Comparison of added mass method with sophisticated analytical BEM-FEM approach using ADAD-IZIIS software

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SUMMARY

This paper presents a comparison of dynamic response of a fluid-arch dam coupled system obtained by use of the classical Westergaard added mass formulation and the more advanced analytical FEM-BEM technique. Herein the influence of the applied techniques on the distribution and intensities of the manifested stresses is presented within the dam body. Dynamic response is achieved assuming that the arch dam has a linear elastic material behavior. For solving the dynamic equation of motion direct integration technique is used, whereat the dam foundation, as well as foundation undergoing the reservoir, is assumed rigid. The FSI is performed following the assumptions that the water in the reservoir is inviscid with irrotational motion of the water particles limited to small amplitudes of velocities and acceleration while the gravity surface waves are neglected. The presented results are obtained by use of FE–BE oriented software ADAD-IZIIS, especially written for analyses of arch dams. The software also has the possibility for calculation of hydrodynamic effects according to Westergaard added mass method.

Keywords: fluid-arch dam interaction, BEM-FEM, added mass, ADAD-IZIIS software, hydrodynamic pressure

1. INTRODUCTION

The phenomenon of FSI that occur during the seismic response of the coupled dam-reservoir system has been for the first time physically explained and mathematically solved by Westergaard (1933). The very first dynamic analyses, considering the hydrodynamic inertial forces, were based on application of added mass method. Besides its simplicity in application and popularity among the analysts, Kuo (1982) and Kotsubo (1960) have emphasized that evaluation of hydrodynamic pressure using the added mass approach is not accurate, since the assumptions used for calculation of the added mass are not in accordance with random nature of the earthquake phenomenon. This type of analysis leads to a conservative design. There is a common belief that in case of a rigid structure, the magnitude of the hydrodynamic pressure becomes high, but this is not always true. The magnitude of hydrodynamic pressure may increase significantly for flexible structures as well. If resonance effect and the energy release mechanism in the fluid occur it can lead to the development of unsound design. Thus it is necessary to study the fluid–structure interaction problem considering the flexibility property of the structure that may alter the behavior of the fluid domain significantly. However, the added mass method is very effective for calculation of Eigen values for coupled dam-reservoir systems. During the last decade consequent and extensive research was carried out for development of more sophisticated approaches and analytical methods aimed towards overcoming of disadvantages of the added mass method as well as lightening the impact of the characteristic features of this phenomenon over the calculation results. They are divided into solution methods concerning FEM-FEM or BEM-FEM numerical techniques based on time or frequency domain approach, Seghir et al.

(2009), Tsai and Lee (1987), Tsai (1992), Tsai et al. (1992). This paper deals with coupled BE-FE analysis of the fluid–structure systems considering the coupled effect of elastic structure and an incompressible and inviscid fluid. The solution of the coupled system is accomplished by solving the motion of the two systems, the dam and the fluid in the reservoir, separately. The interaction effects at the fluid–solid interface are enforced by adding the matrix of hydrodynamic forces to the classic equation of dynamic motion of dam, ADAD-IZIIS (2008). Research was made for concrete arch dam with structural height of H=130m. The fluid–dam system was subjected in upstream direction to El Centro N-S record of the Imperial Valley earthquake. The first seven seconds of the record was used and scaled to the peak acceleration of 0.3g. The time step increment of 0.01s was chosen for the integration process. In a general fluid-soil-structure system, the structure and the reservoir domain are supported by the elastic soil medium. However, in the analysis presented in this paper the rock foundation is assumed to be rigid which means that the dam–foundation interaction is neglected and the motion of all nodes at the fluid–dam-rock interface are following the input ground accelerations without any modification. The dam properties are as follows: Young's modulus E = 31.5 GPa; mass density ρ = 2450 kg/m^3; Poisson's ratio ν = 0.2; the acoustic wave velocity in water c = 1440 m/s.

2. GOVERING EQUATIONS

2.1 Governing equations for coupled BE-FE FSI solution

The incremental form of differential equation of motion for system subjected to the dynamic action including the effect of fluid-dam interaction is as follows:

$$[M]\Delta\ddot{U}(t)+[C]\Delta\dot{U}(t)+[K]\Delta U(t)=-[M]\cdot[R]\Delta a_g(t)+\Delta HDF(t) \tag{2.1}$$

where [M] is the mass matrix, [R] is the acceleration transformation matrix, [C] is the structural damping matrix, [K] is the structural stiffness matrix, $\Delta U(t);\Delta\dot{U}(t);\Delta\ddot{U}(t)$ are vectors of nodal incremental displacements, velocities and accelerations, relative to the ground, $\Delta a_g(t)$ is the vector of incremental ground accelerations, $\Delta HDF(t)$ is time dependant vector of incremental nodal forces at the dam-fluid interface caused by the propagation of the acoustic waves generated in the reservoir domain. Wilson-θ method is used for the solution of the general equation of motion Eqn. 2.1 and for θ=1.38 unconditional stability of the direct integration process is achieved. This method requires the damping matrix to be represented in explicit form. This is accomplished using Rayleigh damping. According to Rayleigh, the structural damping in the system can be included by the Eqn. 2.2.

$$[C]=\alpha_R[M]+\beta_R[K] \tag{2.2}$$

The Rayleigh damping coefficients α_R and β_R are determined using two known damping ratios ξ_i along with the corresponding eigenvalues ω_i Eqn. 2.3 chosen to present energy dissipation ability of the fluid-structure system in the best way.

$$\alpha_R=\frac{2\omega_1\omega_2^2\xi_1-2\omega_2\omega_1^2\xi_2}{\omega_2^2-\omega_1^2} \qquad \beta_R=\frac{2\omega_2\xi_2-2\omega_1\xi_1}{\omega_2^2-\omega_1^2} \tag{2.3}$$

During the dynamic response, at any time step, the vector of incremental hydrodynamic forces is directly added to the vector of seismic force. The vector of HDF is defined applying BEM numerical technique for solution of Laplace's equation of fluid motion as well as a specially written algorithm within the software ADAD-IZIIS.

The governing equation for solving the small amplitude irrotational motion of the impounded incompressible and inviscid fluid is governed by the three-dimensional Laplace's equation as follows:

$$\frac{\partial^2 W}{\partial x^2}+\frac{\partial^2 W}{\partial y^2}+\frac{\partial^2 W}{\partial z^2}=0 \tag{2.4}$$

where W(x, y, z) is the function of the potential in the fluid domain with boundary surfaces Γ_1 and Γ_2 where essential and natural type of boundary conditions exist. Applying BE technique, the discretization of boundary surfaces is by an assembly of eight nodded quadratic “serendipity” type of boundary elements as follows:

$$\frac{1}{2}W_i+\sum_{nel=1}^{NEL}\int_{\Gamma 1}\left(\frac{\partial W}{\partial n}p-\overline{W}\frac{\partial p}{\partial n}\right)d\Gamma_1+\sum_{nel=1}^{NEL}\int_{\Gamma 2}\left(\frac{\partial \overline{W}}{\partial n}p-W\frac{\partial p}{\partial n}\right)d\Gamma_2=0 \tag{2.5}$$

Where: i=1, NBE; NBE - number of nodes in the boundary element model

Father on, despite “direct” Czygan and Estorff (2002), Yu et al. (2001), Estorff and Antes (1991), Fukui (1987), Karabalis and Beskos (1985), or “iterative coupling” methods Soares et al. (2005), Soares and Estorff (2004), Soares et al. (2004), (2005), the software utilize simple and effective numerical technique of matrix of hydrodynamic influence that provides a compatible link of both media which are in interaction. It is accomplished utilizing the concept of virtual work of unit acceleration.

2.2 Governing equations of dynamic motion in case of application of added mass method

If added mass method is used for solving FSI then it is necessary to calculate the amount of the virtual mass fixed against the upstream face of the dam. The added mass should produce inertial forces equivalent to the expected hydrodynamic effects at any time increment. The procedure for this calculation is given in section 3. If added mass method is used then the vector of incremental nodal hydrodynamic forces in Eqn. 2.1 is replaced by the modified mass matrix. The structural mass matrix is modified by the increment of the added mass that produces equivalent increment of hydrodynamic force during the dynamic motion. Therefore, the differential equation of dynamic motion Eqn. 2.1, acquires the following form:

$$[M+Ma]\cdot\Delta\ddot{U}(t)+[C]\cdot\Delta\dot{U}(t)+[K]\cdot\Delta U(t)=-[M+Ma]\cdot[R]\Delta a_g(t) \tag{2.6}$$

The added mass method implies rearranging of the incremental differential equation. After some transformations it acquires the following form:

$$(M+M_A)\left[\frac{6}{\tau_k^2}\hat{\Delta}U_i-\frac{6}{\tau_k}\dot{U}_i-3\ddot{U}_i\right]+[\alpha M+\beta K_i]\cdot\left[\frac{3}{\tau_k}\hat{\Delta}U_i-\frac{\tau_k}{2}\ddot{U}_i-3\dot{U}_i\right]+K_i\hat{\Delta}U_i=\hat{\Delta}P_i \tag{2.7}$$

After arranging known and unknown terms Eqn. 2.6 becomes:

$$\overline{K_i}\hat{\Delta}U_i=\overline{\hat{\Delta}F_i} \tag{2.8}$$

where,

$$\overline{K_i}=K_i+\frac{6}{\tau_k^2}(M+M_A)+\frac{3}{\tau_k}[\alpha M+\beta K_i]_i \tag{2.9}$$

$$\overline{\hat{\Delta}F_i}=\hat{\Delta}P_i+(M+M_A)\cdot\left[\frac{6}{\tau_k}\dot{U}_i+3\ddot{U}_i\right]+[\alpha M+\beta K_i]\cdot\left[\frac{\tau_k}{2}\ddot{U}_i+3\dot{U}_i\right] \tag{2.10}$$

Eqn. 2.8 expresses the dynamic equilibrium of a linear or nonlinear system at the "i"-th time increment and it has the same form as the static equilibrium equation.

3. WESTEGAARD SOLUTION OF HDP – ADDED MASS METHOD

When Westergaard formulation is applied when solving the FSI of arch dam, there is a need for some modification of basic Westergaard assumption. Firstly, it should be considered that earthquake motion is not normal to the dam face. In general the upstream face of the dam is double curved and therefore, the orientation of the faces relative to the ground motion varies from point to point. Also recognition should be given to the influence of dam flexibility over the magnitudes of the manifested hydrodynamic effects. Namely the relative response acceleration of dam face varies from point to point and differs from the earthquake acceleration. According the classic Westergaard solution, the pressure caused by unit acceleration at any point "i" is expressed by:

$$f_{ni} = \frac{8\alpha wh}{\pi^2} \cos\frac{2\pi t}{T} \sum_{1,3,5}^{n} \frac{1}{n^2 c_n} e^{-q_n} \sin\frac{n\pi y_i}{2h} \tag{3.1}$$

where: $$q_n = \frac{n\pi c_n x_{TS}}{2h}, \qquad k = v_s^{\,2}\rho, \qquad c_n = \sqrt{1-\frac{16h^2}{n^2 l^2}} = \sqrt{1-\frac{0.71889}{n^2}\left(\frac{h\,\mathrm{sec.}}{1000\mathrm{Tft.}}\right)^2} \tag{3.2}$$

The parameters in the above expressions are well known Westergaard (1933). Calculation is based on use of the first 10.000 terms of infinite row of Westergaard solution. The period of propagation of the acoustic waves is selected from the Fourier spectrum of the excitation and amounts to T=0.5sec.
The normal acceleration $\ddot{r}_{ni}$ due to Cartesian components is given by the direction cosines between the coordinate axis and the normal.

$$\underline{\ddot{r}_{ni}} = \underline{L_{ni}}\,\underline{\ddot{r}_i} \qquad \underline{L_{ni}} = \{\lambda_{xn}\lambda_{yn}\lambda_{zn}\} \tag{3.3}$$

The normal force at point "i" is obtained as follows:

$$P_{ni} = F_{ni}\,\underline{L_{ni}\,\ddot{r}_i} \tag{3.4}$$

In order to use finite element analysis this normal force must be resolved into Cartesian components:

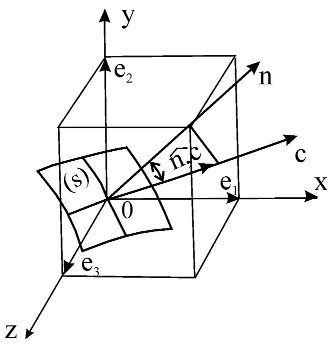
$$\underline{P_i} = \underline{L_{ni}}^T P_{ni} = \underline{L_{ni}}^T F_{ni}\,\underline{L_{ni}}\,\underline{\ddot{r}_i} \tag{3.5}$$

Hence the diagonal terms of the added mass matrix are as following:

$$m_{ai} = \underline{L_{ni}}^T F_{ni}\,\underline{L_{ni}}\,\underline{\ddot{r}_i}/\rho \tag{3.6}$$

Here $\underline{m}_{ai}$ is a diagonal (lumped) mass matrix for each upstream node, and is to be added appropriately to the general mass matrix in order to obtain the full inertial effect. The added mass exhibits some properties of its own. Thus, in distinction of the structural mass, this value is independent of the direction of the degrees of freedom, the value of the added mass depends on the direction in which the excitation develops and the direction of the degrees of freedom. Since the earthquake is assumed to travel upon some direction c and if the structural degrees of freedom are in directions x, y, z, and then the components of the lumped mass have the following expressions:

$$m^h{}_{ni} = F_{ni}/\rho \tag{3.7}$$



$$m^{h}_{cxi} = m^{h}_{ni}\cos(n,c)\cos(n,x)$$
$$m^{h}_{cyi} = m^{h}_{ni}\cos(n,c)\cos(n,y)$$
$$m^{h}_{czi} = m^{h}_{ni}\cos(n,c)\cos(n,z)$$

Figure 1. Computation diagram for added mass of water

3.1 Derivation of approximate Westergaard formulas

For the purpose of approximate computation one might replace the infinite row of Westergaard solution Eqn. 3.1 by a parabola, even if this parabola has a sloping, not vertical, tangent at the bottom.

$$p = C\alpha\sqrt{hy} \qquad C = \frac{P}{\alpha\sqrt{hy}} \tag{3.8}$$

The main source of variation of C is the constant c_1 with n=1, in Eqn. 3.9. As c_1 appears as denominator in the first term in each of the sums in eq. 3.1, it is reasonable to express C via expression Eqn. 3.9, where K is a constant.

$$C = \frac{K}{c_1} \tag{3.9}$$

Using the fact that the coefficient C depends on the ratio of the impounded water depth h and predominate period of excitation T, and by inspection of the numerical results obtained by application of the Westergaard equations [1], it has been concluded that the most convenient value for the coefficient K is K=0.0255 ton/ft^3.
Accordingly the approximate formula for hydrodynamic pressure calculation is:

$$p = C\alpha\sqrt{hy} = \frac{0.0255\,ton/ft^3}{\sqrt{1-0.72\left(\frac{h\sec}{1000Tft}\right)^2}}\alpha\sqrt{hy} \tag{3.10}$$

For the analysis presented herein the value of the coefficient C is C=12.4 kN/m^3.

4. DISCUSSION OF THE RESULTS - WESTEGAARD VERSES BE-FE METHOD

If flexible structure is considered, like an arch dam, added mass principle, according to Westergaard does not give accurate resultants, because Westergaard theory is based on straight rigid dam assumption. Since software ADAD-IZIIS have the possibility for calculation of hydrodynamic pressure according to Westergaard added mass method and according to coupled FE-BE method, fallowing figures present the discrepancies in obtained resultants.
Figures 2, 3, and 4. present the time history responses of relative displacement, velocity and acceleration for selected node in X, Y and Z direction respectively, at dam crest (crown cantilever) where the extremes of the response occurred. Obviously, the response acceleration of the dam is modified by 37% when BEM-FEM numerical method is used for solving FSI. Westergaard added mass method gives 28% modification of the dynamic response of the dam. The flexibility property of

the structures and the influence of the reservoir domain alter the behavior of the fluid significantly and consequently the coupled system has a stronger response. Figure 5. presents the time histories of the three principal stresses at the nodes where the extreme of each stress is achieved when FSI effect is omitted. It also presents the impact of the FSI over the time histories of these stresses and its extremes. Figure 7. presents a snapshot of hydrodynamic pressure distribution over the interface, at time T=4.95sec. It is obtained under the assumptions that the topology of the canyon has a regular shape as indicated in the drawing. However, the insight in the developed hydrodynamic effects is not possible when added mass method is used. According to presented results herein, it is evident that added mass method is not applicable to flexible systems, because the discrepancies are significant, which is best shown in figure 6. The figure shows diagrams of stress distribution with and without included hydrodynamic effects, whereat hydrodynamic effects are calculated according to added mass method and coupled BE-FE method. The stress extreme is increased by 15% if added mass method is used and 49% if coupled BE-FE method is used, which gives error of 23% using added mass method in arch dam design.

There is a common belief that in case of a rigid structure, the magnitude of the hydrodynamic effects becomes high, which can not be generalized. In order to confirm or deny this statement, other analyses have been done for the 70m depth of impounded water. As the dam is more rigid in the lower part and consequently the absolute accelerations at the dam-fluid interface are lower it appears that the above statement is almost valid for this particular case, Figure 8. Namely, despite the case of 100m water depth, here the mismatching of both approaches is negligible.

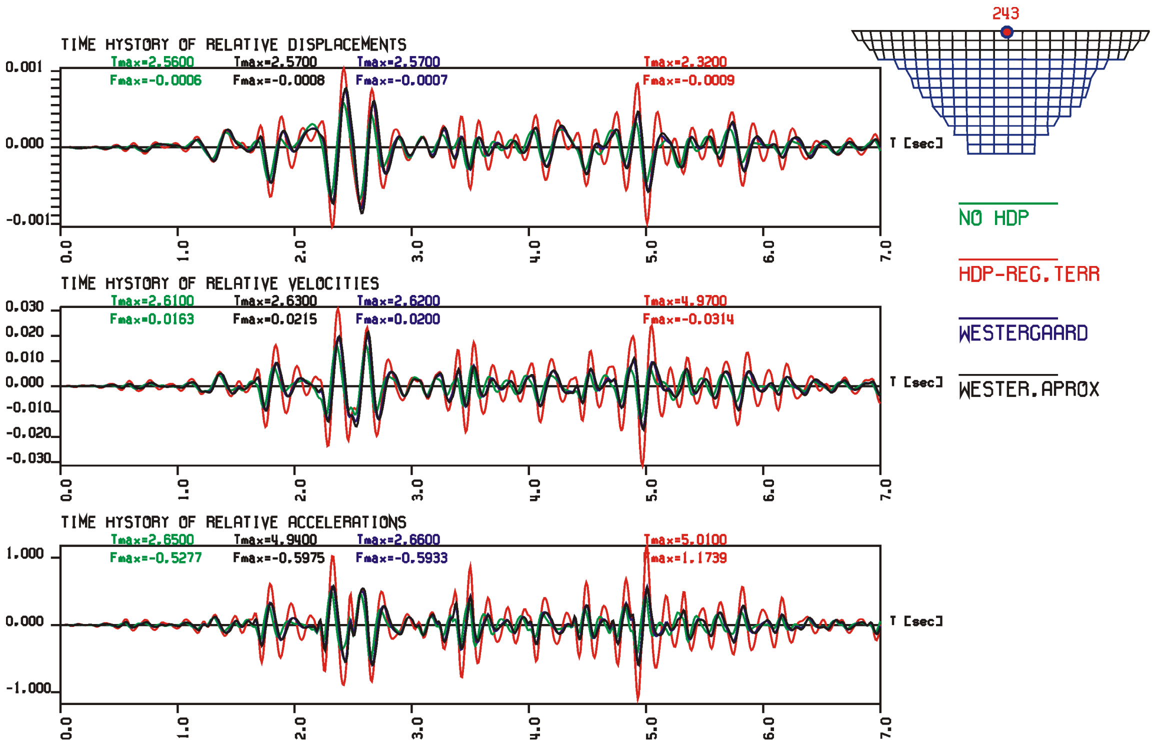


Figure 2. Modification of the dam response in X direction due to the FSI effect

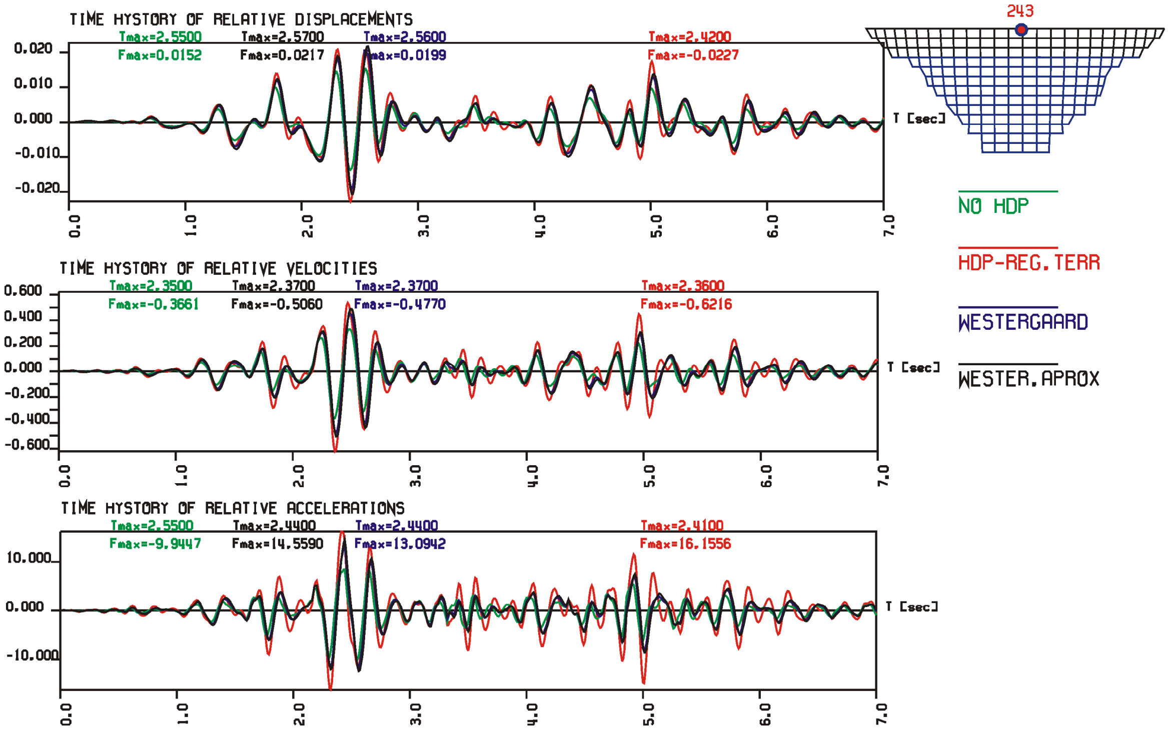


Figure 3. Modification of the dam response in Y direction due to the FSI effect

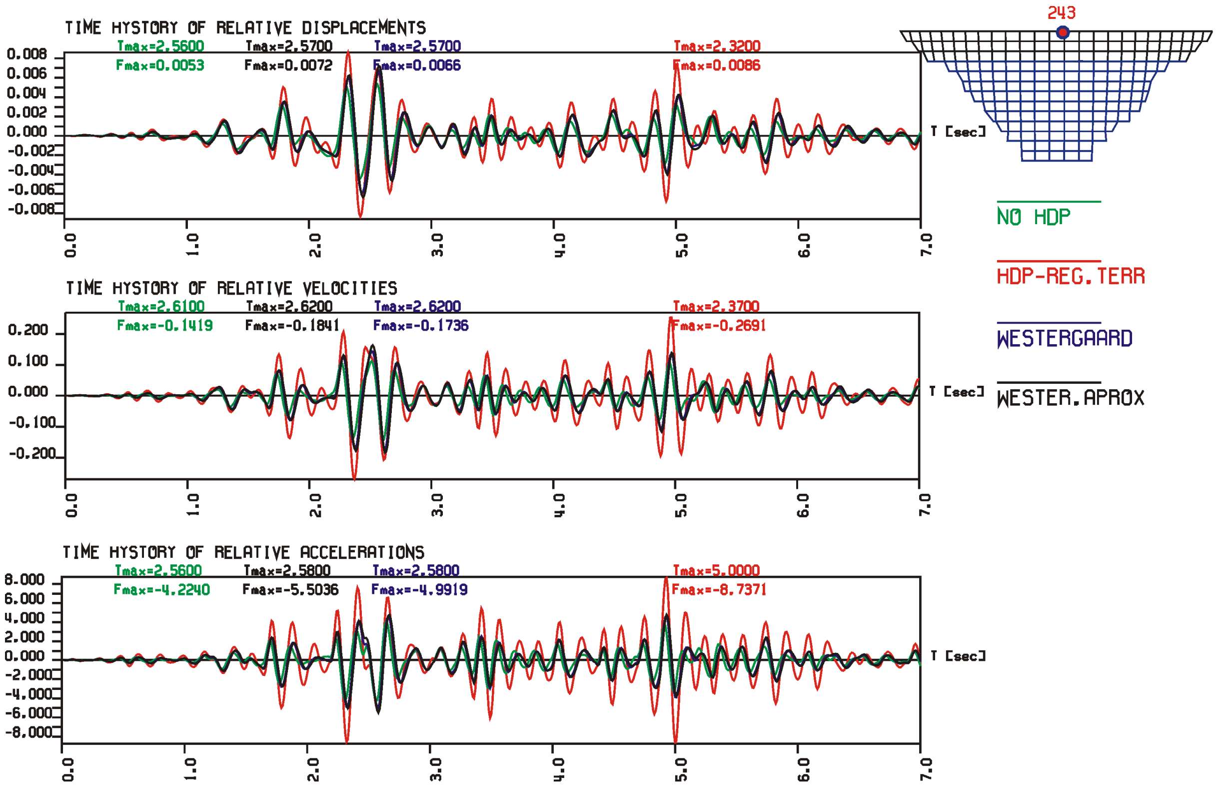


Figure 4. Modification of the dam response in Z direction due to the FSI effect

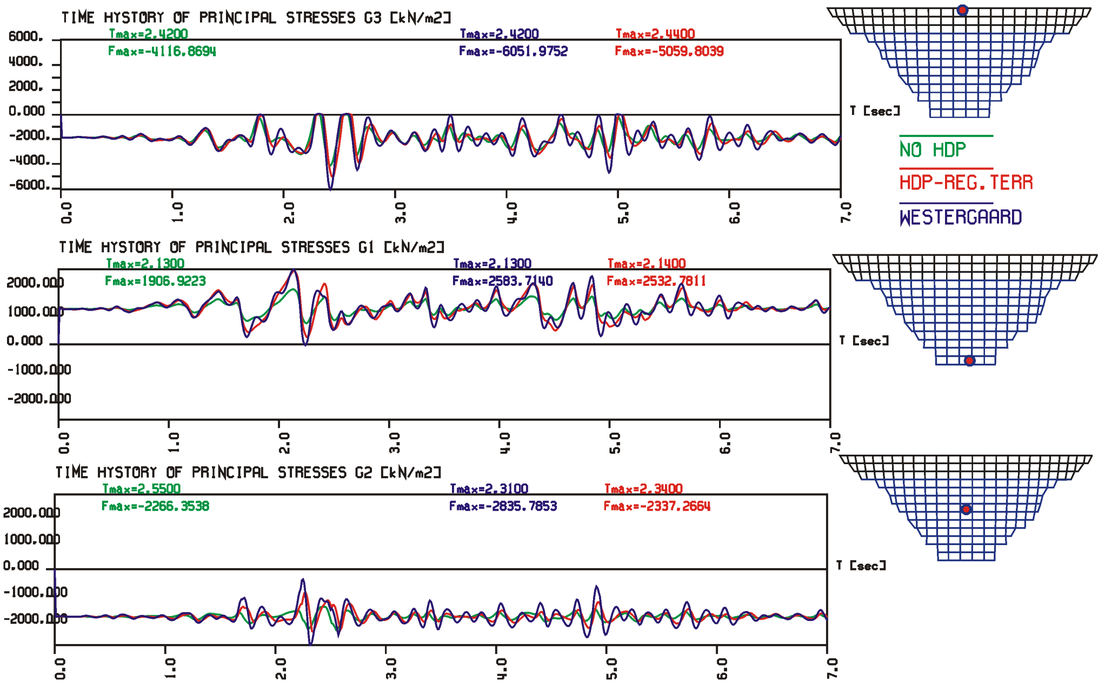
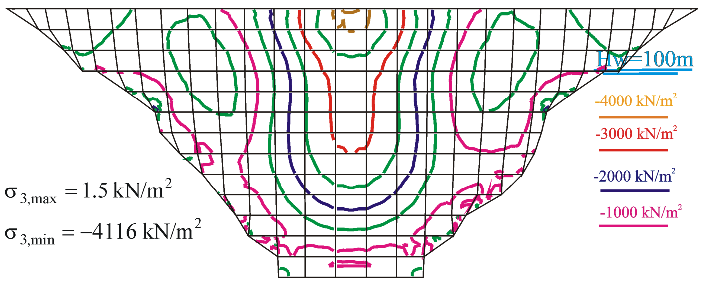
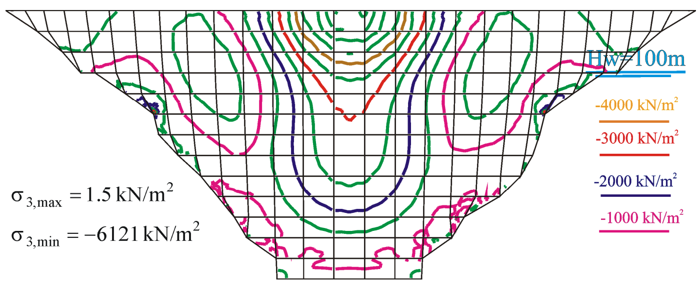


Figure 5. Time histories of extreme principal stresses with and without FSI effect

a) Stress distribution on dam extrados face G3, T=2.42 s, without HDE



b) Stress distribution on dam extrados face G3, T=2.42 s, with HDE using BEM-FEM



c) Stress distribution on dam extrados face G3, T=2.42 s, with HDE using added mass method

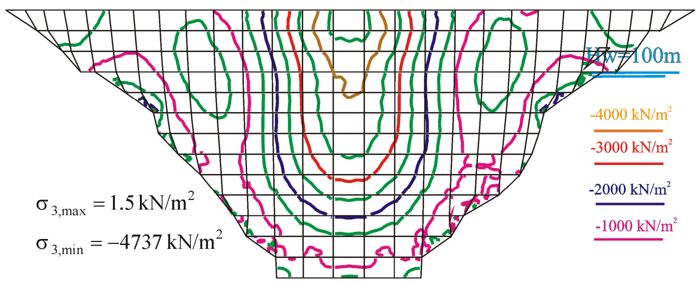


Figure 6. Stress distribution. Modification of the dam response due to the FSI effect

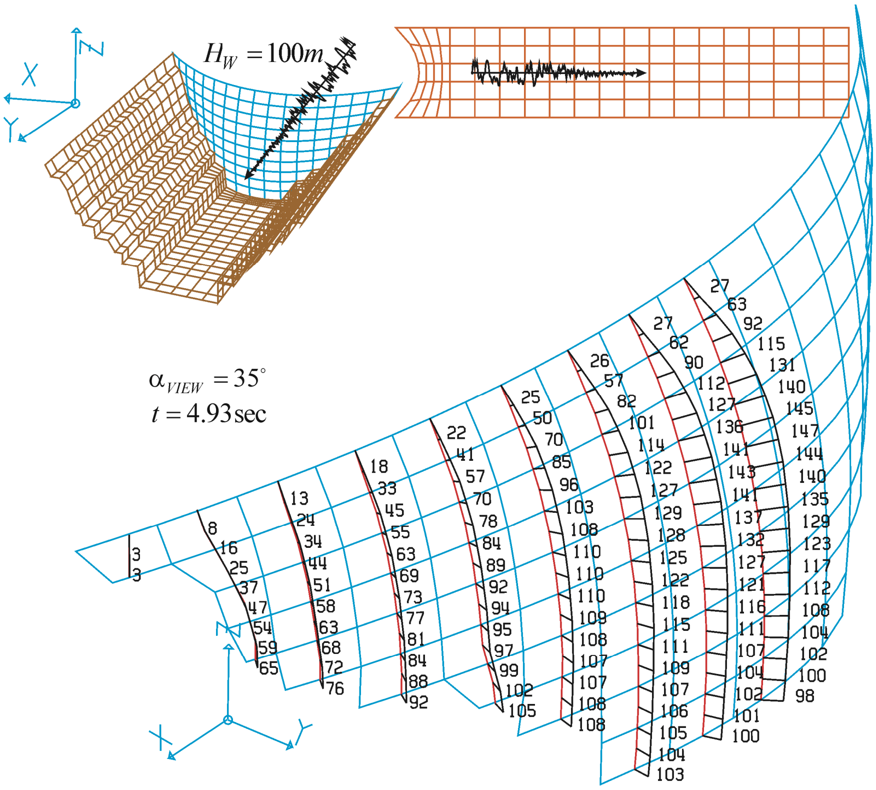


Figure 7. Snapshot of hydrodynamic pressure distribution at time T=4.95 sec

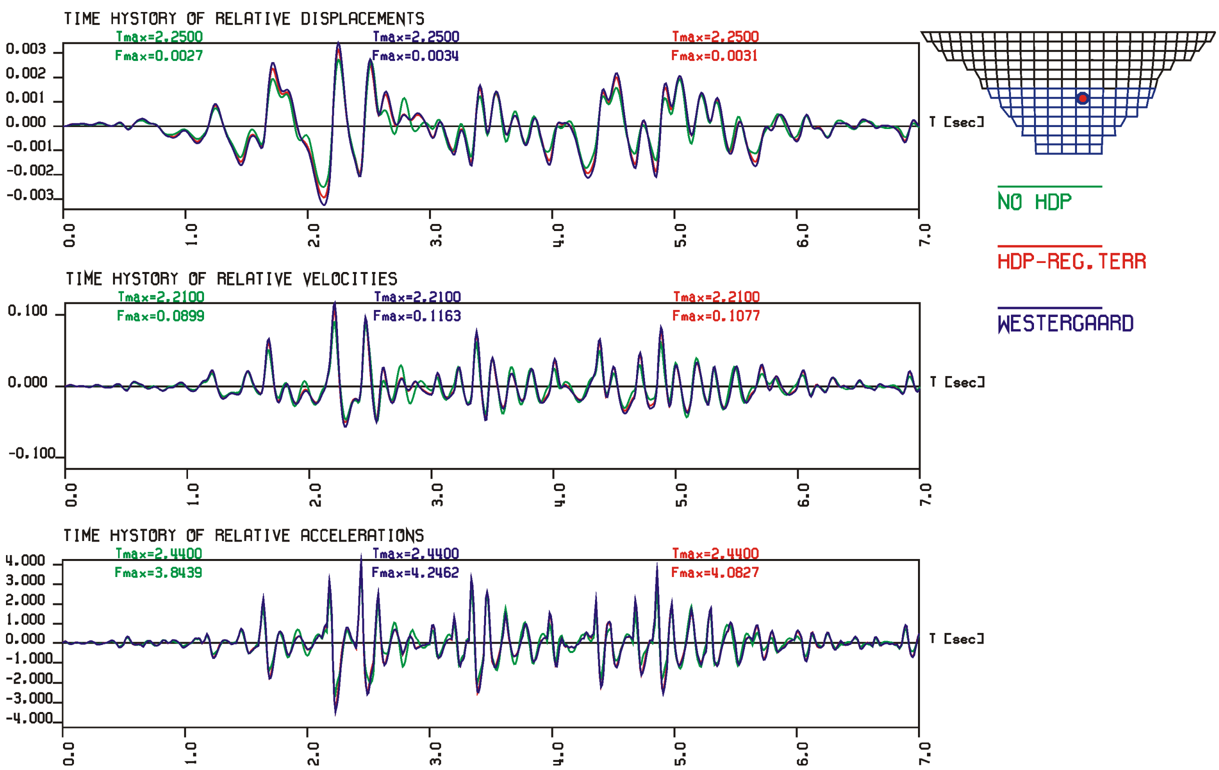


Figure 8. Modification of the dam response in Y direction due to the FSI effect

CONCLUSION

The Westergaard added mass method provides acceptable results only in the range of restricted hypothesis. Since it neglects the dam flexibility and water compressibility and does not require any discretization of the reservoir domain wherever these features have an impact on the magnitude of hydrodynamic effects there will be discrepancy of the obtained resultants. Recently develop BEM-FEM, FEM-FEM or hybrid methods are focused towards overcoming disadvantages of added mass method. They give far more realistic estimates of FSI effects. In this particular case whether added mass method would over or under estimate the hydrodynamic effects depends on the water depth in respect to the flexibility properties of the dam.

REFERENCES

Westergaard H.M. (1933). Water pressure on dams during earthquakes. *Transactions of the American Society of Civil Engineers,* **Vol. 98,** 418–472.

Kotsubo, S. (1960). Dynamic water pressure on dams during earthquake. *2nd World Conference on Earthquake Engineering*, 799- 814.

Kuo J.S.H. (1982). Fluid-structure interactions: Added mass computation for incompressible fluid. *UCB/EERC-82/09 Report*, University of California, Berkeley

Seghir A., Tahakourt A. and Bonnet G. (2009). Coupling FEM and symmetric BEM for dynamic interaction of dam–reservoir systems. *Engineering Analysis with Boundary Elements*. **Vol. 33**, 1201–1210.

Tsai C.S., Lee G.C. (1987). Arch dam–fluid interactions by FEM-BEM and substructure concept. *International Journal of Numerical Methods in Engineering*. **Vol. 24**, 2367–2388.

Tsai C.S. (1992). Analyses of three-dimensional dam-reservoir interaction based on BEM with particular integrals and semi-analytical solution. *Computers and Structures*. Vol. 43:5, 863-872.

Tsai C.S., Lee G.C. and Yeh C.S. (1992). Time-domain analyses of three-dimensional dam-reservoir interactions by BEM and semi-analytical method, *Engineering Analysis with Boundary Elements*. **Vol. 10:2**, 107-118.

Czygan O. and Estorff von O. (2002). Fluid–structure interaction by coupling BEM and nonlinear FEM, *Engineering Analysis with Boundary Elements*. **Vol 26**, 773–779.

Yu G., Mansur W.J., Carrer J.A.M., and Lie S.T. (2001). A more stable scheme for BEM/FEM coupling applied to two-dimensional elastodynamics. *Computers & Structures*. **Vol. 79**, 811–823.

Estorff von O. and Antes H. (1991). On FEM–BEM coupling for fluid–structure interaction analysis in the time domain. *International Journal for Numerical Methods in Engineering*. **Vol. 31**, 1151–1168.

Fukui T. (1987). Time marching BE–FE method in 2D elastodynamic problems. *In: Proceedings of the international conference on boundary element methods IX*, Stuttgart.

Karabalis D.L and Beskos D.E. (1985). Dynamic response of 3D flexible foundations by time domain BEM and FEM. *Soil Dyn Earthq Eng*. **Vol. 4**, 91–101.

Soares D.J., Carrer J.A.M. and Mansur W.J. (2005). Non-linear elastodynamic analysis by the BEM: an approach based on the iterative coupling of the D-BEM and TD-BEM formulations. *Engineering Analysis with Boundary Elements*. **Vol. 29**, 761–774.

Soares D.J. and Estorff von O. (2004). Combination of FEM and BEM by an iterative coupling procedure, *4th European Congress on Computational Methods in Applied Sciences and Engineering, ECCOMAS*, Jyva¨skyla¨ , Finland,.

Soares D. J., Estorff von O. and Mansur W.J. (2004). Iterative coupling of BEM and FEM for nonlinear dynamic analyses. *Computational Mechanics*. **Vol. 34**, 67–73.

Soares D. J., Estorff von O. and Mansur W.J. (2005). Efficient nonlinear solid–fluid interaction analysis by an iterative BEM/FEM coupling, International Journal for Numerical Methods in Engineering. Vol. 64, 1416–1431.

Mircevska V. and Bickovski (2008). ADAD-IZIIS software: Analysis and Design of Arch Dams, IZIIS- univ. “Ss. Cyril and Methodius”, User’s Manual,