Contributions to the Chilean Code for Seismic Design of Buildings with Energy Dissipation Devices

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SUMMARY:

A code for seismic design of buildings with energy dissipation devices is being developed in Chile. The studies carried out to validate some code dispositions are presented here. First, an evaluation of the damping modification factor as function of damping and natural period of the building is performed. This parameter reduces the design spectrum of pseudo acceleration based on the responses for 5% damping. Next, an evaluation of approximate responses of inelastic single-degree-of-freedom systems with energy dissipation devices is carried out. The approximate method of analysis is validated by comparing to exact non-linear dynamic analysis responses. This is done for different structural characteristics as well as several damper types. All of them use the Chilean earthquake records data-base.

Keywords: Energy dissipation, damping modification factor, approximate non-linear methods, Chilean code.

1. INTRODUCTION

Several buildings have been designed in Chile which includes energy dissipation devices. Therefore, it is urgent to develop a special code for regulating the engineering practice in this matter. The studies that are presented here aim to help the work of the Committee that is adapting the ASCE-7 code to Chilean conditions, including the corresponding seismic risk and the liaison with other national codes. One important fact is the calculation of the damping modification factor that corrects the displacement response of a SDOF system from 5% damping to a given value of damping. The value of this parameter has to be evaluated for Chilean earthquakes, which differ from North American ones.

Another important issue is the use of approximate methods of analysis in the design process. One of these methods models the structure by an equivalent SDOF system with viscous damping, following the works of, for example, Ramirez et al (2002a). Using the concept of the Capacity Spectrum Method and a SDOF bi-linear representation of the structure, several cases of damper characteristics are evaluated (viscous linear, viscous non-linear, and hysteretic). Sets of Chilean earthquakes selected from the complete data set for different soil conditions are scaled to fit to design spectra and the approximate responses are compared to the exact integration responses.

Conclusions are obtained about the dependence of the damping modification factor and the errors committed by using the approximate method of analysis.

2. DATA BASE OF CHILEAN EARTHQUAKE RECORDS AND SCALING PROCEDURE

2.1. Data base of Chilean earthquakes records

In order to develop local code dispositions, it is necessary to calibrate the parameters used in the code with results obtained from analyses using the Chilean earthquakes data base. The earthquake considered for the data base of acceleration records are described in Table 2.1. The records were classified considering the soil type at the different sites, according to Chilean code NCh433of1996 (2009). Information about soil conditions at the sites was taken from Riddell et al. (1993) and Arango et al. (2010). Whenever a contradiction was observed between these two sources, or when there was no enough information, the classification was done according to the similitude of the response spectrum to the elastic design spectrum of the Chilean code NCh2745 (2003) for base isolated buildings.

Using this procedure, 28 records were classified as soil type I (hard soil), with PGA ranging from 87.6 cm/s^2 to 581 cm/s^2, 76 records were considered soil type II (intermediate soil), with PGA ranging from 84.3 cm/s^2 to 696 cm/s^2, and 26 records were in soil type III (soft soil), with PGA from 78.5 cm/s^2 till 614 cm/s^2. The upper period cut off for the records is 3 s.

Table 2.1. Earthquakes considered for the data base.

Earthquake[1]	Date	M_w	Type	Epicenter		Depth [km]
				Latitude [°S]	Longitude [°W]	
Valparaíso	03-03-1985	8.0	Inter-plate	33.240	71.850	33.0
Punitaqui	15-10-1997	7.1	Intra-plate	30.773	71.315	56.0
Ocoña	23-06-2001	8.4	Inter-plate	17.280	72.710	29.6
Tarapacá	13-06-2005	7.8	Intra-plate	19.896	69.125	108.0
Tocopilla	14-11-2007	7.7	Inter-plate	22.314	70.078	47.7
Maule	27-02-2010	8.8	Inter-plate	36.290	73.239	30.0

[1]Information obtained from Seismological Institute of the University of Chile, except Ocoña's earthquake, where the CMT catalog was taken as reference.

2.2. Selection and scaling procedure

The scaling procedure described by Kottke and Rathje (2008) was used to select the records. Therefore, the root mean square error in the logarithm space was calculated, for all possible combinations of 7 acceleration records for each soil type, respect to the elastic design spectrum of the Chilean code NCh2745 (2003). One hundred periods of vibrations equally spaced between 0.03 and 4 s were used in this process. The sets of records with least mean square errors, which ensured some variability with regard to earthquake type and site location was finally selected. The individual scaling factors were calculated using the Centroid Method with a target standard deviation equal to zero, because the design spectrum was obtained through a probabilistic procedure and, therefore, already incorporates some level of standard deviation.

The resulting response spectra of the scaled records are summarized in Figure 2.1, for soils type I, II, and III.

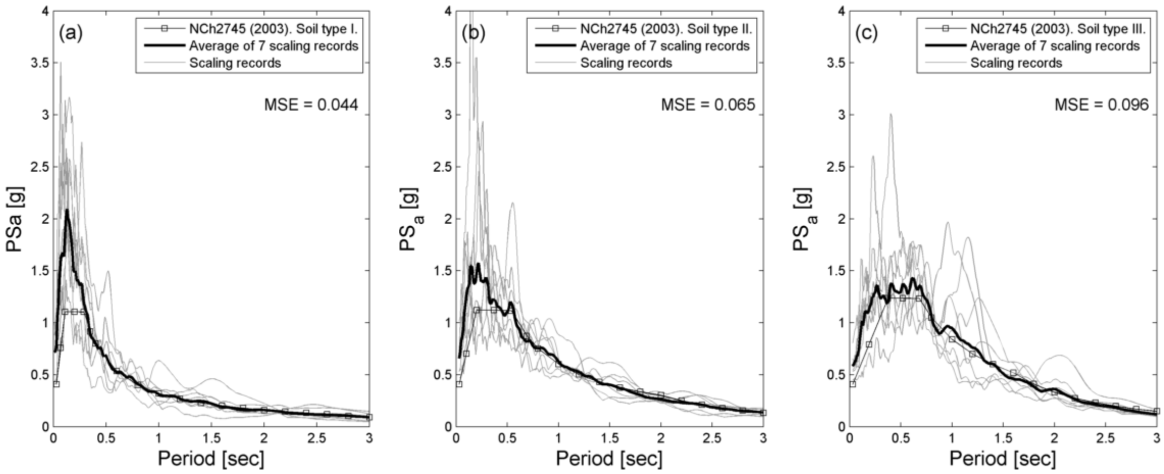

Figure 2.1. Average response spectra of the scaled records for soil type: (a) I, (b) II, and (c) III.

3. DAMPING MODIFICATION FACTOR

Actual seismic design codes for building use as basic design spectrum an elastic pseudo acceleration spectrum for 5% critical damping. However, for the analysis and design of structures with energy dissipation devices it is necessary to build a design spectrum for damping larger than 5%. The classical approach for modifying the spectrum for larger amount of damping is to consider a *damping modification factor* (DMF) of the pseudo-acceleration spectrum, B_d, such that $PS_a(\beta, T) = B_d PS_a(5\%, T)$. Note that B_d is also valid for the pseudo-relative velocity and the displacement spectra, but it differs for the relative velocity, B_v, and the absolute acceleration, B_a, spectra.

The damping modification factor, DMF, strongly depends on the level of damping of the system and the period of vibration, although it has been found that it also depends on the duration of motion, distance to the fault, magnitude, tectonic characteristics, type of soil, and directivity near the source.

Several authors have studied this parameter. Noteworthy is the work by Newmark and Hall (1982) that was used to determine the coefficients incorporated in the UBC 1997, NEHRP 1997, and IBC 2000 codes for the design of structures with seismic isolation and/or energy dissipation systems. They were also included in ATC-40, 1996, FEMA 273, 1997, and FEMA 356, 2000. The values proposed by Ramirez et al. (2002b) were adopted in NEHRP 2000, 2003, and 2009 and in ASCE-7, 2005, 2010.

Lin and Chang (2003) using a data base of 1053 accelerographs of 102 North American seismic events, obtained the expression for B_d (Eqn. 3.1) for damping values between 2 and 50%, valid for periods between 0.1 and 10 s.

$$B_d = 1 - \frac{aT^{0.30}}{(T+1)^{0.65}}, \quad a = 1.303 + 0.436 \ln \beta \qquad (3.1)$$

Other authors, like Bommer and Mendis (2005), included in their studies additional variables: Magnitude of the earthquake, hypocentral distance, type of soil, directivity effect and duration of motion. They concluded that the value of B_d is lower when the magnitude of the earthquake is larger. The same is true for the distance to the source. With respect to the type of soil, the value of

DMF was found lower for medium to soft soils than for rock. As to directivity at close-to-source events, it was found that the velocity pulses reduce the effectiveness of damping.

Cameron and Green (2007) studied the same variables than Bommer and Mendis but using a larger data base. Their conclusions ratify the previous results. However, the authors explain the observations considering the frequency content of the records instead of their duration. The conclusion was that B_d is lower when the period of vibration is near the periods with more energy in the records. Accordingly, the DMF depends on the geotechnical conditions. The soil layers filter the high-frequencies and amplify the frequencies near the dominant period of the soil. It also depends on the magnitude because larger earthquakes imply larger energy in the longer period zone

3.1. DMF for different spectral quantities

In today practice the seismic analysis and design of structures is based on the pseudo values of the relative velocity and absolute acceleration. This is correct for small damping but is not true for larger values, because the pseudo and the real values differ considerably, especially for long periods, when the differences are substantial. This is important when additional damping is provided by viscous dampers that respond to relative velocity, and when the non-structural components are going to be protected against damage related to the absolute acceleration.

The differences between B_v and B_d and between B_a and B_d can be appreciated in Figure 3.1 that shows the average results for the earthquakes of the data base described in section 2. Note that B_v does not differ much from B_d for a range of periods (0.2 to 1.0 s), but the difference can be substantial for lower and longer periods, especially for large amounts of damping. In the case of B_a, the difference respect to B_d is small for low amounts of damping, but it is substantial when damping is larger than 20%, especially for long periods (>1.0 s). For 50% damping and periods longer than 2.5 s, B_a becomes larger than 1.0.

Given that the simplified method of analysis that will be evaluated later is based on the non-linear static method of ATC-40 (1996) and FEMA 274 (1997) and that this method considers the equality between the displacement of an equivalent viscous-elastic system and the real system, the relevant DMF is B_d. For other situations B_v and B_a can also be important and, therefore, they were evaluated using the complete set of Chilean earthquakes.

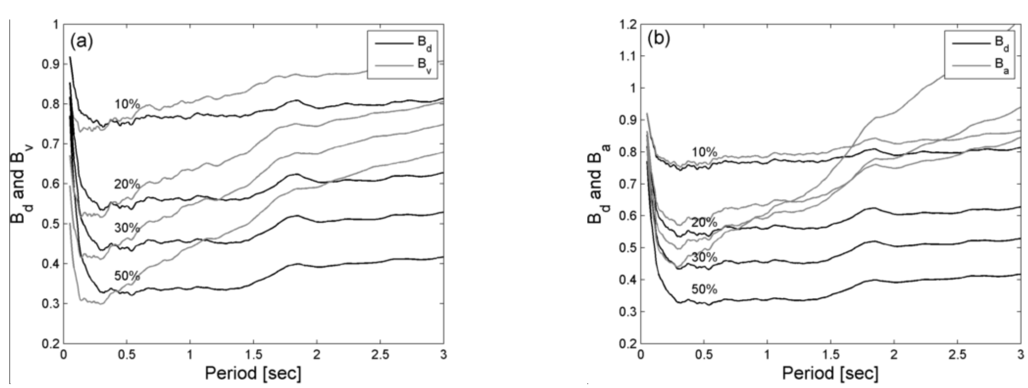

Figure 3.1. Differences between: (a) B_v and B_d and (b) B_a and B_d.

3.2 Influence of the types of earthquake and soil

Earthquakes in Chile are governed by the subduction of the Nazca Plate under the South American Plate. This mechanism produces mainly two kind of dangerous earthquakes: *inter-plate* earthquakes –by friction between plates– and *intra-plate* earthquakes –originated by fracture of the Nazca Plate that is subducting–. There are also other earthquakes of less magnitude produced by motion of superficial faults that are less frequent and with very localized damage and will not be considered here. Inter-plate and intra-plate earthquakes, because of their different source mechanisms, have different characteristics of frequency content, duration, rupture length and tension fall. The most important characteristic is the frequency content because intra-plate earthquake have larger energy at higher frequencies than inter-plate ones, and frequency content, as it has been said, is an important factor on the value of B_d.

Because frequency content depends also on the soil type, the set of earthquake records was first divided in two groups, inter-plate and intra-plate, and then each group was separated by soil types I and II, according to Chilean code NCh433of 1996. The results for B_d logarithmic average values are plotted in Figure 3.2. The conclusion is that the DMF is smaller for very short periods and larger for long periods in the intra-plate case than in the inter-plate's. This effect, although more relevant in soils type I, it is also true for soil type II.

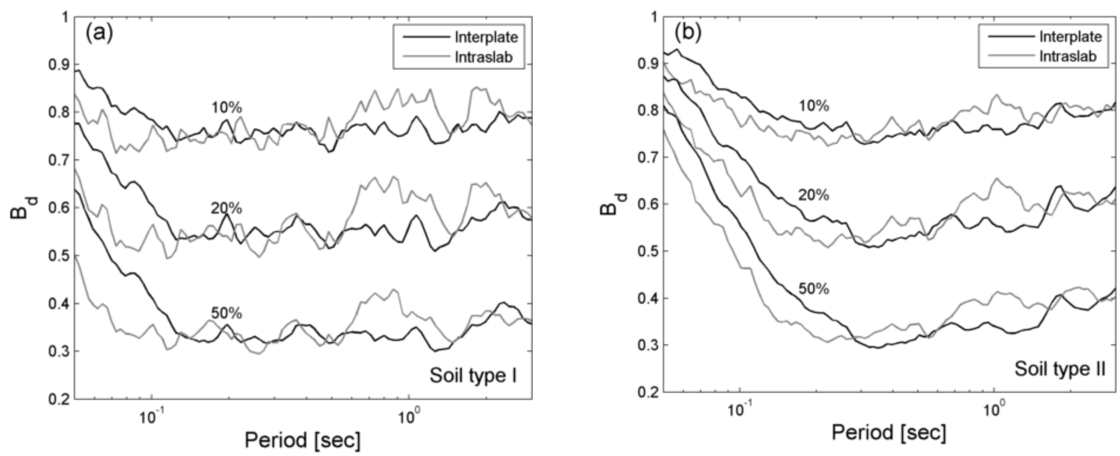

Figure 3.2. DMF for inter-plate and intra-plate earthquakes in: (a) soil type I and (b) soil type II.

It is also clear that B_d tends to 1.0 –as the period goes to zero– faster for softer soils. This is because of, as it was mentioned before, the lower frequency content of the records for softer soils. For large periods the differences between this two types of soils is not relevant.

3.3. Influence of duration of motion

The dependence of B_d on duration of motion has been studied by Bommer and Mendis (2005) and Stafford et al. (2008). Both of them found large differences between the values of B_d for motions very short and very long. However, Cameron and Green (2007) found that the duration is important only for very low damping, near 1%.

The effect of duration is studied here considering the duration of motion as the time elapsed between 5 and 95% of the Arias intensity ($D_{5\text{-}95\%}$). The results obtained for the complete data base of Chilean earthquakes are plotted in Figure 3.3 in terms of their mean values (points) and standard deviations (error bars). The conclusion is that for increasing damping the effect of duration is less relevant, coinciding with what it has been observed by other researchers. The variation is, in general, few significant for the range of durations analyzed. It is only important for very low damping ratios, of the order of 1%.

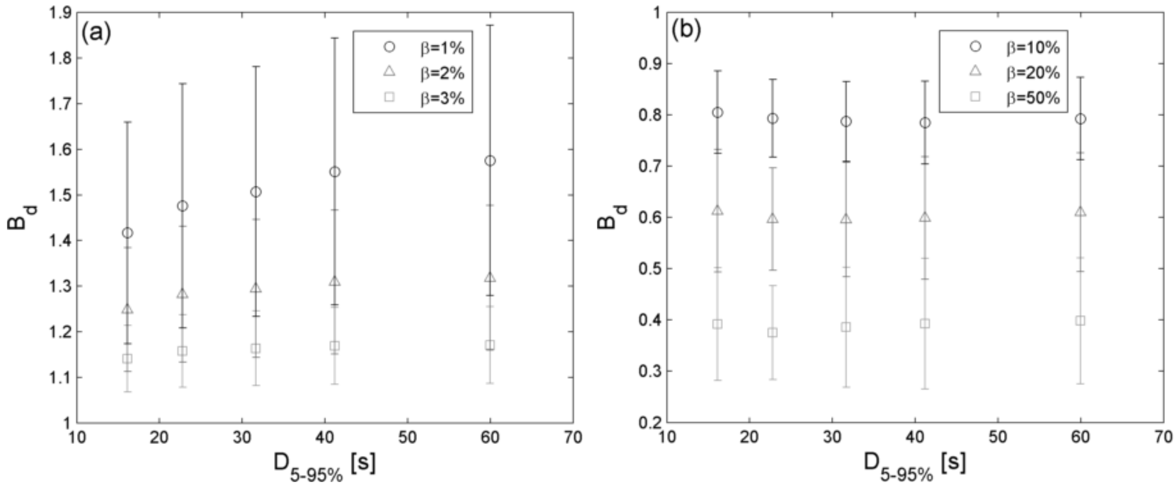

Figure 3.3. DMF as function of the significant duration of motion in (a) β<5% and (b) β>5%.

3.4. Equation for the DMF

Based on the discussion already presented in previous section, Eqn. 3.2 is proposed to calculate the DMF. It was obtained by non-linear regression analysis of average values. This equation is similar to the one proposed by Lin and Chang (2003) but considers a squared logarithm and it imposes the condition $B_d(5\%, T) = 1$. Figure 3.4 shows how the formula fits to the average values.

$$B_d = 1 - f(\beta)\frac{T^{8.76}}{(T+0.01)^{8.94}}\ ,\ f(\beta) = -0.031\ln^2\left(\frac{\beta}{0.05}\right) + 0.386\ln\left(\frac{\beta}{0.05}\right) \tag{3.2}$$

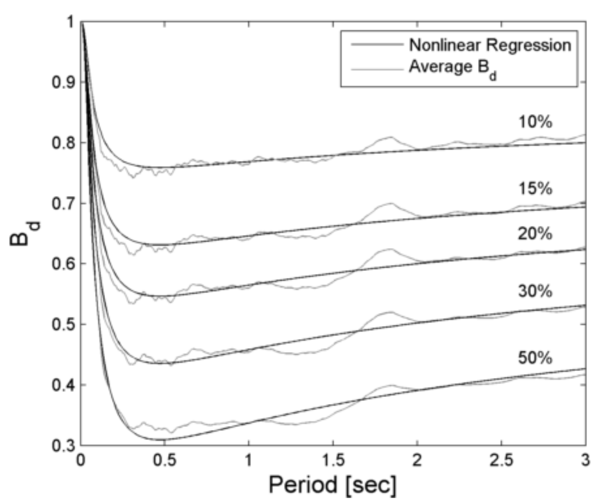

Figure 3.4. Proposed DMF for different damping ratios.

4. EVALUATION OF SIMPLIFIED METHOD OF ANALYSIS OF INELASTIC SDOF SYSTEMS WITH ENERGY DISSIPATION DEVICES

Seismic provisions for design of buildings equipped with energy dissipation devices, like ASCE-7 (2010), include simplified methods of analysis of inelastic buildings with viscous linear, viscous nonlinear, viscous-elastic and hysteretic. Evaluation of these procedures for SDOF inelastic systems with linear viscous dampers was previously done by Tsopelas et al. (1997) and afterwards was extended by Ramirez et al. (2002a) to systems with viscous non-linear and hysteretic dampers. The evaluation of these types of systems gives valuable information respect to the accuracy of the procedures that are based on the Capacity Spectrum Method (CSM) used by ATC-40 and FEMA 274, that consider the response of the fundamental mode of vibration of the structure as the response of a SDOF inelastic system that in turn is approximated by an equivalent viscous elastic SDOF system with a secant period of vibration and an equivalent viscous damping.

The application of the CSM method requires getting the intersection of the demand and capacity curves for obtaining the maximum relative displacement. The demand curve is in this case the elastic design spectrum of the NCh2745 Chilean design code for base isolated structures, reduced by the corresponding B_d value (Eqn. 3.2). Amount of damping in the equivalent system is a function of the structure's displacement, therefore the procedure is iterative. Moreover, the capacity curve is defined by the parameters used in the evaluation, such as the elastic period of vibration, the strength reduction factor, and the post-yielding stiffness ratio. In the case of hysteretic dampers the stiffness, strength, and post-yielding stiffness ratio of the damping system are also important in setting the capacity curve.

The approximate response is compared with the exact non-linear response for each case that is being evaluated. The mathematical model used for bi-lineal hysteretic structure and viscous linear, viscous nonlinear, and hysteretic dampers are the ones proposed by Tsopelas et al. (1997) and Ramirez et al. (2002a). Numerical integration of the motion equations was done by the Dormand-Prince procedure implemented in routine ODE45 of MATLAB. The dynamic systems were subjected to three sets of records scaled as to fit the demand spectra of the three types of soil considered. Inherent damping (β_i) was taken as 5% for all cases.

4.1 Parameters that define the seismic response

The structural system is defined here by the elastic period of vibration (T_e), the post-yielding stiffness ratio (r_s) and the strength reduction factor (R_μ), defined as the ratio between the spectral acceleration for the inherent damping and the elastic period of vibration, and the absolute yield acceleration (equal to the yield force divided by the mass). The viscous dampers are characterized by the critical damping (β_v), given by Eqn. 4.1, where C is the dashpot constant and m is the mass. The non-linear viscous dampers are defined by the velocity exponent (α) and the added equivalent critical damping (β_{vn}), given by Eqns. 4.2 to 4.4, where C_{nl} is the non-linear proportionality constant, and D_e is the maximum relative displacement under elastic conditions. Finally, the hysteretic dampers are characterized by the post-yielding stiffness ratio (r_d), the ratio between the total elastic stiffness of the system and the elastic stiffness of the structure (r_K), and the ratio between the yield force of the damper and the yielding force of the structure (r_F). The values of the parameters considered in this study are shown in Table 4.1. For the case of hysteretic dampers the period 0.3s was not considered and a period of 2.5s was added.

$$\beta_v = \frac{CT_e}{4\pi m} \tag{4.1}$$

$$\beta_{vn} = \frac{\lambda(\alpha) C_{nl} D_e^{\alpha-1} (2\pi)^{\alpha-2}}{2\pi m T_e^{\alpha-2}} \tag{4.2}$$

$$\lambda(\alpha) = \frac{2^{\alpha+2}\Gamma\left(1+\frac{\alpha}{2}\right)}{\Gamma(2+\alpha)} \tag{4.3}$$

$$\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt \tag{4.4}$$

Table 4.1 Parameters considered for inelastic systems with viscous linear, viscous non-linear, and hysteretic dampers.

Parameters	Values
T_e [s]	0.3, 0.5, 0.7, 1.0, 1.5, 2.0
r_s	0.02, 0.05, 0.1, 0.2, 1.0
R_μ	1.5, 3.0, 5.0
β_v	0, 0.05, 0.1, 0.15, 0.20
α	0.4, 0.6, 0.8
β_{vn}	0.05, 0.1, 0.15, 0.20
r_d	0.02, 0.05, 0.1, 0.2
r_K	2, 4, 6
r_F	0.1, 0.3, 0.5

4.2 Equivalent linear properties of approximate model

Simplified methods of analysis consider the response of an equivalent viscous elastic system with secant period and equivalent viscous damping obtained from the energy dissipated by the system in one cycle of harmonic maximum displacement. For inelastic systems with viscous linear damping the equivalent linear properties are defined by Eqns. 4.5 to 4.7, where T_{sec} is the secant period, D the maximum relative displacement, A_s the acceleration at maximum displacement, β_{eq} the total viscous damping, A_{ys} the yielding absolute acceleration of the structure, and D_{ys} the yielding displacement; the rest of the variables have already been defined.

$$T_{sec} = 2\pi\sqrt{\frac{D}{A_s}} \tag{4.5}$$

$$\beta_{eq} = \beta_i + \beta_v \frac{T_{sec}}{T_e} + \frac{2}{\pi}\frac{(A_{ys}D - A_s D_{ys})}{A_s D} \tag{4.6}$$

$$A_s = A_{ys} + r_s \frac{A_{ys}}{D_{ys}}(D - D_{ys}) \tag{4.7}$$

For bi-linear hysteretic structure with nonlinear viscous dampers the equivalent damping is

$$\beta_{eq} = \beta_i + \beta_{vn}\left(\frac{D_e}{D}\right)^{1-\alpha}\left(\frac{T_{sec}}{T_e}\right)^{2-\alpha} + \frac{2}{\pi}\frac{(A_{ys}D - A_s D_{ys})}{A_s D} \tag{4.8}$$

Finally, for bi-linear hysteretic structure with hysteretic dampers the expressions are

$$T_{sec} = 2\pi\sqrt{\frac{D}{A_s+A_d}} \tag{4.9}$$

$$\beta_{eq} = \beta_i\sqrt{\frac{A_s}{A_s+A_d}} + \frac{2}{\pi}\frac{(A_{ys}D-A_sD_{ys})+(A_{yd}D-A_dD_{yd})}{(A_s+A_d)D} \tag{4.10}$$

$$A_d = A_{yd} + r_d\frac{A_{yd}}{D_{yd}}(D-D_{yd}) \tag{4.11}$$

Where A_d is the acceleration of the hysteretic damper for the maximum displacement.

4.3. Limit of maximum displacement

In the approximate methods proposed by Ramirez et al. (2002a), as well as the one established by ASCE-7 (2010), the displacement obtained by the CSM is arbitrarily limited to the maximum displacement for elastic conditions. This condition gives better results for the approximate spectra of displacement, velocity, and acceleration. The elastic displacement is calculated as $S_{de}(\beta_i + \beta_v, T_e)$ for systems with linear viscous dampers, as $S_{de}(\beta_i + \beta_{vn}, T_e)$ for systems with non-linear viscous damping and as $S_{de}(\beta_h, T_e{}')$ for systems with hysteretic damping, where $T_e{}'$ and β_h are obtained from the expressions

$$T_e' = 2\pi\sqrt{\frac{D}{A_d+\frac{A_{ys}D}{D_{ys}}}} \tag{4.12}$$

$$\beta_h = \beta_i\sqrt{\frac{A_{ys}}{A_{ys}+A_{yd}+r_dK_d(D_{ys}-D_{yd})}} + \frac{2}{\pi}\frac{A_{yd}D_{ys}-\left(A_{yd}+r_dK_d\left(D_{ys}-D_{yd}\right)\right)D_{yd}}{\left(A_{ys}+A_{yd}+r_dK_d\left(D_{ys}-D_{yd}\right)\right)D_{ys}} \tag{4.13}$$

4.4. Evaluation of the simplified analysis methods

The three dynamic variables of interest (displacement, velocity, and acceleration) for soils type II records are plotted in Figures 4.1 and 4.2 for non-linear viscous damping and hysteretic damping. The horizontal axis represents the values obtained by the approximate procedure and the vertical axis represents the "exact" non-linear response. Points under a 45º line correspond to overestimation (conservative values), whereas points over that line are sub-evaluation (un-conservative values). The influence of soil type was not significant in the evaluation of the CSM, except in the prediction of the relative velocity for which the estimations for medium to short structural periods are more conservative for softer soils. Displacement spectra are compared using the ASCE procedure, which includes the elastic displacement limit, and the original CSM that not include such limit.

5. CONCLUSIONS

The evaluation of the damping modification factor, DMF, for different spectral quantities was addressed considering parameters as damping, period, types of earthquake and soil, and duration of motion, using the Chilean earthquake records data base. The conclusion was that for practical applications, in the range of interest for energy dissipation technology, only damping and period were relevant. A formula for the DMF for relative displacement was proposed for the Chilean code.

Then, the Capacity Spectrum Method, CSM, for approximate non-linear analysis of structures was evaluated aimed to be included in the code. The findings were that the CSM, in its original form, produces conservative results for displacements only in structures where the inelastic action is not relevant, or when they remain in the elastic range. The CSM end up with erratic or non-conservative results as a consequence of over-evaluation of the equivalent viscous damping, when the inelastic effect in the structure is more important. The introduction by Ramirez et al. (2002a) of a limit of displacement of the inelastic response as the elastic response of the structure for initial stiffness comes out to be very important in the evaluation of the displacement response. It improves the estimations especially in structures that respond significantly in the inelastic range and in structures with medium to large natural periods. Inclusion of an elastic response limit for displacements generally leads to larger displacements than the original method. This fact could lead to too conservative values in structures very flexible that respond significantly in the inelastic region. The influence of soil type was not significant in the evaluation of the CSM, except in the prediction of the relative velocity, for which the estimations for structural periods medium to short are more conservative when the soil is softer.

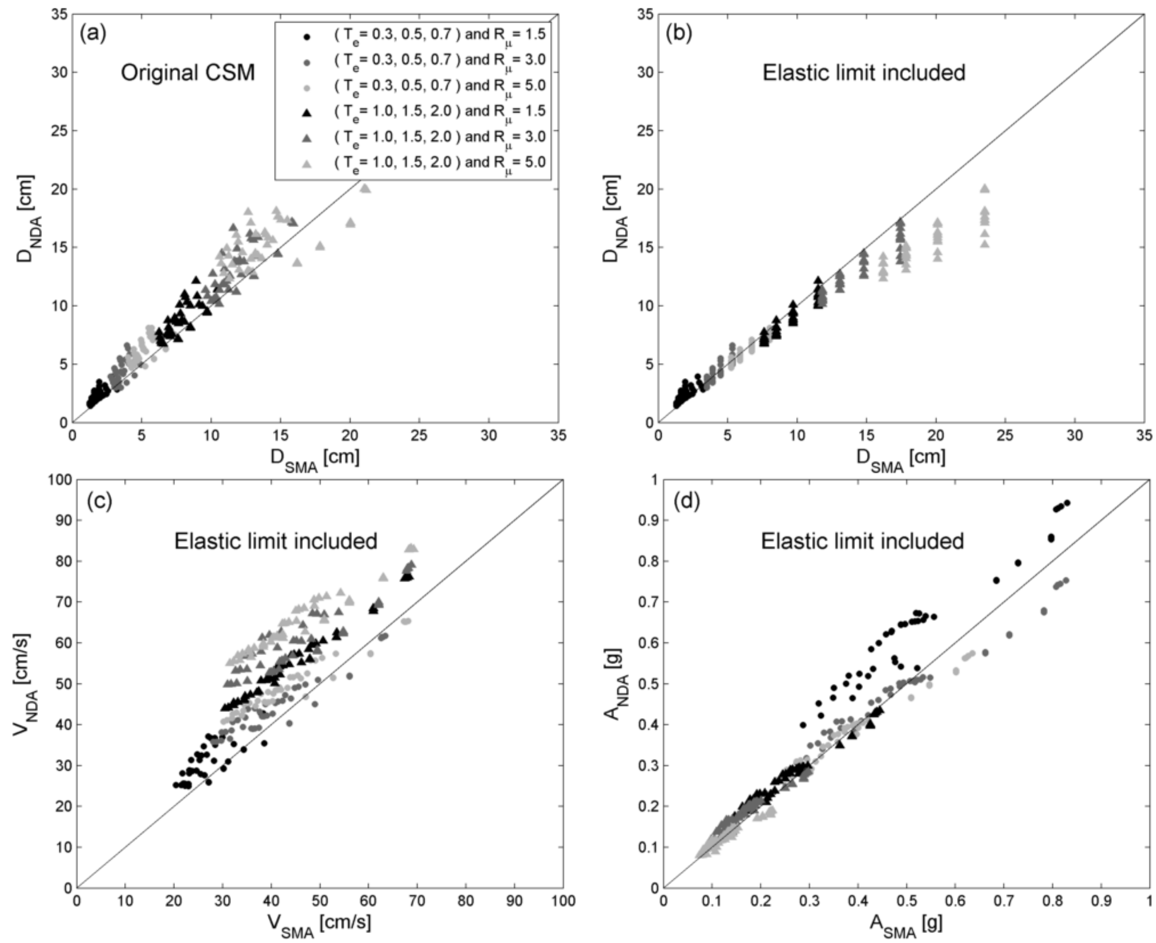

Figure 4.1. Approximate method versus exact response for systems with non-linear viscous dampers for α=0.6.

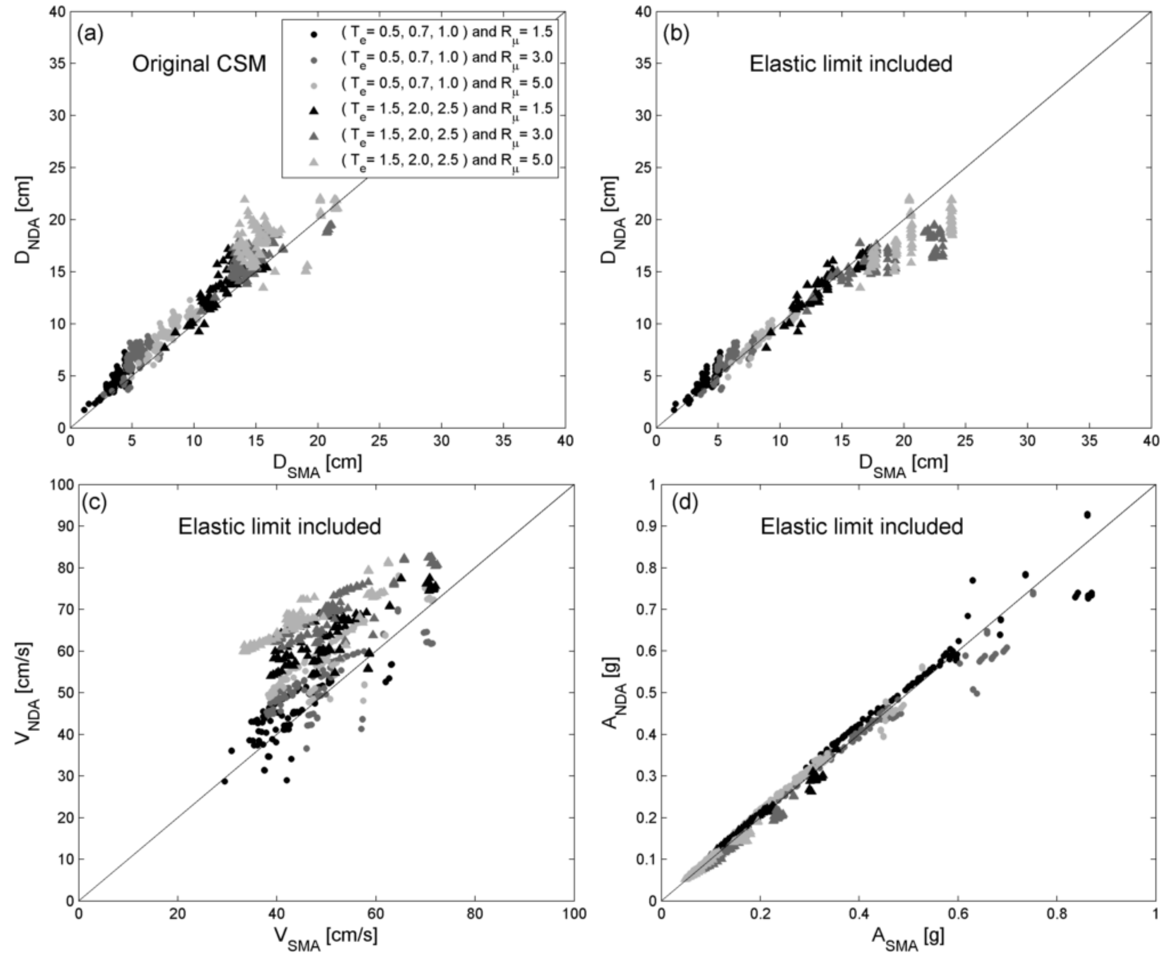

Figure 4.2. Approximate methods versus exact response for systems with hysteretic dampers with 2% post-yielding stiffness ratio.

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