition and signal processing for laboratory dynamic testing of soils. The system controls the equipments, acquires data and processes the information automatically for a variety of tests. Spectral analysis done with software tools instead of electronic apparatuses reduces the cost of equipment.

The automated program is a powerful tool to carry out parametric studies on the dynamic behaviour of soils. Since all information regarding the soil specimen, the stress-strain state and the test results are centralized, the chances of mistake in recording and analysis the data are highly reduced.

The effect of the frequency on the shear wave velocity was confirmed. The evidence obtained in this work showed that this effect could be more important in coarse soils.

The differences in shear wave velocities between RC, CTS and BE in tests performed on saturated sand could not be completely attributed to the effect of the frequency. Regardless of the simplicity or sophistication, there is still the human component in BE testing, which leads to subjectivity. In consequence, there is always a degree of uncertainty about the correct travel time. Thus, it requires further investigation.

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