# Estimation of Inelastic Response Spectra Using Artificial Neural Networks

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#### **SUMMARY:**

An Artificial Neural Network (ANN) model is presented to estimate inelastic response spectra for earthquake acceleration records. The moment magnitude, fault mechanism, Joyner-Boore distance, shear-wave velocity, fundamental period of the structure and the system ductility are used as input values of the ANN. Fifty earthquake ground motions taken from the NGA Database and recorded at sites with different type of soils were used during the training phase of the Feedforward Multilayer Perceptron model. The Backpropagation algorithm was used to train the Network. The ANN results present acceptable concordance with the actual seismic response spectra preserving a minimum error between the actual and the estimated spectra using ANN.

Keywords: Artificial neural network, Backpropagation algorithm, inelastic response spectra

## **1. INTRODUCTION**

A response spectrum is one of the most useful tools for Earthquake Engineering purposes. In fact, earthquake-resistant design is based commonly in the use of response spectra to estimate the seismic demand in structures. The response spectrum concept was used for first time by Benioff (1934), subsequently by Housner (1941) and Biot (1941). Nowadays, seismic design codes around the world use response spectra to estimate the design forces of a structure. Hence, several studies have tried to establish methodologies to estimate elastic and inelastic response spectra; in particular, the estimation of inelastic seismic response spectra requires complex numerical analysis. This study proposes the use of Artificial Neural Networks as an alternative to estimate inelastic response spectra.

In the last years, ANN have been used widely in Earthquake Engineering, and their versatility to solve nonlinear and complex problems turn them in an alternative tool to search for solution for problems that require complex analysis. For example, Alves (2006) used neural networks for predicting seismic events. Lee and Han (2002) used five models of ANN to generate artificial seismic response spectra. Kerth and Ting (2005) trained a Backpropagation model to estimate the peak ground acceleration as a function of the distance to the epicenter, the hypocenter deepth and the earthquake magnitude; with this study they designed the high speed lines of railways in Taiwan. Using a neuronal network model, the aftershocks related to large earthquakes were predicted by Barrile (2006); at the same time, studies have been developed to estimate the peak ground acceleration (Günaydın 2008, Arjun and Kumar 2009).

Motivated by all the successful applications of ANN to solve complex nonlinear problems, in this study ANN were selected as a tool to estimate inelastic seismic response spectra. In what follows some basics concepts of Artificial Neural Networks are described.

# 2. ARTIFICIAL NEURAL NETWORKS

Artificial Neural Networks theory arises from the need to solve complex problems, not as a sequence of steps, but as the evolution of computational systems inspired by the human brain, and therefore endowed with certain "intelligence". An ANN is a mathematical model or computational model inspired by the structure and functional aspects of biological neural networks. The structure of a neural network is as follows: neurons are the main processing elements, these are connected to other neurons via a signal weight (synapses), the entries are the dendrites and the result is the axon (see Fig 2.1). Like biological neural networks, the ANN need a process of learning to establish relationships between the variables that define a specific phenomenon. The processing power of an ANN is due to its distributed parallel structure and ability to learn from some examples, obtaining acceptable results for patterns never shown to it.



Figure 2.1. Schematic of biological neuron

There are many topologies established by different authors to define the structure of the ANN. In this work the Feedforward Multilayer Perceptron (FMP) was selected (Shepherd 1997). Fig. 2.2 illustrates the architecture of the FMP. The architecture begins with an input layer which is connected to a hidden layer; this can be connected to another hidden layer or directly to the output layer.



Figure 2.2. Feedforward Multilayer Perceptron

The training of the ANN used in this study was carried out using the "Backpropagation" algorithm proposed by Rumelhart et al. (1986). The procedure is as follows:

1) A set of patterns consisting of pairs of inputs and outputs is applied to the network.

2) The input data information is entered on the first layer. This information is propagated over the network through a propagation rule. The inputs are multiplied by the weights of the connection between layers, the output is transformed by a nonlinear function and transferred to the next layer (there are several transfer functions, for example: step, linear, mixed, Gaussian, hyperbolic tangent, secant hyperbolic and sigmoid). It is noteworthy that many investigations have used a sigmoid transfer function in the hidden layers and a linear function in the output layer, getting satisfactory results. For this reason herein the sigmoid function was selected as propagation rule. The same procedure is applied to the following layers until the output of the network is obtained.

3) The result is compared with target values and the error is calculated.

4) The error in the output layer is propagated backwards from the output layer through the hidden layers until reaching the first, so that all the neurons receive a certain percentage of error.

5) Considering the received amount, each neuron makes an adjustment to their connection weights.

6) The procedure is repeated with other input pairs until the error is smaller than a given tolerance.

# 3. EARTHQUAKE GROUND MOTIONS USED FOR THE ANALYSES

A total of fifty ground motion records obtained from the NGA Database, corresponding to worldwide earthquakes were used for the analyses. The records used for this study correspond to earthquakes with J-B distances (Joyner and Boore 1993) between 18 and 194 km; moment magnitudes ( $M_w$ ) from 5.9 up to 7.7 (representing moderate and large earthquakes). The distribution of the records in terms of moment magnitude and J-B distance is presented in Fig. 3.1, where it can be observed records obtained at different distances and from different events as moment magnitudes indicate (a vast record selection is very important for training the Artificial Neural Networks).



Figure 3.1. Moment magnitude and J-B distance distribution of the selected 50 records taken from the NGA Database

Tables 3.1 through 3.5 summarize the main characteristics of the seismic records used in this study (corresponding to all the type of soil zones according with shear-wave velocity). In the tables the first column refers to the record number, the second column corresponds to the moment magnitude of the event, the fourth column refers to the J-B distance, and the last column is the shear-wave velocity (Vs) above a depth of 30m at the site. In the next subsections, the definition of the failure type (column 3 in Tables 3.1-3.5) and the soil type are described.

| Record | Moment    | Failure | J-B      | Vs (m/s) |
|--------|-----------|---------|----------|----------|
|        | Magnitude | Туре    | distance |          |
| 1      | 6.61      | 2       | 61.79    | 1735.0   |
| 2      | 6.90      | 1       | 52.93    | 1562.2   |
| 3      | 6.36      | 2       | 40.01    | 1676.1   |
| 4      | 6.88      | 1       | 100.22   | 1559.6   |
| 5      | 6.19      | 0       | 23.23    | 1862.2   |
| 6      | 6.06      | 3       | 38.22    | 1684.9   |
| 7      | 6.93      | 3       | 41.68    | 1815.3   |
| 8      | 7.28      | 0       | 144.13   | 1946.0   |
| 9      | 6.69      | 2       | 46.65    | 1821.7   |
| 10     | 7.68      | 0       | 106.71   | 1659.6   |

Table 3.1. Ground motion records for soil type A

|  | Table 3.2. | Ground | motion | records | for soil | l type B |
|--|------------|--------|--------|---------|----------|----------|
|--|------------|--------|--------|---------|----------|----------|

| Record | Moment    | Failure | J-B      | Vs (m/s) |
|--------|-----------|---------|----------|----------|
|        | Magnitude | Туре    | distance |          |
| 1      | 6.63      | 0       | 129.11   | 1328.6   |
| 2      | 6.5       | 2       | 49.13    | 1274.4   |
| 3      | 6.88      | 1       | 82.6     | 1274.4   |
| 4      | 6.06      | 3       | 27.21    | 1112.4   |
| 5      | 7.14      | 0       | 45.16    | 1413     |
| 6      | 7.13      | 0       | 41.82    | 814.3    |
| 7      | 6.36      | 2       | 45.49    | 1315     |
| 8      | 6.36      | 2       | 33.42    | 1015.6   |
| 9      | 6.88      | 1       | 83       | 1274.4   |
| 10     | 6.06      | 3       | 29.56    | 1015.6   |

 Table 3.3. Ground motion records for soil type C

| Record | Moment    | Failure | J-B      | Vs (m/s) |
|--------|-----------|---------|----------|----------|
|        | Magnitude | Туре    | distance |          |
| 1      | 6.61      | 2       | 173.16   | 388.5    |
| 2      | 7.35      | 2       | 24.07    | 368.2    |
| 3      | 6.9       | 1       | 29.79    | 532.7    |
| 4      | 6.2       | 1       | 22.68    | 530      |
| 5      | 6.36      | 2       | 44.82    | 438.5    |
| 6      | 6.7       | 0       | 65.67    | 548.6    |
| 7      | 6.19      | 0       | 29.35    | 738.5    |
| 8      | 6.93      | 3       | 40.85    | 449.6    |
| 9      | 7.28      | 0       | 62.98    | 645.4    |
| 10     | 6.4       | 1       | 79.33    | 598.6    |

| Table 3.4  | Ground     | motion | records  | for soil | type D |
|------------|------------|--------|----------|----------|--------|
| 1 and 5.7. | v ii o unu |        | TCC/UIU3 | 101 5011 |        |

| Record | Moment    | Failure | J-B      | Vs (m/s) |
|--------|-----------|---------|----------|----------|
|        | Magnitude | Туре    | distance |          |
| 1      | 7.36      | 2       | 114.62   | 253.2    |
| 2      | 6.19      | 0       | 17.64    | 327.1    |
| 3      | 6.63      | 0       | 45.12    | 180.8    |
| 4      | 6.5       | 2       | 33.32    | 319.6    |
| 5      | 7.35      | 2       | 193.91   | 219.6    |
| 6      | 6.2       | 1       | 43.5     | 359.6    |
| 7      | 6.93      | 3       | 24.27    | 191.8    |
| 8      | 6.69      | 2       | 82.03    | 296.6    |
| 9      | 7.62      | 3       | 24.98    | 188.1    |
| 10     | 6.9       | 1       | 22.54    | 320      |

| Record | Moment    | Failure | J-B      | Vs (m/s) |
|--------|-----------|---------|----------|----------|
|        | Magnitude | Туре    | distance |          |
| 1      | 6.9       | 0       | 21.35    | 151.2    |
| 2      | 7.51      | 0       | 68.09    | 175      |
| 3      | 6.9       | 0       | 19.14    | 164      |
| 4      | 6.93      | 3       | 77.32    | 155.1    |
| 5      | 6.93      | 3       | 94.56    | 169.7    |
| 6      | 6.93      | 3       | 43.77    | 116.4    |
| 7      | 6.19      | 0       | 51.68    | 146      |
| 8      | 5.9       | 2       | 56.17    | 176      |
| 9      | 5.9       | 2       | 109.02   | 173      |
| 10     | 5.9       | 2       | 124.87   | 156      |

Table 3.5. Ground motion records for soil type E

# 3.1 Failure mechanism

The selected ground motion records were generated by four different failure mechanisms: Longitudinal (0), Normal (1), Reverse (2) and Oblique (3). In the selection of the record accelerations there were considered an average of 12 records for each type of failure, so that ANN could generalize the estimation of spectra corresponding to different type of failure mechanisms. This is very helpful in order to reduce the error in the estimation of the inelastic response spectra.

# 3.2. Soil type

In this study five soil types have been considered as a function of the shear-wave velocity. Table 3.6 shows the soil type as a function of the shear-wave velocity, as classified by NEHRP. Note that soil type A represents a very hard soil and soil type E corresponds to a soft soil. Ten records were selected for each soil type.

| Soil | Soil      | Vs (m/s)   |
|------|-----------|------------|
| Туре | Condition | v s (m/s)  |
| Α    | Hard Rock | > 1500     |
| В    | Rock      | 760 - 1500 |
| С    | Soft Rock | 360 - 760  |
| D    | Deep Rock | 180 - 360  |
| Е    | Clay      | < 180      |

Table 3.6. Soil types under consideration

#### 4. ARTIFICIAL NEURAL NETWORK MODEL

The objective of this part of the study is to implement Artificial Neural Networks to estimate inelastic response spectra using the Multilayer Perceptron (Backpropagation) algorithm. To achieve this objective; in first place, appropriate parameters must be selected for training, and in second place we try to find the optimal network architecture. The most important limitation of ANN is that their efficiency depends on the training algorithm and the network architecture. Unfortunately there are no guidelines for determining these characteristics. By using the procedure of trial and error it is possible to try to find the optimal network, but this approach does not guarantee that the network is optimal. In order to obtain satisfactory results with the Backpropagation algorithm it is necessary that the values of the training vector represent the entire domain of the output. The proper selection of training parameters is an important factor in training artificial neural networks.

The seismic response spectrum can be displayed graphically as an element that integrates the maximum response of a single-degree-of-freedom system having certain level of damping and

ductility when exposed to an excitation at the base. The seismic response may be defined by different parameters (displacements, velocities, etc.). In this study, pseudo-acceleration will be used as a parameter to estimate the earthquake response spectra, consequently the parameter of the output network will be the pseudo-acceleration, which is directly related with the structural resistance.

# 4.1 Estimation of input and output parameters for the ANN

An appropriated selection of the parameters that represent an ANN is fundamental for the functioning of the network. The parameters needed to estimate the seismic response spectra in terms of pseudo-acceleration are: magnitude, mechanism of failure, J-B distance, shear-wave velocity, fundamental period of vibration of the system, and ductility. According to the above, inputs to the network are defined by the values of M, MF, J-B, Vs, To and Du, respectively. The output node is represented by the pseudo-acceleration (Sa). The values considered for the input parameters are shown from Tables 3.1 to 3.6, and the range of the input values of the network are shown in Table 4.1.

 Table 4.1. Range of input parameters of the neural network

|    | Input | Range        |
|----|-------|--------------|
| М  |       | 5.9-7.7      |
|    | MF    | 0-3          |
|    | JyB   | 18 a 194 Km  |
| Vs |       | 116-1946 m/s |
|    | То    | 0-3 sec      |
|    | Du    | 1-6          |

# 4.2 Architectures

A large number of architectures was tested in order to obtain the optimum ANN model architecture. The format for the arrangements of the architectures shown in the first column of Table 4.2 is as follows:  $IxH_1xH_2x \dots H_ixO$ . Where I represents the number of neurons in the input layer;  $H_i$  represents the number of neurons in the output layer. Table 4.2 shows some of the models that were used. In this table we can see that models with more than one hidden layer have a smaller total error in the training phase as compared with those with one hidden layer total error than models with one hidden layer, this is because the model is overtrained and the network could approach efficiently to estimate the spectra in the training phase; nevertheless, the last is not valid in the testing phase; which implies that the network could not be generalized.

| Table 4.2. Elaborated architectures of ANN |            |       |  |  |  |
|--------------------------------------------|------------|-------|--|--|--|
| Architecture                               | Iterations | Total |  |  |  |
|                                            | iterations | error |  |  |  |
| 6x50x1                                     | 73         | 0.13  |  |  |  |
| 6x75x1                                     | 56         | 0.12  |  |  |  |
| 6x100x1                                    | 165        | 0.082 |  |  |  |
| 6x125x1                                    | 186        | 0.093 |  |  |  |
| 6x30x60x1                                  | 338        | 0.095 |  |  |  |
| 6x50x60x1                                  | 498        | 0.087 |  |  |  |
| 6x90x60x1                                  | 657        | 0.084 |  |  |  |
| 6x100x50x1                                 | 865        | 0.077 |  |  |  |
| 6x30x30x30x1                               | 1823       | 0.069 |  |  |  |
| 6x50x50x50x1                               | 1435       | 0.073 |  |  |  |
| 6x100x100x100x1                            | 2181       | 0.057 |  |  |  |

Table 4.2. Elaborated architectures of ANN

It turned out that the optimal architecture for this problem is a multilayer neural network with feedforward. The selection of the optimal architecture of an ANN is not an easy task as it is necessary to test a large number of architectures to achieve one as the best. Currently, there are training methods of ANN based on evolutionary algorithms that are able to find the optimal architecture in a more efficient way than the trial an error process. In this paper, it was selected that the optimal model architecture consists of one input layer, one hidden layer and output layer, with the following topology: 6x100x1 it means that the ANN has 6 neurons in input layer, 100 neurons in the hidden layer and one neuron in the output layer (see Table 4.2).

# 5. ANN MODEL (TRAINING AND TESTING PHASES)

As it was mentioned before, Backpropagation algorithm was selected for the training phase. Transfer functions in the network layers were: sigmoid for the hidden layer and linear function for the output layer. During training it was found that the ANN had trouble for estimating the inelastic response spectra so it was necessary to test a large number of architectures. In order to train and test the ANN models a computer program was developed that includes routines for Matlab Neural Network Tool Box (Demuth et al. 2009). The selection of training parameters was such that the ANN had a predictive capability to predict events not used during training. Fig. 5.1 compares the regression results in the networks training, which compares the value estimated by the network and the actual value. While Fig. 5.1b shows the model 6x100x100x100x1, Fig. 5.1a illustrates the architecture 6x100x1. The regression line indicates the precision of the ANN in the estimation of the actual value of Sa. The error in the training phase can be seen in the slope of the line, an ideal model of ANN should have 1:1 slope. It is noticed that for case b the error estimated in the training is slightly smaller than for case a, however, in the test phase the error produced by the network for case b was much greater than for case a. This is due to overlearning. The overlearning problem was observed in all the networks with two or more hidden layers.



Figure 5.1. Comparison of the regression results of the ANN for the training phase

# 5.1 Training results

Figs. 5.2a to 5.2d compare the inelastic seismic response spectra for different ductility levels and soil types obtained during the training phase (dashed line) with the actual spectra (solid line). The training results stage shows an acceptable approximation of the predicted with respect to the actual spectra. The maximum error between the actual response spectra and using ANN was 12%. It is observed that the spectral shape presents a clear definition throughout the range of periods. This is very important because the spectral shape of the pseudo-acceleration spectrum is an indicator of the ground motion potential of an earthquake (Bojórquez and Iervolino 2011).



Figure 5.2. Comparison of earthquake response spectra obtained during the training phase (ANN vs actual).

#### 5.2 Test results

After the training gives adequate results, in the test phase some records are randomly chosen from the NGA Database in order to test the network (these records were never shown to the ANN model). The test phase results are shown in Figs. 5.3a to 5.3d. Note that in the testing stage the ANN results do not reach the levels obtained during the training stage; however, the results still show a good approximation between the predicted spectra via ANN and the actual spectra, and this is valid for different type of soils and ductility levels. The mean square error between model and actual spectra ANN is about 15%. Results in Figs. 5.3a to 5.3d are presented for zone A, B, C and E, respectively, using only one record for each one of them. Similar results were found for other records analyzed.

The results of the ANN model to estimate inelastic response spectrum corresponding to different acceleration records are acceptable. It is noticed that the error in the estimation increases when ductility tends to increase. When estimating response spectra for different soil types, the network has more difficulties in estimating spectra in soil type E than in type A. This means that the estimation of the response spectra for soft soils generates a higher error than for intermediate or for hard soils. The most important observation is that the ANN model retains all the shape characteristics of the earthquake inelastic response spectra.



Figure 5.3. Comparison of earthquake response spectra obtained during the testing phase (ANN vs actual)

# 6. CONCLUSIONS

An Artificial Neural Network model to estimate inelastic seismic response spectra taking into account different type of soils and seismic zones in the world, is proposed. The moment magnitude of the event, mechanism of failure, Joyner and Boore distance, shear-wave velocity, fundamental period of the structure, and ductility were selected as the inputs for the ANN model. The output was represented by the spectral pseudoacceleration. The input parameters selected are usually used to propose ground motion attenuation prediction equations.

It is concluded that the ANN model proposed to estimate inelastic seismic response spectra has acceptable concordance between the actual spectra and those obtained by the ANN approach. The ANN has good approximation for estimating the elastic response spectra, as these present the smallest error. In the case of the inelastic spectra, the error increases when the ductility grows; however, the estimation of inelastic spectra using the ANN model is acceptable; and more important, it preserves the spectral shape of the actual response spectra.

It is noteworthy that for the successful operation of the ANN model it is essential to select properly the architecture, method and learning algorithms, as well as the parameters that represent it.

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