Calculating Modal Target Epsilons from multiple GMPE Models: A case study for Montreal

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SUMMARY:
In Probabilistic Seismic Hazard Analysis (PSHA), the target epsilon "ε_o" represents the departure of the target ground motion, at a specific hazard level, from that predicted by the ground-motion prediction equation (GMPE). Target epsilon values are commonly used in seismic performance evaluations for identifying design events and selecting ground motion records for vulnerability and liquefaction studies. There are few methods for calculating the modal (most likely) target epsilons and values are usually reported at different locations by considering a single GMPE in the seismological model. A single GMPE model provides an independent set of epsilons. For cases where multiple GMPEs are being considered, a method is required to calculate robust weighted modal epsilon values that account for epistemic uncertainty. A method is proposed for calculating target epsilons when multiple GMPE models are being considered. The proposed method, along with other procedures, is demonstrated for the case of Montreal Island.

Keywords: Target Epsilon - GMPE - Seismic Hazard – Epistemic Uncertainty

1. INTRODUCTION

An important element of PSHA is the incorporation of ground-motion uncertainty from earthquake sources (Harmsen, 2001). Uncertainty of the predicted ground-motion (ex. Spectral acceleration, *Sa*) at each period is represented by a lognormally distributed random variable. This variable represents the aleatory uncertainty of the ground-motions predicted by the GMPE. The target epsilon, ε_o, (also called "proper ε" or "expected ε") is a parameter that represents the number of, logarithmic, standard deviations by which the, logarithmic, target ground motion (at a target return period) at a specific hazard level deviates from the median predicted value by an attenuation function for a given magnitude (*M*) and distance (*R*) (Bazzurro and Cornell, 1999). Target epsilons are generally calculated using Eqn. 1.1 as a function of spectral acceleration.

$$\varepsilon_o = \frac{\ln SA_o - \ln Sa}{\sigma_{\ln Sa}} \tag{1.1}$$

Where, σ_{lnSa} is the standard deviation of the logarithm of the predicted spectral acceleration, SA_o is the target spectral acceleration at a specified hazard level and *Sa* is the median value predicted by a GMPE. Fig. 1.1 shows the graphical representation of the target epsilon along with the probability of exceeding the target spectral acceleration (area under the curve from SA_o to ∞). By definition, ε_o depends on the GMPE, the period and the hazard level of interest. Calculating target epsilon values is common practice and there are several methods available to do so. The main difference between these methods is related to how the probability of exceeding the target ground motion is assigned to the different epsilon bins during the PSHA. These methods, as discussed in the next section, are mostly implemented when a single GMPE is being considered in the analysis. In some cases, multiple GMPE models are considered in PSHA to quantify epistemic uncertainty and the expected variability in the results. A single GMPE model provides a single set of target epsilons and as a consequence multiple GMPEs will provide multiple sets of target epsilons. When multiple GMPEs are considered, estimating a single set of weighted target epsilon values is required.

The Canadian city of Montreal is considered an appropriate example since it is located in the moderately seismic region of Eastern North America (ENA) where GMPEs are considered a high source of epistemic uncertainty and since there are not enough documentation for target epsilon in previous and current codes for this city. Four different GMPEs are used in order to update and propose new hazard values along with target epsilons for Montreal. Target epsilons are calculated for each GMPE model independently and then a weighing method is proposed to calculate a single weighted set of epsilon values for all the GMPE models.

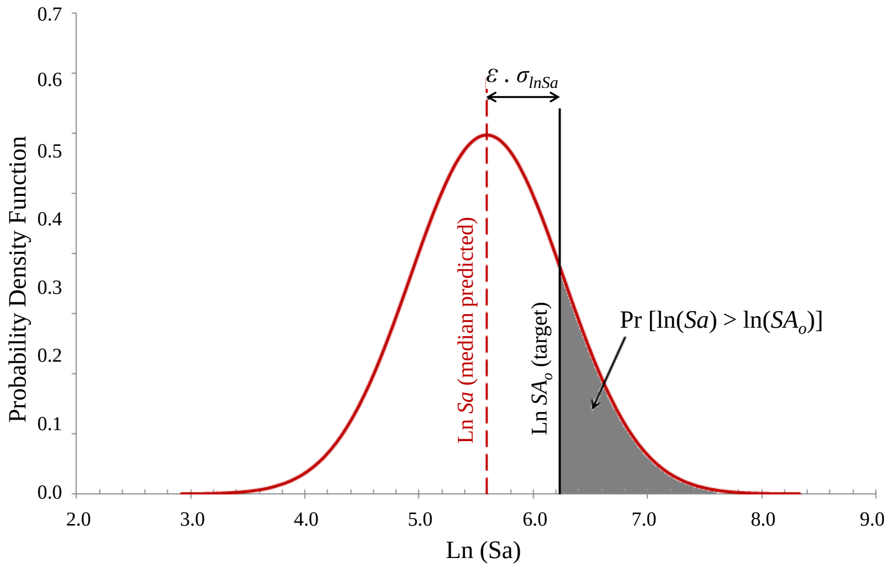

Figure 1.1. Sample normal distribution of *ln Sa* showing the departure of *ln* SA_o from median predicted value *ln Sa* and the probability of exceeding *ln* SA_o (shaded area)

2. METHODS FOR CALCULATING TARGET EPSILON

Deaggregation is known as the process of disintegrating the PSHA hazard values into its constituting components in order to observe the relative hazard contribution for different ranges of the three main variables in the PSHA (i.e. M, R, ε). In traditional PSHA studies, deaggregation was performed only relative to magnitude and distance, M and R, while the random variable ε was almost always neglected (Bazzurro and Cornell, 1999). The epsilon values have many uses such as assessing collapse capacity for structures and selecting ground-motions for dynamic analysis. There are several methods to calculate target epsilons when a single GMPE is being considered. In order to better understand these methods, it is necessary to review the definition of the target epsilon. Assume that during the PSHA for calculating the probability of exceedance (*PE)* at a specific target spectral acceleration SA_o, an event of magnitude M at distance R is being considered. For this event, the logarithm of the target SA_o is one standard deviation less than the logarithm of the median predicted spectral acceleration (Sa$|$ M, R) obtained by substituting (M, R) in the GMPE (the corresponding ε is equal to -1) (Fig. 2.1). This implies that the spectral acceleration created by this event has a probability of 84% (area under the standard normally distributed curve from ε = -1 to ∞) of exceeding the target acceleration SA_o. Different methods of calculating epsilons are available depending on the way by which this probability (i.e. 0.84) is assigned to the different epsilon bins.

The first method assigns this probability of exceedance to each epsilon bin in proportion to the probability content of each epsilon bin, for example, the ε = 0 bin receives the portion of the exceedance probability (0.84) relative to the probability content of this epsilon bin (area under the curve from ε - $\Delta\varepsilon/2$ to ε + $\Delta\varepsilon/2$ where, $\Delta\varepsilon$ is the width of the epsilon bin). Target epsilons are calculated by this method during the deaggregation process and then presented in the joint distribution of (M, R, ε). The modal epsilon (epsilon bin with highest contribution to hazard) can then be determined from this joint distribution. The disadvantage of this method is that the reported modal event (M, R, ε) of the joint distribution is the event that will most likely exceed SA_o but not match it. This means that the target ground motion is not obtained when substituting (M, R, ε) in the GMPE. This method is the most conventional and most used by researchers (ex. USGS 1996).

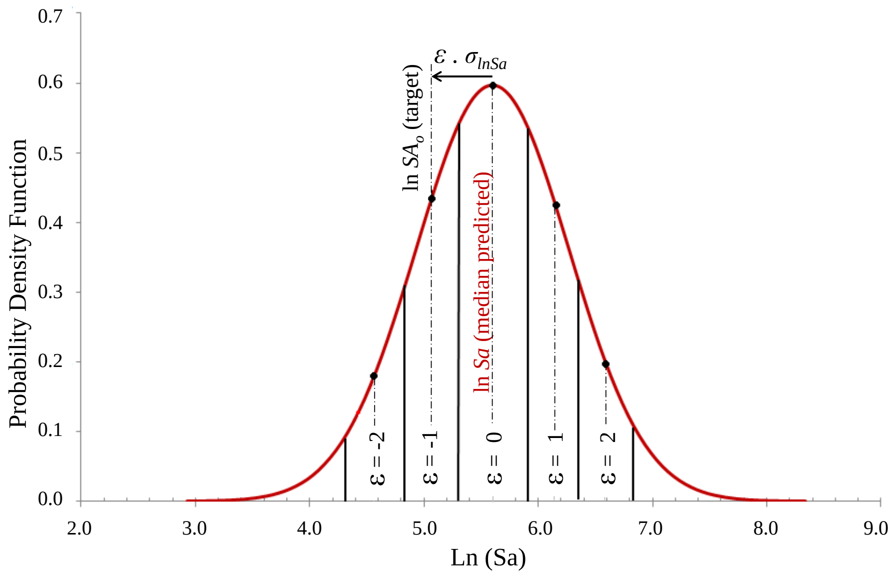

Figure 2.1. Schematic illustration of the epsilon bins ($\Delta\varepsilon = 1.0$)

The second method (McGuire, 1995) assigns the probability of exceedance to the ε = -1 bin. Unlike the previous method, this method ensures that the reported epsilon value (ε = -1) will make the predicted spectral acceleration Sa match the target spectral acceleration SA_o when (M, R, ε) are substituted in the GMPE, however, it has a disadvantage since hazard contributions from epsilon bins larger than -1 for the same event (M, R) are neglected even when they do result in higher Sa.

A third method can be devised which blends the characteristics of both of the previous methods. This method is similar to the second method and yields the same epsilon values, the only difference is that this method takes into consideration the modal M and R values taken from the joint distribution of (M, R, ε) rather than the joint distribution of (M, R), and then adjusts the target epsilon value to ε'_o, the value necessary to obtain the target SA_o when substituted in the attenuation function (Bazzurro and Cornell, 1999). The justification for this proposal is that the spectral shape depends primarily on M and, secondarily, on R but does not significantly depend on ε (Bazzurro and Cornell, 1999). This method ensures that the adopted controlling event has the most likely magnitude and distance. This method is similar to the one proposed by Chapman (1995), the only difference being that Chapman proposes the use of (M, R) values taken from the M-R joint distribution instead of that from the full M-R-ε joint distribution (Bazzurro and Cornell, 1999).

In next section, a method similar to that of Chapman is used since several earthquake hazard analysis software (e.g. CRISIS) only provide the M-R joint distribution. Eqn. 2.1 is used to calculate the modal target epsilons ε^*_o that match the target ground-motion at each period for each GMPE independently.

$$\left(\varepsilon_o^*\right)_i = \frac{\ln\left(SA_o\right)_i - \ln\left(\left(Sa\right)_i \middle| M^*, R^*\right)}{\left(\sigma_{\ln Sa}\right)_i} \tag{2.1}$$

Where, ε^*_o is the modal target epsilon, i refers to a specific GMPE and M, R are the modal magnitude and distance.

3. WEIGHTED TARGET EPSILONS

As previously mentioned, most PSHA studies incorporate a single GMPE in the seismological model. Multiple GMPEs models are often considered when conducting PSHA in regions with moderate or low seismicity to account for the epistemic uncertainty. Epistemic uncertainty is a result of incomplete or insufficient knowledge of the seismicity of such regions. Eqn. 1.1 and 2.1 show that target epsilon is sensitive to GMPEs and therefore it is necessary to provide a single weighted value of epsilon at each period when multiple GMPE models are being considered. All methods discussed in the previous section are appropriate when a single GMPE is being considered. S. Harmsen (2001) proposed Eqn. 3.1 which defines the average or weighted target epsilon when multiple GMPE models are used.

$$\varepsilon_o = \frac{\sum_A \sum_S \varepsilon_{oA} Wt(A) \lambda_S \Pr\left[SA \geq SA_o \left| S, \mu_A, \sigma_A \right.\right]}{\sum_A \sum_S Wt(A) \lambda_S \Pr\left[SA \geq SA_o \left| S, \mu_A, \sigma_A \right.\right]} \tag{3.1}$$

Where, $\Sigma_A \Sigma_S$ are the summations over the GMPE models and seismic sources respectively, *Wt(A)* is the assigned weight to each model *A*, ε_{oA} is the target epsilon for each model separately, λ_s is the mean annual rate of occurrence of earthquakes from source *S* and $\Pr[SA \geq SA_o | S, \mu_A, \sigma_A]$ is the conditional probability that earthquake *S* will produce a ground-motion exceedance, given the maximum likelihood parameters (μ_A, σ_A) of the ground-motion.

For example, if ε_{o1} and ε_{o2} are the epsilons for a given earthquake (*M, R*) from source *S* corresponding to attenuation models 1 and 2, respectively, then ε_o is in general not equal to $(\varepsilon_{o1} + \varepsilon_{o2})/2$ even though the two attenuation models have equal weights. Eqn. 3.1 implements another weighting factor which is the conditional probability of exceeding the target spectral acceleration given the occurrence of this earthquake. This factor is often considerably larger for one of the attenuation models being used in the PSHA (Harmsen, 2001). Bender and Perkins (1993) noted that the hazard curve that results from averaging over attenuation functions is closest to the curve for the function that predicts the highest response and question if the corresponding attenuation function should be given a lower weight when averaging. Since our objective is not to find the average epsilon but to find the most representative epsilon, higher weights are to be applied to more robust and up to date GMPEs. The same weighting scheme can also be used to determine other weighted PSHA statistics such as mean and modal magnitude and distance. In this study, the proposed procedures are only applied to calculate weighted target epsilons.

4. CASE STUDY: CITY OF MONTREAL

The island of Montreal is used as a case study to illustrate the procedures for calculating the target epsilon. Montreal is located in ENA; a region with moderate seismic hazard. GMPEs generated by different researchers for ENA show significant differences relative to seismic-wave attenuation and considerable variability between predictions. Four GMPEs are selected for this study; Atkinson and Boore 1995 (*AB95*), Campbell 2003 (*C03*), Atkinson and Boore 2006 (*AB06*) and Atkinson 2008 (*A08*). Fig. 4.1 illustrates the difference between the attenuation functions for *Class C* site condition (see National building code of Canada (NBCC) 2005 for *Class C* definition). The seismic hazard program, CRISIS, was used to conduct PSHA for each GMPE model individually.

As a start, the deaggregation modal values, (M^*, R^*), for each model and at each period, are obtained from CRISIS output files along with the target spectral acceleration SA_o at a probability of exceedance of 2% in 50 years. Using these PSHA measures obtained from CRISIS output, modal target epsilons ε^*_o are easily calculated for each GMPE model at each period using Eqn. 2.1. These epsilon values are labelled as "independent target-epsilons" and are summarized in Table 4.1. It is observed that AB95 produces positive values unlike AB06 which produces negative values; positive ε values occur when the return period of the target ground motion (i.e. 2475 years for a 2% in 50 year motion) is much longer than the return period of the modal event (*M*, *R*) that causes the ground motion (ATC-63). On the other hand, negative ε values for a 50% in 5 year motion stems from the fact that the return period of the ground motion (i.e. 10 years) is much shorter than the return period of the event that causes the ground motion (ATC-63). Eastern America has low positive ε values because seismic events are less frequent, but the return periods are still typically shorter than the return period of a 2% in 50 year motion (i.e. 2475 years) (ATC-63). Harmsen, 2001 states that modal ε for the 2% in 50 year *PE* may be less than zero in areal fault zones of the Central and Eastern United States (CEUS) which explains the values of the more robust *AB06* GMPE.

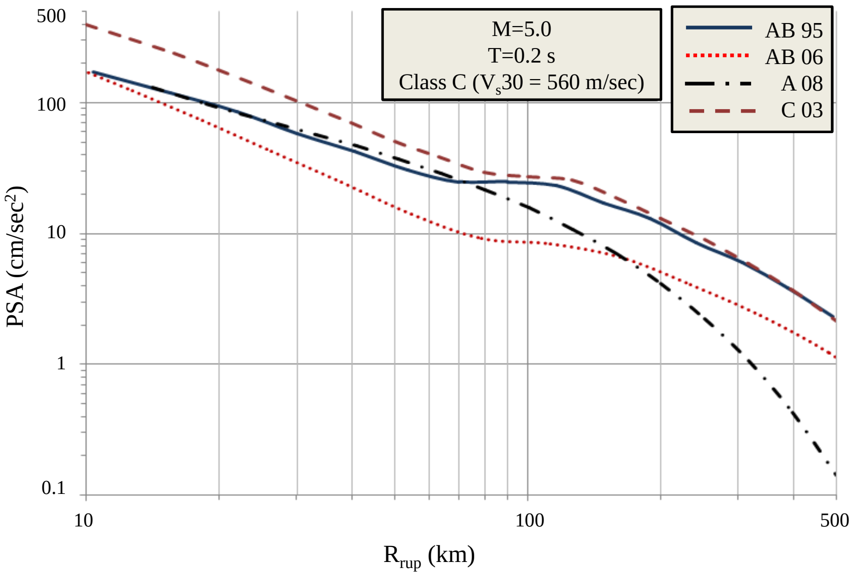

Figure 4.1. Comparison of spectral acceleration as a function of closest to rupture distance R_{rup} for T=0.2s at *Class C* site condition as predicted by the four selected GMPEs

These four sets of independent target epsilons are appropriate to be used individually with their corresponding GMPE model. For example, the 2005 version of the NBCC used *AB95* as a reference GMPE for conducting PSHA for the city of Montreal; therefore it is appropriate to use the *AB95* epsilon set shown in Table 4.1 with the other PSHA results reported in NBCC 2005. However, when considering alternative GMPE models, uniform hazard spectrum (UHS) and deaggregation values (*M*, *R*) are obtained by weighing the PSHA results of each of the GMPE models and as a consequence weighted target epsilons are needed to be reported along with other weighted PSHA results. The procedures of calculating the weighted epsilons and the steps of the propose methods are outlined in Fig. 4.2 and discussed in detail next.

Table 4.1. Independent target-epsilon values at different periods for the four GMPE models for Montreal at PE of 2% in 50 years

	Modal target epsilon, ε^*_o			
Period	AB95	AB06	C03	A08
0.01 (PGA)	1.8	-0.22	-0.84	0.98
0.1	1.03	-0.19	-0.35	1.15
0.15	1.17	-0.13	-0.29	0.78
0.2	0.73	-0.53	-0.61	0.78
0.3	0.78	-0.45	-0.64	0.46
0.4	0.78	-0.4	-1.03	0.94
0.5	0.8	-0.33	-1.02	0.57
1	0.96	-0.17	-0.85	0.25
2	0.89	0.07	-0.67	0.56

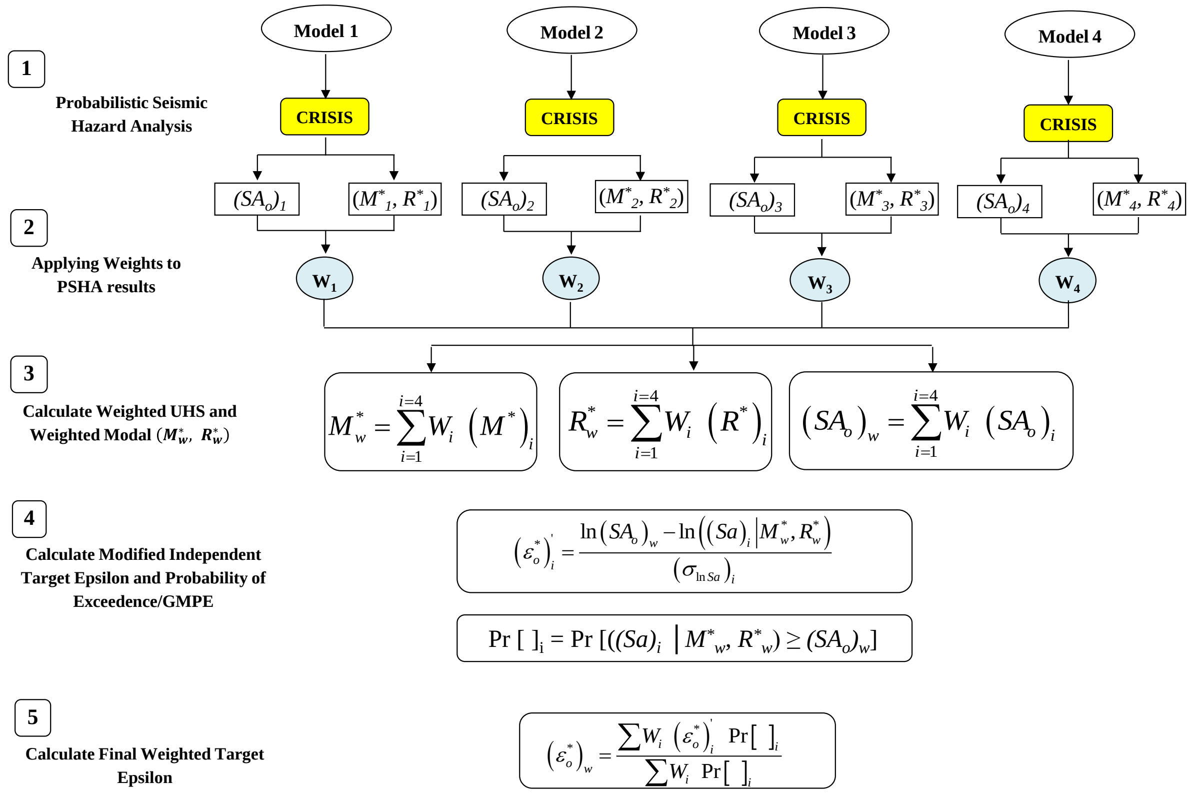

Figure 4.2 Outline of the method proposed for calculating weighted target epsilon

First, the target epsilons are calculated for each GMPE using Eqn. 4.1 which is a modified version of Eqn. 2.1 where weighted PSHA measures (Eqn.4.1a, Eqn.4.1b and Eqn.4.1c) are used instead of independent ones. These epsilons are labeled here as "modified independent target-epsilons", $(\varepsilon^*_o)'$, and are calculated relative to the weighted hazard function that accounts for epistemic uncertainty.

$$\left(\varepsilon_o^*\right)_i' = \frac{\ln\left(SA_o\right)_w - \ln\left(\left(Sa\right)_i \middle| M_w^*, R_w^*\right)}{\left(\sigma_{\ln Sa}\right)_i} \tag{4.1}$$

$$\left(SA_o\right)_w = \sum_{i=1}^{i=4} W_i \left(SA_o\right)_i \tag{4.1a}$$

$$M_w^* = \sum_{i=1}^{i=4} W_i \left(M^*\right)_i \tag{4.1b}$$

$$R_w^* = \sum_{i=1}^{i=4} W_i \left(R^*\right)_i \tag{4.1c}$$

Where, $(SA_o)_w$ is the weighted target spectral acceleration for a target return period, M^*_w is the weighted modal (most likely) magnitude, R^*_w is the weighted modal (most likely) distance and W_i is the weighing factor that reflects the degree of confidence in a GMPE ($\Sigma W_i=0$).

In the previous equation, a weighing factor that reflects the degree of confidence in the different GMPEs is applied to SA_o, M^* and R^* to obtain the weighted $(SA_o)_w$, M^*_w and R^*_w respectively. Weights of 0.1, 0.5, 0.25 and 0.15 are assigned to *AB95*, *AB06*, *C03* and *A08* respectively. These weights are subjective and they reflect the degree of confidence in each GMPE. The weights may differ from one analyst to another, however, slightly different values of weights, than the ones proposed, do not affect the results significantly. The modified independent target-epsilons for all four models are summarized in Table 4.1. It is observed that these modified independent target-epsilons experience less variability in their values compared to independent target-epsilons.

Table 4.1. Modified independent target-epsilons at different periods for the four GMPE models for Montreal at PE of 2% in 50 years

	Modified independent epsilons $(\varepsilon^*_o)'$			
Periods	AB95	AB06	C03	A08
0.01 (PGA)	-0.119	0.330	-0.307	-0.010
0.1	0.145	0.229	-0.537	-0.170
0.15	0.146	0.319	-0.324	-0.337
0.2	-0.403	-0.041	-0.636	-0.731
0.3	-0.418	-0.094	-0.630	-0.992
0.4	-0.453	-0.249	-0.713	-1.092
0.5	-0.354	-0.270	-0.678	-1.121
1	-0.071	-0.235	-0.832	-1.233
2	-0.384	-0.375	-0.725	-1.656

Finally, the weighted epsilon values are calculated using Eqn. 4.2 by a method similar to that of Harmsen (Eqn. 3.1) but without considering multiple seismic sources. This expression for epsilon is a modified version of the weighted epsilon equations presented in Harmsen (2001) and McGuire (1995).

$$\left(\varepsilon_o^*\right)_w = \frac{\sum W_i\ \left(\varepsilon_o^*\right)_i'\ \Pr[\ ]_i}{\sum W_i\ \Pr[\ ]_i} \tag{4.2}$$

Where, $\Pr[\ ]_i$ is the probability of exceeding SA_o using GMPE i and $(\varepsilon^*_o)'_i$ is the modified independent epsilon (Eqn. 4.1). In Eqn. 4.2, the modified epsilons are used along a second weighing factor corresponding to the probability of exceeding the target spectral acceleration. The probability of exceedance is calculated using Eqn. 4.3 for each GMPE by integrating the normal density function from $(SA_o)_w$ to ∞ as shown in Fig. 4.3.

$$\Pr[\ ]_i = Pr\left[\left(Sa\right)_i \geq \left(SA_o\right)_w\right] = \frac{1}{\sigma_{\ln Sa}\sqrt{2\pi}} \int_{(SA_o)_w}^{\infty} \exp\left(\frac{-\left(y - Sa\right)^2}{2\sigma_{\ln Sa}^{\ 2}}\right) dy \tag{4.3}$$

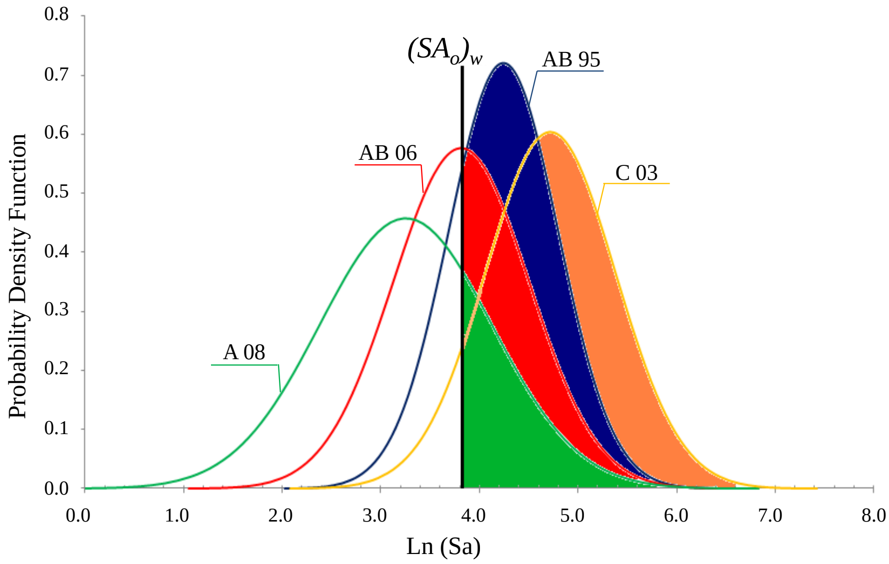

Figure 4.3. Typical ground-motion uncertainty functions at (M^*_w, R^*_w) for the four GMPEs where the shaded areas represent the probability of exceeding $(SA_o)_w$

The additional weighting factor incorporated by the proposed method reduces the variability of the estimate of epsilon. The additional factor is the probability that the event (M^*_w, R^*_w) at a specific period will produce a ground-motion that exceeds the weighted target spectral acceleration $(SA_o)_w$, given the parameters of the GMPE ($lnSa$, σ_{lnSa}) and ($(Sa)_i \mid M^*_w, R^*_w$). This factor is directly related to the calculated epsilon value and the standard deviation of the GMPE. The values of the weighted modal target epsilons are given in Table 4.2. Also, the probabilities of exceeding the weighted spectral acceleration for each GMPE model are presented to indicate which GMPEs affect most the epsilon value.

Table 4.2. Probability of exceeding $(SA_o)_w$**,** the equivalent weight factor for the four GMPEs and the set of weighted target epsilons

Periods	AB95		AB06		A08		C03		Weighted Epsilons $(\varepsilon^*_o)_w$
	$Pr[\ ]_i$	W	$Pr[\ ]_i$	W	$Pr[\ ]_i$	W	$Pr[\ ]_i$	W	
PGA	0.55	0.27	0.37	0.18	0.62	0.30	0.50	0.25	0.054
0.1	0.44	0.21	0.41	0.19	0.70	0.33	0.57	0.27	-0.055
0.15	0.44	0.21	0.38	0.18	0.63	0.30	0.63	0.30	-0.039
0.2	0.66	0.24	0.52	0.19	0.74	0.28	0.77	0.29	-0.395
0.3	0.66	0.24	0.54	0.19	0.74	0.27	0.84	0.30	-0.507
0.4	0.68	0.23	0.60	0.21	0.76	0.26	0.86	0.30	-0.606
0.5	0.64	0.22	0.61	0.21	0.75	0.26	0.87	0.30	-0.609
1	0.54	0.19	0.59	0.21	0.80	0.28	0.89	0.32	-0.649
2	0.65	0.22	0.65	0.21	0.77	0.25	0.95	0.32	-0.841

5. SUMMARY

PSHA was performed for a site in Montreal using four different GMPE model. The modal target epsilons are calculated for each of the four GMPE models independently. Emphasis was put mainly on the modal epsilon values rather than the mean values. The target epsilons exhibit high variability when calculated independently from different GMPE models; this is observed from the results presented earlier. This sensitivity of the epsilon values to model changes makes it hard to recommend representative set of values that accounts for all possible models. Weighted target-epsilons are needed in order to provide values that are consistent with weighted hazard functions. A method is proposed to generate these weighted epsilon values from the different GMPE models. The proposed method uses a weight factor related to the degree of confidence in each GMPE model and an additional weight factor that depends on the probability of exceeding the weighted target spectral acceleration. The second weighting factor assigns more weight to the epsilon value corresponding to the GMPE with the higher probability of exceeding the target spectral acceleration. The two weighing parameters are applied to the epsilon values calculated using constant weighted target spectral acceleration and weighted modal events (M, R) for all GMPEs at each period. The modified independent epsilon values (calculated from weighted measures) exhibit less variability compared to the independent epsilons (calculated from independent measures).

After applying the weighing step, the final weighted target epsilons values for Montreal ranged between 0.0 and -0.8. This range of values was expected since Montreal is located in a region with areal faults and low seismic activity. Needless to say that the epsilon values can only be used with the corresponding UHS and (M, R) deaggregation values. For example, the weighted modal epsilon of -0.61 calculated earlier at T=0.5s, cannot be used with or linked to the current UHS value of $SA_o(0.5)=0.34g$ available in NBCC 2005. This value can only be used with the UHS values and (M, R) modal values reported for the proposed weighted scheme (i.e. $[SA_o(0.5)]_w$). These values of the UHS and the modal magnitudes and distances for the weighted scheme are not presented here due to limited space.

It should be noted that all the sets of different epsilons that were presented were calculated for the *Class C* site condition. Nonetheless, the epsilon values will not change if recalculated for other site condition such as *Class A* site condition. This statement is true for Montreal and it may not be valid for other localities. Also, regarding the epsilon value variability for the weighted scheme, it was observed that small changes to the weights assigned to the GMPEs models will not affect the epsilon value drastically. An acceptable deviation of about 20% is to be expected when considering different logical weighting schemes or when comparing results for the same model but for different researchers (i.e. different PSHA integration parameters).

6. CONCLUSIONS

This study is shedding light on the epsilon parameter. Target epsilons have many uses such as assessing collapse capacity for structures and selecting ground-motions for dynamic analysis. The target epsilons were investigated for the city of Montreal and the findings can be summarized as the following:

1) The target epsilon values are mainly dependent on the corresponding GMPE especially when high level of epistemic uncertainty is involved due to the considerable variability between ground-motion predictions of different GMPEs.

2) Epsilon values obtained from a specific GMPE model or any other weighing method should be used with the corresponding hazard products such as the UHS and the deaggregation values. In other words, the epsilon values presented in this study are not to be used in an arbitrary way but to be used with the equivalent reported UHS and modal bivariate values.

3) Target epsilon obtained from different GMPEs but with respect to a common weighted target spectral acceleration $(SA_o)_w$ and common bivariate modal values (i.e. modified independent epsilons $(\varepsilon^*_o)'$ are more consistent and less sensitive to weight or model changes compared to the independent epsilons ε^*_o.

4) In order to report modal target epsilon that can be used with the proposed weighted UHS and the proposed weighted deaggregation values, a weighting scheme is applied which does not only uses simple weights but also incorporate the probability of exceeding $(SA_o)_w$ for each GMPE as an additional weighing factor. Final weighted target epsilon values are in the range of 0.0 to -0.8. Negative target epsilons are expected for low and moderate seismicity zones with areal faults such as ENA.

5) Modal target epsilons were calculated for the *Class C* site condition, however, for Montreal, the modal target epsilons for *Class A* are similar if not the same.

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