Generation of Elastic as well as Inelastic Design Spectra Compatible Accelerogram Using Time-Frequency Analysis

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SUMMARY:
A new methodology is proposed to simulate accelerogram which is compatible to both linear and nonlinear design spectra. For this purpose, a wavelet-based parametric characterization of earthquake accelerogram is adopted and the parameters are optimized to tune its non-stationary features. A modified form of L-P basis is considered for wavelet transform which has the unique feature to decompose the signal in non-overlapping bands. The parameters in each frequency bands are then optimized in such a way that the simulated accelerogram is compatible with linear and non-linear design spectra. In this context, the non-linear design spectrum is obtained by varying the initial period of an elasto-perfectly-plastic oscillator for a given ductility demand. As the duration of a motion influences its energy based damage causing capacity, the proposed model allows the designer to obtain different time histories with different strong motion durations for a given pair of linear and non-linear design spectra.

Keywords: spectrum compatible accelerogram, inelastic design spectrum, wavelet transform, genetic algorithm

1. INTRODUCTION

It is a common practice nowadays to modify a recorded time-history to match its response spectrum with the specified design spectrum so that the ground motion corresponds to the specified seismic hazard (Hancock et al. 2006). Such a spectrally matched modified time-history forms basis for linear and nonlinear time-history analyses to check adequacy of seismic design. It is true that several recorded accelerograms having very different nonstationary characteristics (both amplitude and frequency) can be modified to match their individual response spectra with a target design spectrum. Nevertheless, nonlinear response spectra (e.g. constant ductility spectra) obtained from each of such spectrally matched time-histories are likely to be significantly different (Mukherjee and Gupta 2002). Hence, for carrying out nonlinear analyses with the help of spectrally matched time-histories to check adequacy of design, the selection of recorded time-histories are crucial and that requires subjectivity. Previous study shows that artificial spectrally matched accelerograms produce varied nonlinear responses and a proper suite of them may produce average nonlinear response comparable to the nonlinear design spectrum (Barenberg 1989). However, it is not straight forward to choose a suite of spectrally matched time-histories such that in average sense their nonlinear spectra will represent a target design spectrum for a wide range of natural periods.

The subjectivity involved in the selection of recorded ground motion can be obviated if nonlinear design spectrum is also used as a basis for generating spectrally matched ground motions. It should be mentioned here that matching with respect to a linear design spectrum cannot be bypassed as it is the standard practice to characterize the seismic hazard at a site through linear spectrum only (Hancock et al. 2006). Several researches have been carried out on ductility-based nonlinear design spectrum that can be used for seismic design (Newmark and Hall 1982, Miranda and Bertero 1992, Borzi et al. 2001, Chopra and Chintanapakdee 2004). In fact, currently constant ductility-based inelastic design spectra in terms of strength reduction factors are commonly used in various design codes in the framework of performance-based design (Giaralis and Spanos 2010), but in future more rigorous nonlinear design spectra are expected to come (Yong and Jianwu 2008). Thus, it is highly opportune to develop a procedure which will generate artificial time-histories that will be compatible to both linear and nonlinear design spectra in order to maintain consistency between response quantities and codal provisions. In a recent study, Yazdani and Takada (2009) proposed a wavelet-based method to obtain linear/nonlinear response spectrum-compatible earthquake time-histories from a recorded time-history by internal substitution and scaling of wavelet coefficients while maintaining a target total energy of signal. However, besides such perturbation of wavelet coefficients may result in artificial accelerograms having unrealistic nonstationary characteristics, it cannot generate artificial accelerogram compatible with linear and nonlinear design spectra simultaneously.

In the present study a wavelet-based methodology is proposed to obtain linear and nonlinear design spectra compatible artificial earthquake accelerograms. Wavelet-based approaches have proved their superiority over other techniques in terms of preserving realistic nonstationary features of a seed motion while making it compatible to a target design spectrum (Mukherjee and Gupta 2002, Das and Gupta 2008, Spanos et al. 2009). Hence, wavelet-based methodology is adopted in the present study. Further, mathematical model of wavelet coefficients for earthquake accelerograms are available and such model through its parameters can control the nonstationary characteristics of the simulated accelerogram (Das and Gupta 2011). Considering recorded motion as a seed motion restricts further change in nonstationary features while modifying it for spectral matching with respect to a linear design spectrum (Mukherjee and Gupta 2002), thus, the compatible motion may not match with the nonlinear design spectrum satisfactorily. Hence, in the present study the mathematical model for wavelet coefficients, as mentioned above, involving all parameters controlling the nonstationary behaviour has been directly used to simulate spectra compatible time-histories. A simple and efficient optimization-based procedure is adopted to get the optimal parameters of the model for wavelet coefficients from which the ground motion is simulated by inverse wavelet transform. In this study, the nonlinear design spectrum is assumed to be for elasto-perfectly-plastic oscillators having different initial periods and for a given ductility demand. Since duration of ground motion plays an important role in the energy-based damage causing capacity (Hancock and Bommer 2007), the present methodology also leaves a possibility for a designer to have multiple time-histories with different strong motion durations for a given pair of design spectra.

2. MATHEMATICAL DETAILS

The proposed methodology has two major mathematical components related to (i) the modelling of wavelet coefficients of an accelerogram and (ii) optimization of the parameters of modelled wavelet coefficients to achieve matching with respect to design spectra.

2.1. Modelling of wavelet coefficient envelope

The time-frequency descriptions that characterize the nonstationary features of earthquake accelerograms, belonging to a certain class (uni-modal nature of wavelet coefficient envelope), can be efficiently modelled by means of wavelet coefficient envelopes as (Das and Gupta 2011):

$$W_{\psi}f\left(a_j,b\right) = \begin{cases} u_j(b)\left|\sin(\omega_{2j}b+\phi_{2j})\right|\sin\left(\omega_{1j}b+\phi_{1j}\right);\ -21 \le j \le\ -18 \\ u_j(b)\left(0.7\left|\sin\left(\omega_{2j}b+\phi_{2j}\right)\right|+0.3\right)\sin\left(\omega_{1j}b+\phi_{1j}\right);\ -18 < j \le 6 \\ W_j\left(0.7\left|\sin\left(\omega_{2j}b+\phi_{2,6}\right)\right|+0.3\right)\sin\left(\omega_{1j}b+\phi_{1,6}\right);\ 6 < j \le 10 \end{cases} \tag{1}$$

where, $W_{\psi}f\left(a_j,b\right)$ is the wavelet coefficient (transform) of an accelerogram $f(t)$ with $a_j = 2^{j/4}$ being the scale parameter for jth level and b being the shift parameter. The mother wavelet is assumed to be modified L-P with $\sigma = 2^{1/4}$ (Basu and Gupta 1998). In Eqn. (1), $u_j(b)$ is the modulation of the wavelet coefficient envelope with its peak, W_j, realized at $b = p_j$ and is expressed as (Das and Gupta 2011):

$$u_j(b) = W_j e^{0.5+\frac{p_j-b}{2q_j}-0.5e^{(p_j-b)/q_j}} \tag{2}$$

where, q_j is a measure for the characteristic width of $u_j(b)$. Further, in Eqn. (1), ω_{2j} is the slowly varying frequency for level j and it is defined as:

$$\omega_{2j} = \frac{(\sigma-1)\pi}{4a_j} \tag{3}$$

whereas, ω_{1j} is the pseudo frequency of the wavelet basis corresponding to jth level and it is defined as:

$$\omega_{1j} = \frac{(\sigma+1)\pi}{2a_j} \tag{4}$$

and ϕ_{1j}, ϕ_{2j} are two phase angles such that the peaks of the corresponding sinusoids occur at $b = p_j$. $\phi_{1,6}$ and $\phi_{2,6}$ as in Eqn. (1) are ϕ_{1j} and ϕ_{2j} respectively for $j = 6$.

It is clear from Eqns. (1) & (2) that p_j and q_j account for the relative arrival and duration of the component wave respectively while W_j controls its strength. Thus, these three parameters for all j govern the amplitude and frequency nonstationarities of the artificial accelerogram.

2.2. Optimization of wavelet-based parameters

It is clear from Eqn. (1) that there are three level-dependent parameters p_j, q_j and W_j for $-21 \leq j \leq 10$ to completely characterize the nonstationary features of a synthetic realistic earthquake accelerogram. There are 28 distinct values for each of p_j and q_j ($j = -21, -20, \ldots, 6$), and 32 distinct values for W_j ($j = -21, -20, \ldots, 10$). Hence, it is quite logical to assume that the total 88 independent parameters can be optimized such that the reconstructed accelerogram obtained from the modelled wavelet coefficients will match with a given pair of linear and nonlinear response spectra provided the given pair is realistic (i.e., either they are obtained based on real records or obtained from linear design spectrum using realistic law for strength reduction formula (Newmark and Hall 1982)). For the purpose of optimization realistic bounds on p_j and q_j are imposed as [5, 25]s and [1, 20]s respectively because these bounds are likely to cover a wide range of realistic accelerograms (Das and Gupta, 2011). Since p_j accounts for relative arrival of different component waves, in order to achieve realistic frequency nonstationarity the bounds of p_j is made j dependent for five different ranges of j based on previous findings (Das and Gupta, 2011) as: p_j[5, 10] ∀ $j[-21, -19]$, p_j[8, 15] ∀ $j[-18, -12]$, p_j[12, 20] ∀ $j[-11, -4]$, p_j[16, 25] ∀ $j[-3, 3]$, p_j[12, 20] ∀ $j[4, 6]$.

Presently, wavelet-based approach is available to re-scale different decomposed time-histories (i.e., wavelet inverse transform of wavelet coefficient for any particular level) of unit peak acceleration by a scalar quantity, such that the modified time-history becomes compatible to a given design spectrum (Mukherjee and Gupta 2002). Since this level dependent scaling factor is fully correlated to W_j, for any trial value for p_j and q_j, randomly chosen within their allowable bounds, the trial value of W_j, W_j^{trial}, is first obtained using the existing technique (Mukherjee and Gupta 2002). Since, W_j has maximum influence on response spectral ordinates than the other two parameters (Das and Gupta 2011), for the optimization technique the bounds of $W_j$is considered as $[0.9W_j^{trial}, W_j^{trial}/0.9]$. This will ensure that during a direct search based optimization procedure the matching with respect to the linear spectrum is not getting considerably disturbed during any intermediate step.

2.2.1. Objective function

For any optimization technique, defining the objective function is the most crucial component as the efficiency of the final optimized result will depend on how good the objective function has been formulated from a physical problem.

Let $DSA(T)$ denotes the linear design spectrum as a function of natural periods, T and $DSA_{NL}(\mu, T)$ denotes the nonlinear design spectrum as a function of initial natural periods, T, of elasto-perfectly-plastic (EPP) oscillators each having ductility demand μ. In the present case, T correspond to the pseudo frequencies of wavelet bases, thus the design spectra are evaluated at 32 different time periods. Further, let $RSA(T)$ denotes the linear response spectrum of the artificial accelerogram, obtained from

Eqn. (1) by wavelet inverse, and $RSA_{NL}(\mu, T)$ denotes the nonlinear response spectrum. Other details are same as those for the design spectra. Unlike design spectra, response spectra change sharply with change in (initial) natural periods and for that both $RSA(T)$ and $RSA_{NL}(\mu, T)$ are evaluated by taking average of response spectral ordinates in the neighbourhood of any given T considering finer sampling within adjacent Ts.

Ideally the present problem is a multi-objective minimization problem and the objective functions are

$$OF_1 = \sum_{i=1}^{32}(DSA(T_i) - RSA(T_i))^2 \tag{5}$$

and

$$OF_2 = \sum_{i=1}^{32}(DSA_{NL}(T_i) - RSA_{NL}(T_i))^2. \tag{6}$$

However, it is found convenient and effective to merge the two objective functions into a single objective function without any loss of generality as:

$$OF = \sum_{i=1}^{32}\sum_{j=1}^{32}(DSA\left(T_j\right) - RSA(T_j))^2 + (DSA_{NL}(T_i) - RSA_{NL}(T_i))^2 \tag{7}$$

2.2.2. Optimization method

It is quite intuitive that the objective function OF is very much nonlinear with respect to the 88 wavelet-based parameters and hence it is found that conventional gradient and/or curvature based optimization techniques are unable to come close to the global minimum, instead they are prone to converge into a local minimum. Thus, given the dimension and nature of the problem it may be more logical to go for population-based direct search methods. For that reason, genetic algorithm (GA) is employed to obtain the minimized values of the parameters in the objective function in Eqn. (7). Since, GA algorithm maximized the fitness function; the fitness function is defined as:

$$F_t = 1/(1 + OF) \tag{8}$$

Hence, maximum value of F_t coincide with minimum value of OF.

3. RESULTS AND DISCUSSIONS

The wavelet based mathematical model discussed in the previous section is used for numerical analysis. In this context, first two different seed motions (e.g. measured time histories of El-Centro and Park-field ground motions) with given nonstationary characteristics are considered to simulate linear design spectrum compatible ground motions. It may be noted that S00E component recorded at El-Centro site, Imperial Valley Irrigation District during the 1940 Imperial Valley earthquake and N05W component recorded at Array No. 5, Cholame, Shandon, California during the 1966 Parkfield earthquake are used here. Fig. 1 shows the target as well as simulated linear and nonlinear design spectra. In this context, the linear design spectrum given in Eurocode (2003) for B type soil is considered and nonlinear spectrum is obtained due to Newmark and Hall (1982). The ductility demand for the nonlinear design spectra is considered to be 2. The simulated spectra are obtained by changing the energy scaling parameters in each frequency bands as proposed by Mukherjee & Gupta (2002). From these figures it is clear that although the linear spectrum matches with the target one in mean sense, there are significant differences in the nonlinear design spectra. As discussed earlier, this is due to the fact that given non-stationary features of the seed motions are insufficient to closely match simultaneously with the linear as well as nonlinear design spectra. This, in turn, advocates the need for a better mathematical description for earthquake ground motions as proposed in Eqn. (1) which allows the designer to control three important aspects of realistic ground motion viz., the energy content in each scale, arrival time of different frequencies and their durations.

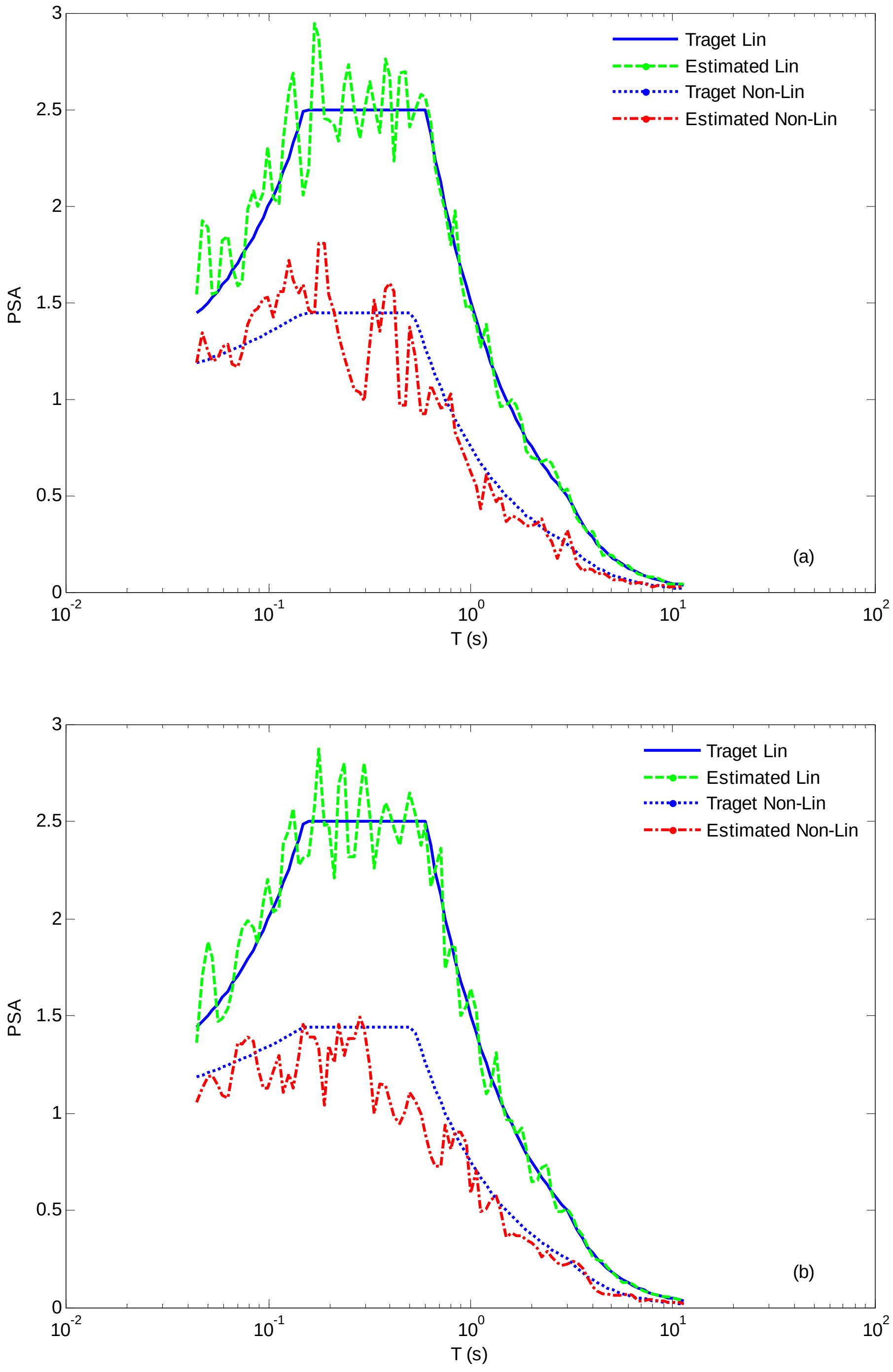

Figure 1. Response Spectrum with predefined nonstationarity; (a) El-Centro Motion and (b) Parkfiled Motion

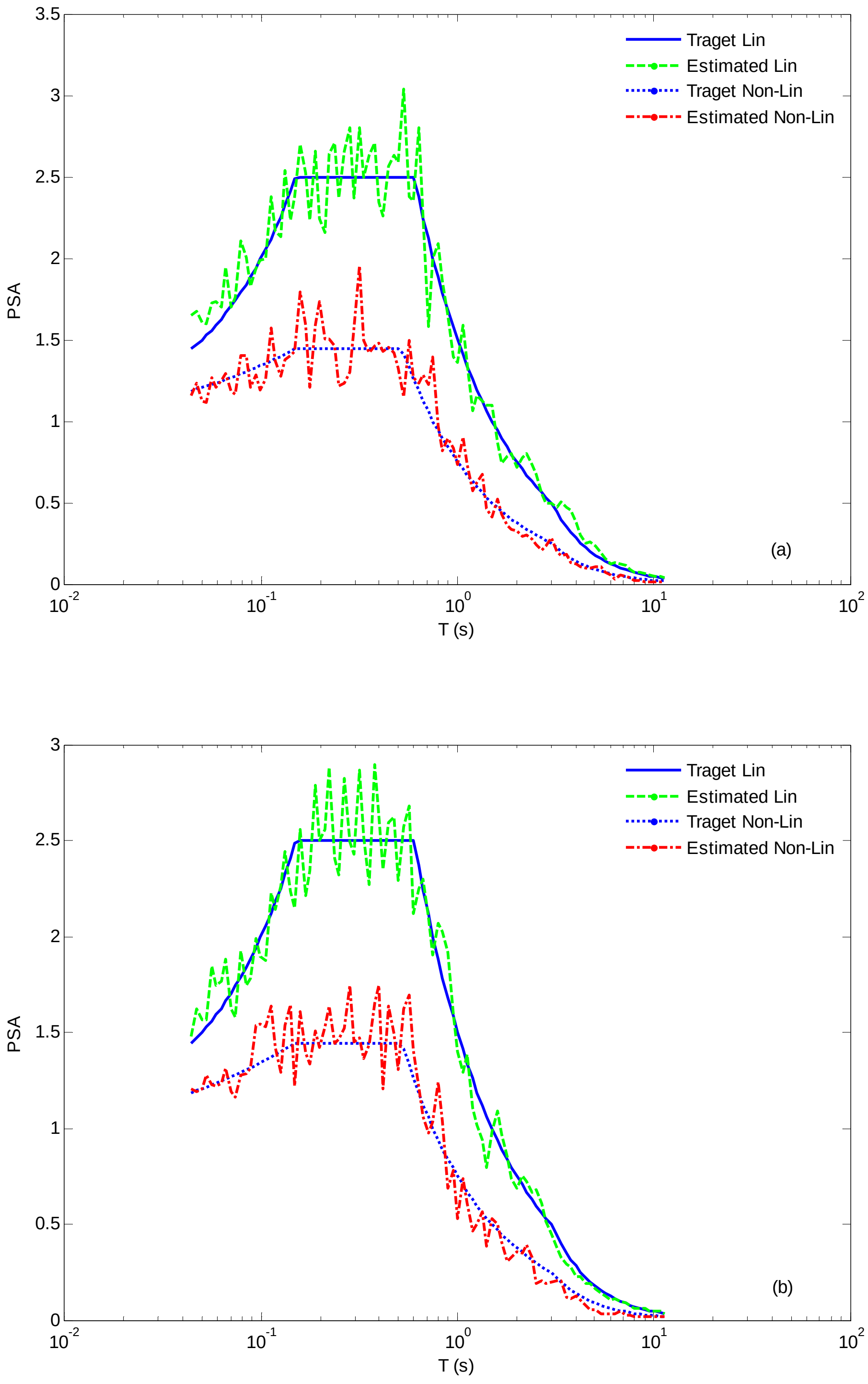

Figure 2. Response Spectrum with optimized nonstationarity; (a) $q_j \in [1, 20]$ and (b) $q_j \in [4.8, 8.6]$

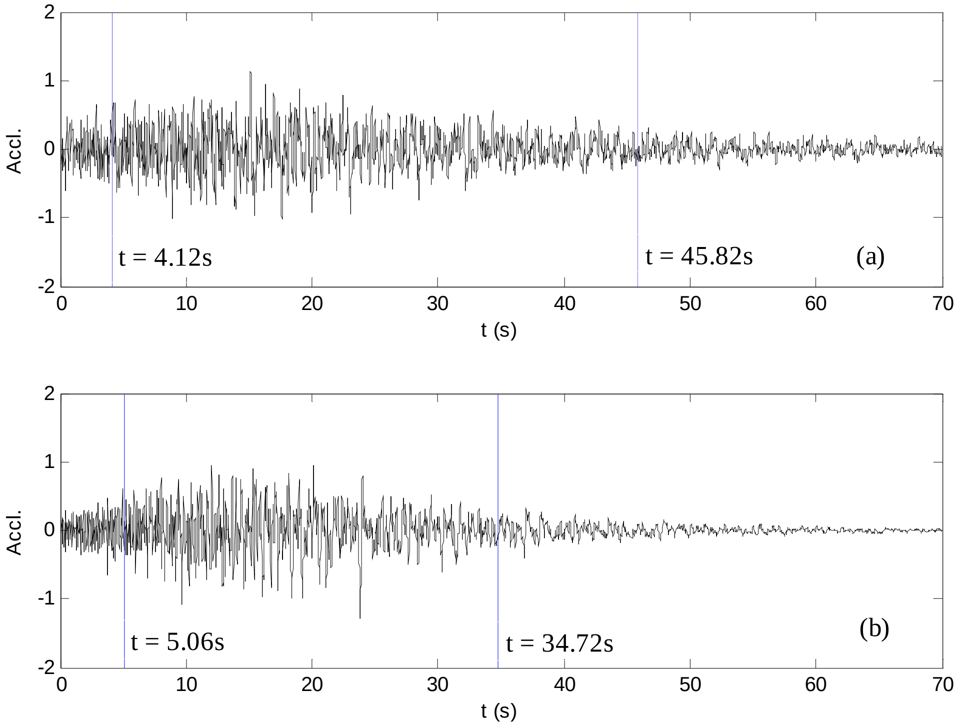

Figure 3. Simulated spectrum compatible motion; (a) $q_j \in [1, 20]$ and (b) $q_j \in [4.8, 8.6]$

As proposed earlier, these parameters can be optimized in such a way that the linear and nonlinear design spectra evaluated from the simulated motions obtained by inverse wavelet transform closely match with their target spectra. With this in view, the proposed wavelet based non-stationary model described in Eqn. (1) is used to simulate the ground motion compatible to both linear and non-linear design spectra. To achieve this, the objective function described in Eqn. (7) is minimized using genetic algorithm. For this purpose, 75 populations and 200 generations are used to estimate global optimal values of the parameters p_j, q_j and W_j. A single point crossover is used in the proposed optimization. Fig. 2a shows simultaneous matching with the target and simulated linear and nonlinear design spectra. The simulated ground motion is shown in Fig. 3a which has strong motion duration of 41.7s (Trifunac and Brady 1975). As explained earlier, the proposed wavelet based model allows the designer to simulated ground motions of different durations. To show this, the bounds of the parameter q_j is kept between [4.8, 8.6] corresponding to 20% and 40% of feasible range as described in section 2.2. Fig. 2b shows the target and simulated design spectra for this bounds on q_j. A close match between the target and the simulated design spectra can be noticed which advocates the possibility and existence of a unique nonstationary signal of different durations as per the need of the designer which will simultaneously match with the linear and nonlinear design spectra. The respective time history is shown in Fig. 3b which shows the strong motion duration to be 29.66s.

4. CONCLUSIONS

The numerical study presented in the previous section clearly shows that simulated accelerogram from a given non-stationary characteristics (i.e. seed motion) do not simultaneously match with the linear and non-linear design spectra. This, in turn, justifies for the development of a robust model of earthquake ground motion which is compatible to both the design spectra. With this in view, the paper presents a wavelet based model to generate a given set of linear and non-linear spectrum compatible accelerogram. In this context, the wavelet based parametric description helps to decompose the signal into orthogonal frequency bands. The parameters in each frequency bands are optimized using evolutionary algorithm in such a way that the error between the given and the simulated design spectra are minimized. The numerical study presented here shows that the simulated spectra closely match with the given set of spectra. Besides this, the proposed model also allows the designer to generate accelerogram with different durations which is essential form the point of view of damage based design.

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