A Stochastic Simulation Based Approach for Seismic Loss Analysis and Probability Function Computation

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SUMMARY:

Estimation of losses in a structure due to future earthquakes is essential to reduce potential losses and assist recovery. A new stochastic simulation based approach for the estimation of seismic loss probability function is proposed with the adoption of stochastic ground motion models coupled with nonlinear stochastic dynamic models. This proposed approach allows for a more comprehensive characterization of the probability distribution of the loss including the tail parts due to combinations of scenarios which can lead to extreme and catastrophic consequences. Those combinations of scenarios do not need to be pre-specified but are automatically included during the proposed simulation process. Another contribution of the proposed approach is that it allows for the simultaneous consideration of multiple types of losses and the evaluation of the exceedance probability of these losses at different combinations of thresholds. Seismic loss probability function estimation problem essentially becomes a reliability estimation problem with multiple performance objectives at different threshold combinations. The effectiveness and efficiency of the proposed method are shown by an illustrative example involving a multi-storey inelastic building.

Keywords: seismic loss estimation, failure probability, stochastic dynamics, stochastic simulation

1. INTRODUCTION

Performance-based engineering aims to quantify the performance of systems based on quantifiable and probabilistic performance objectives. Performance objectives are statements of acceptable performance of the system, defined by the performance quantities of interest attached to certain specified thresholds. The quantity of interest can take the form of conventional system response parameters (e.g., stress, deflection, drift) or their derivatives (e.g., dollar losses, downtime). Probabilistic performance objectives need to take into consideration any uncertainty that may arise because of the uncertainty in the future excitations, the imperfection or lack of accurate information in the modeling of physical problem, or a combination of these.

Moehle and Deierlein (2004) presented a framework for probabilistic seismic assessment of structures that involves four fundamental steps: hazard analysis, response analysis, damage assessment, and loss evaluation. First, the seismic hazard is characterized by adopting intensity measures that correspond to a specific annual rate of return. These intensity measures are then used to scale a suite of ground motion recordings in order to capture ground motion uncertainty. Using the generated ground motions, dynamic analyses are carried out to obtain the conditional distribution of structural response quantities. The structural response quantities are next linked to damage measures that describe the condition of the structure and its components. Finally, given a probabilistic description of damage, the process culminates with the calculations of exceedance (failure) probabilities of decision variables (direct dollar loss, casualties etc.) that can be used to make risk related decisions. Over the last few years, several studies have demonstrated the implementation of this framework (e.g., Ramirez, 2010, Aslani and Miranda, 2005, Mitrani-Reiser, 2007, Yang et al., 2009). However, since only a small suite of scaled, ground motion recordings is used to compute the structural response quantities, the more complete probabilistic information (especially the tail parts) of the response indices or performances of the structure is not obtained. The past application of the methodology focuses on the high probability re-

gions of each individual loss quantity (which may be sufficient for estimating the first and secondorder statistics such as mean and standard deviation), and falls short of accurately considering lower probability regions that characterize rarer events which can lead to significant consequences and probabilistic dependence among multiple loss quantities.

Here the proposed method performs stochastic simulation based reliability analysis where failure is defined as unsatisfactory performance of the system with multiple performance objectives expressed in terms of different types of losses. Reliability analysis by stochastic simulation based techniques has been proved to be very efficient and reliable in problems involving high stochastic dimension of interest, complex dynamic systems and rarer events (Schuëller and Pradlwarter, 2007). In the literature, a few stochastic simulation based techniques that are applicable to solving the problem of interest can be found, such as importance sampling (Au and Beck, 2003a), subset simulation (Au and Beck, 2001, Au and Beck, 2003b), auxiliary domain method (Katafygiotis and Cheung, 2007), line sampling (Pradlwarter et al., 2007), spherical subset simulation (Katafygiotis and Cheung 2007; Katafygiotis et al., 2010). These techniques require the probability distribution function (PDF) of all uncertain quantities and model parameters prior to analysis. The complete probabilistic representation of ground motion in this case is obtained by adopting stochastic ground motion models with uncertain seismic source parameters. Stochastic ground motion models provide more reliable prediction for small distance large magnitude events which contribute the most to inducing large responses in structures (Jalayer and Beck, 2008).

Au and Beck (2003b) presented the application of subset simulation for efficiently computing small failure probabilities encountered in seismic risk problems involving structural dynamic analysis. In their study, they obtained failure probability estimates for the case where the performance objectives were specified in terms of structural response parameters (e.g. story drift ratio). In this paper, failure probability estimates are obtained at various combinations of decision variable thresholds (e.g. direct dollar loss, downtime) using a new algorithm developed by the authors. The proposed method involves the modification of the simulation algorithms in the subset simulation to tackle the estimation problem of seismic loss probability function.

2. STOCHASTIC GROUND MOTION MODEL

The uncertainty in the ground motion characterization is by far the dominant source of uncertainty in calculating seismic losses. The stochastic point-source ground motion model characterized by the moment magnitude M and the distance R from source motion is selected to quantify the uncertainty in ground motion. It is defined by the deterministic radiation spectrum and stochastic noise. The deterministic radiation spectrum is calculated by the following equation

$$Acc(M, R, f) = Source(M, f)Path(R, f)Site(f)$$
(2.1)

where Acc(M, R, f) is the total radiation spectrum obtained at site; Source(M, f) is the source spectrum at unit distance; Path(R, f) is the path effect that includes the effect of both geometrical spreading and inelastic attenuation; Site(f) is the site response operator that includes the effect of both site (de)amplification and high frequency deamplification; and f is the frequency.

The stochastic aspects are treated by modeling the motion as noise with the above specified underlying spectrum. To obtain a sample ground motion record $u_g(t)$ for a given scenario event, first a discrete-time white noise sequence **Z** with unit spectral intensity for the sampling interval Δt is generated. The noise is then windowed by multiplying it by an envelope function e(t, M, R). The windowed noise is transformed into frequency domain and the spectrum is normalized by the square-root of the mean square amplitude spectrum. The resulting spectrum is multiplied by the point-source spectrum Acc(M, R, f) which is then transformed back to the time domain to yield a sample of the ground acceleration time history. The synthetic ground motion $u_g(t)$ generated from the model is thus a function of the additive excitation parameters Z, and model parameters M and R (Boore, 2003, Rezaeian and Der Kiureghian, 2010).

3. NONLINEAR STOCHASTIC DYNAMIC RELIABILITY ANALYSIS FOR LOSS ESTI-MATION

Stochastic dynamic analysis provides the mean for probabilistic assessment of seismic demand on structures subjected to uncertain excitation modeled by stochastic processes. It allows the determination of various statistics of the structural performance, such as the probability distributions of maximum responses or the first-passage probability. Given the performance function $D(\theta)$ and the corresponding threshold $C(\theta)$, the failure probability can be expressed by a multi-dimensional (reliability) integral of the form

$$P_F = \int I_F(D(\mathbf{\theta}) > C(\mathbf{\theta})) p(\mathbf{\theta}) d\mathbf{\theta}$$
(3.1)

where $\theta \in \mathbb{R}^n$ is the vector of uncertain variables of the problem; $p(\theta)$ is the joint PDF quantifying the relative plausibility of values that the set of uncertain variables θ may assume; and I_F is the indicator function: $I_F = 1$, if $D(\theta) > C(\theta)$ and otherwise $I_F = 0$.

To evaluate the performance of the system for any possible future event, different threshold levels of performance quantity of interest need to be considered and the corresponding reliabilities need to be estimated. In this study, the aim is to obtain probability distribution of the loss including the tail parts due to combinations of scenarios which can lead to extreme consequences. Loss may refer to dollar loss (repair and restoration cost), loss in functionality (downtime), or human loss (casualties) etc. Let $\mathbf{L} \in \mathbb{R}^{N_L}$ denote the loss vector in which the *i*-th component is given by the *i*-th loss quantity under consideration. To estimate the probability of any loss L_i exceeding the threshold c_i , the integral in Eqn. 3.1 can be modified to the following for the case of multiple performance objectives expressed in terms of \mathbf{L} :

$$P_{F} = \int I_{F} (\bigcup_{i=1}^{N_{L}} \{L_{i} > c_{i}\}) p(\mathbf{L}) d\mathbf{L}$$

$$= \int I_{F} (\bigcup_{i=1}^{N_{L}} \{L_{i} > c_{i}\}) p(\mathbf{L} \mid M, R, \mathbf{Z}) p(M, R) p(\mathbf{Z}) d\mathbf{L} dM dR d\mathbf{Z}$$
(3.2)

where $p(\mathbf{L}|M, R, \mathbf{Z})$ is the PDF of **L** for a specific seismic event; p(M, R) define the uncertainty in regional seismicity; and $p(\mathbf{Z})$ is the PDF of excitation parameters. It should be noted that in the above equation, in addition to accounting for ground motion uncertainty, uncertainty resulting from the size and location of the earthquake are also directly introduced. Distribution of any loss for a scenario event can be obtained by fixing *M* and *R*.

In Eqn. 3.2, $p(\mathbf{L}|M, R, \mathbf{Z})$ is equivalent to $p(\mathbf{L}|\text{EDP})$ where EDP denotes engineering demand parameter (e.g., peak interstory drift) vector obtained from the structural response analysis using the ground motion generated using stochastic ground motion model. Then, the PDF of **L** conditioned on stochastic ground motion parameters and variables is given by

$$p(\mathbf{L} \mid M, R, \mathbf{Z}) \equiv p(\mathbf{L} \mid \text{EDP}) = \sum_{\forall dm} p(\mathbf{L} \mid \text{DM} = dm) p(\text{DM} = dm \mid \text{EDP})$$
(3.3)

$$p(DM = dm \mid EDP) = p(\bigcap_{j=1}^{N_c} \{DM_j = dm_j\} \mid EDP)$$
(3.4)

$$p(L_i \mid \text{DM} = \text{dm}) = p(\sum_{i=1}^{N_c} L_{i,j} \mid \text{DM} = \text{dm})$$
(3.5)

$$p(L_i | DM = dm) = p(g(L_{i,1}, ..., L_{i,N_c}) | DM = dm)$$
 (3.6)

where $DM = \{DM_1, DM_{2_1}, ..., DM_{N_c}\}$ is a damage state random vector and $dm_j \in \{1, 2..., nds_j\}$; N_c is the total number of damageable components and nds_j is the total number of discrete damage states for component *j*. There are $nds_1 \cdot nds_2 \cdot ... \cdot nds_{N_c}$ possible realizations of DM and the PDF of total loss is given by the summation over each possible realization dm. $p(DM_j=dm_j|EDP)$ can be obtained using the fragility functions for component *j* (fragility functions are probability distributions that are used to indicate the probability that a component will be damaged to a given or more severe damage state as a function of demand parameters). $p(L_{i,j}|DM_j=dm_j)$ is the PDF of the *i*-th loss quantity for component *j* in damage state dm_j and is defined using loss functions (which is the probability of occurrence of a certain level of loss when a certain damage state has been observed in the component). Depending on the type of loss quantity, $p(L_i|DM=dm)$ can take the form in Eqn. 3.5 or a general form as in Eqn. 3.6. It can be expected that the dimension of random variables involved is high and the failure region has complicated geometry. To evaluate the integral in Eqn. 3.2, a subset simulation based approach modified for the estimation of seismic loss PDF is presented in the following section.

3.1. Proposed Subset Simulation Based Method for Loss Analysis and Loss Probability Function

The basic idea of subset simulation is to consider a sequence of *m* failure events (one being the subset of another) $F_1 \supset F_2 \supset ... \supset F_m = F$ converting a rare-event problem into a problem with a sequence of more frequent events that are conditioned on failing successively increasing threshold levels. This enables the computation of the failure probability as a product of conditional probabilities $P(F_{i+1}|F_i)$ and $P(F_1)$ as

$$P_{F} = P(F_{m}) = P(F_{m} | F_{m-1})P(F_{m-1} | F_{m-2}) \cdots P(F_{2} | F_{1})P(F_{1})$$
(3.7)

$$P(F_1) \approx \frac{1}{N} \sum_{k=1}^{N} I_{F_1}(\boldsymbol{\theta}_k)$$
(3.8)

$$P(F_{i+1} \mid F_i) \approx \frac{1}{N} \sum_{k=1}^{N} I_{F_{i+1}}(\boldsymbol{\theta}_{i,k}) \qquad F_i = \left\{ \boldsymbol{\theta} : D(\boldsymbol{\theta}) > c_i \right\}$$
(3.9)

where $P(F_1)$ is estimated by simulating samples by Monte Carlo Simulation (MCS) and $P(F_{i+1}|F_i)$ is estimated using samples distributed as the conditional PDF $p(\cdot|F_i)$. Samples satisfying the conditional PDF $p(\cdot|F_i)$ are generated by a Markov Chain Monte Carlo simulation technique based on the modified Metropolis–Hastings (MH) method using samples distributed according to $p(\cdot|F_i)$ obtained from the previous simulation level. The MH method consists of the following steps (Au and Beck, 2001):

1. Given a current state $\mathbf{\theta}_k = \{\mathbf{\theta}_k^j : j = 1, ..., n\}$ distributed as $p(\cdot | F_i)$, generate a candidate state $\tilde{\mathbf{\theta}}_{k+1}$

For the *j*-th group random vector θ_k^j , a pre-candidate component ξ_{k+1}^j from the proposal distribution $q_j^*(\cdot \mid \theta_k^j)$ and compute the acceptance ratio

$$r_{k+1}^{j} = \frac{p^{j}(\xi_{k+1}^{j})q_{j}^{*}(\theta_{k}^{j} \mid \xi_{k+1}^{j})}{p^{j}(\theta_{k}^{j})q_{j}^{*}(\xi_{k+1}^{j} \mid \theta_{k}^{j})}$$

Set $\tilde{\theta}_{k+1}^{j} = \begin{cases} \xi_{k+1}^{j} & \text{with probability } \min(1, r_{k+1}^{j}) \\ \theta_{k}^{j} & \text{with probability } 1 - \min(1, r_{k+1}^{j}) \end{cases}$

2. Accept $\boldsymbol{\theta}_{k+1} = \tilde{\boldsymbol{\theta}}_{k+1}$ if $\tilde{\boldsymbol{\theta}}_{k+1} \in F_i$, else set $\boldsymbol{\theta}_{k+1} = \boldsymbol{\theta}_k$

The intermediate thresholds b_n : n = 1,..., m-1 are chosen "adaptively" so that the conditional probabilities are approximately equal to a some specified value, p_0 .

For applying the subset simulation, a proposal PDF q^* is required for each random variable in F_i to generate Markov chain samples following $q(\cdot|F_i)$ using the modified MH method. q^* affects the distribution of the candidate state given the current state (transition PDF) and consequently the efficiency of the MH algorithm. In the original subset simulation approach, the transition of individual random vector are considered independent, so the transition PDF of the Markov chain between two states in F_i can be expressed as a product of the individual transition PDFs. However, not all the random variables in Eqn. 3.2 are independent, loss vector **L** is a probabilistic vector-valued function of other independent random variables $[M, R, \mathbf{Z}]$. But it can be shown that if in F_i , $[M_{k+1}, R_{k+1}, \mathbf{Z}_{k+1}]$ are simulated according to step 1 in the modified MH algorithm given above and \mathbf{L}_{k+1} is simulated according to the PDF $p(L_{k+1} | M_{k+1}, R_{k+1}, \mathbf{Z}_{k+1})$ given in Eqn. 3.2 and then step 2 in the modified MH algorithm is carried out, then the resulting sample will also be distributed as $p(\cdot|F_i)$. Due to space limitations, the corresponding proof is presented here.

The output of subset simulation analysis is the exceedance probability of loss at various threshold levels. Additional uncertainties arising from structural modeling can be easily incorporated in the proposed stochastic simulation based methods without necessarily increasing the computational effort. The effectiveness and efficiency of the proposed method are shown by an illustrative example in the following section. An efficient approach has been developed by the authors to calculate the exceedance probability of loss vectors at various threshold combinations. Due to space limitations, this will be presented in the journal version of this paper.

4. ILLUSTRATIVE EXAMPLE

The example used in the study is the hotel structure located in Van Nuys. This building is a seven story reinforced concrete (RC) structure that was severely damaged during the 1994 Northridge Earthquake. In this study, it was assumed that the building is in its original condition prior to the occurrence of the Northridge earthquake. Simplified structural model as developed by Ching, et al. (2004) is used for structural dynamic analysis. Economic loss (building repair cost) and downtime (building repair time) of earthquake induced damage are considered as risk related decision variables.

4.1. Stochastic ground motion model

The stochastic ground motion records are generated by adopting Atkinson and Silva (2000) stochastic ground motion model developed for California seismicity, modified for soil site. The model belongs to the class of point-source models characterized by the moment magnitude M and the epicentral distance r. The total point-source spectrum observed at any site is given by

$$A(f) = A_0(f)V(f)G(f)\exp(-\gamma(f)R)\exp(-\pi f\kappa)$$
(4.1)

where A_0 is the 'equivalent point-source spectrum' based on two corner frequencies defined at a unit distance described by

$$A_{0}(f) = C(2\pi f)^{2} M_{0} \left\{ (1-\varepsilon) / [1+(f/f_{a})^{2}] + \varepsilon / [1+(f/f_{b})^{2}] \right\}$$
(4.2)

The constant C is given by $C = \Re V F_s / (4\pi\rho\beta^3)$, where \Re is the radiation pattern, V is the partition onto two horizontal components, F_s is the free surface amplification, and ρ and β are the density and shearwave velocity, respectively, in the vicinity of the source; M_0 is the seismic moment; f_a and f_b , the lower and higher corner frequencies are related to the size of the finite fault and subfault size, and ε is a relative weighing parameter. In Eqn. 4.1, V describes the amplification factor for the crustal velocity gradient, *G* is the geometric spreading factor, γ is the anelastic attenuation factor, κ is the near-surface attenuation factor, and $R = \sqrt{(r^2 + h^2)}$, *h* is the 'equivalent point-source depth'. Non stationarity in the ground motion amplitude is achieved by using an empirical window function *e*(*t*) (Boore, 2003)

$$e(t) = a(t/t_{\eta})^{b} \exp(-ct/t_{\eta})U(t)$$
(4.3)

where U(t) is the unit step function and parameters *a*, *b*, and *c* are determined such that e(t) has a peak with value of unity when $t=\varepsilon \times t_{\eta}$ and $e(t)=\eta$ when $t=t_{\eta}$. The time t_{η} is given by $t_{\eta}=2T_w$ where T_w is the duration of the ground motion, expressed as a sum of a path dependent component and a source dependent component. The distribution of earthquake sizes is modeled by bounded Gutenberg-Richter recurrence law (Kramer, 1996) and uncertainty in epicentral distance is described by a triangular distribution given by

$$p(M) = \frac{\beta \exp(-\beta M)}{\exp(-\beta M_{\min}) - \exp(-\beta M_{\max})}$$
(4.4)

$$p(r) = 2r / r_{\text{max}}^2 \quad r \in [0, r_{\text{max}}]$$
(4.5)

where β =2.303b and b=1 that describes the relative size distribution of seismic. It is assumed that $[M_{\min}, M_{\max}] = [5, 8]$ and $r_{\max} = 50$ km.

4.2. Fragility and Loss Functions

Loss due to earthquake induced damage to drift sensitive structural and nonstructural components is considered here as the performance function. The mapping between EDP and economic loss adopted here follows the ABV framework proposed by Porter et al. (2001). Peak interstory drift ratio (IDR) at each story is used as the EDP. For illustration and convenience, it is assumed in this example that (a) once an individual structural component has experienced extensive damage (loss of vertical carrying capacity), a local or global collapse mode has occurred in the structure that triggers the need to replace the building, (b) the damage experienced by different components for a given EDP vector are statically independent. Component specific fragility functions are defined using the lognormal distribution and their statistical parameters, logarithmic mean θ and logarithmic standard deviation β , were taken directly from the study conducted by Aslani & Miranda (2005). Given the fragility functions, the probability of component *j* being in a damage state *k* (i.e. $DM_j = k$) is given by

$$P(DM_{j} = k \mid \text{EDP}) = \Phi\left(\frac{\ln(\text{EDP}) - \theta_{j,k}}{\beta_{j,k}}\right) - \Phi\left(\frac{\ln(\text{EDP}) - \theta_{j,k+1}}{\beta_{j,k+1}}\right)$$
(4.6)

where Φ is standard normal cumulative distribution function (CDF). The direct economic loss i.e. the repair cost (*L=EL*) is calculated by summing the economic loss for all components conditioned on no local/global collapse and is equal to the replacement cost of the building in case of local/global collapse. Economic loss functions follow a lognormal distribution with logarithmic mean μ and logarithmic standard deviation σ (Aslani and Miranda, 2005) and the CDF when component *j* is in damage state *k* is given by

$$P(EL_{j} \le l \mid DM_{j} = k) = \Phi\left(\frac{\ln(l) - \mu_{j,k}}{\sigma_{j,k}}\right)$$

$$(4.7)$$

Samples of *EL* distributed according to $p(EL | M, R, \mathbb{Z})$ can be generated by MCS by first simulating a random damage state for each component given the EDP using Eqn. 4.6, and then given the damage state, random economic loss sample for each component is generated using Eqn. 4.7. Finally the sum

of economic losses in individual components gives a sample of *EL* distributed according to $p(EL | M, R, \mathbb{Z})$.

Downtime is defined here as the period of time between the occurrence of seismic event and the completion of the building repair efforts. There are various factors that affect building downtime: building inspection, damage assessment, financial planning, resource mobilization, repair duration etc. The repair time data associated with individual components are obtained from Mitrani-Reiser (2007). It is assumed that they also follow a lognormal distribution and their CDF is given by an equation similar to Eqn. 4.6. Repair time required for every floor is calculated by summing downtime for all damaged components in that floor and the total repair time for the building is assumed equal to the maximum of repair time required for every floor. The repair time data used here are for a different structure but with similar types of components and are adopted here only for the purpose of illustration.

4.3. Results

Subset Simulation is applied with a conditional failure probability at each level equal to $p_0 = 0.1$ and with the number of samples set to N = 500 at each conditional level. Levels 1, 2 and 3 correspond to the failure probability of 0.1, 0.01 and 0.001, respectively. One-dimensional chain adaptive symmetric uniform distribution is adopted as proposal for each addictive excitation random variable and level adaptive bivariate Gaussian distribution with mean and covariance matrix estimated from samples from the most recent simulation level is adopted as the proposal for M and r. Fig. 4.1 shows the estimates of exceedance probability for different threshold levels of economic loss normalized by the replacement cost of the building. Mean exceedance probability obtained using 50 independent simulation runs and the result for each simulation method. It can be seen that the probability of no damage and local/global collapse from Fig. 4.1 is 0.18 and 0.001, respectively. Fig. 4.2 shows the scattering of M, r samples shift towards the large magnitude and small distance region. The samples shown in solid are the combination of M, r that leads to local/global collapse from the last simulation level.



Figure 1. Exceedance probability at various loss thresholds



Figure 2. Conditional *M*, *r* samples at conditional levels 0,1,2,3



Figure 3. Exceedance probabilities for different loss forms

Typically, exceedance probabilities for individual loss quantities are obtained independently. Exceedance probabilities as a function of multiple thresholds can yield valuable insights into how all the different loss quantities come together to affect the seismic risk and system performance as a whole. It is thus appealing to obtain the exceedance probability estimates for different combinations of loss threshold. Fig. 4.3 shows the exceedance probabilities if either of the two thresholds i.e. economic loss or downtime is crossed. A very efficient algorithm has been developed to obtain this result. Due to space limitations, the details are not presented here but will be presented in the journal version of this paper. As one of the performance threshold takes the extreme value (=1), the failure probability is the same as the one that depends on the other performance parameter. To further illustrate the type of information which can be obtained using the proposed method, the contribution of each story to the total loss is shown in Fig 4.4 for each failure probability level.



Figure 4. Contribution to total loss from each story (conditioned on no collapse)

5. CONCLUSIONS

A new stochastic simulation based approach for the estimation of seismic loss probability function is proposed with the adoption of stochastic ground motion. Exceedance probabilities at various loss thresholds are obtained using the modified subset simulation that efficiently computes small failure probabilities. The approach is robust to the number of random variables and is directly applicable even when considering both seismic uncertainty and modeling uncertainty. An example was presented to show the applicability, effectiveness and efficiency of the proposed approach.

The performance of the proposed method depends on (a) the stochastic ground motion model to provide realistic description of the characteristic of the ground motions expected to happen at the specific site; (b) modeling of the physical system and collapse; (c) the quality of fragility functions and loss functions for structural and nonstructural components.

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