Response Spectrum analysis for floor acceleration

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SUMMARY:
The peak floor acceleration (PFA) is a critical parameter influencing the performance of non-structural elements in buildings. This paper develops a response spectrum analysis method based on the Complete Quadratic Combination (CQC) rule to estimate the PFA. The method approximately accounts for the contribution of truncated higher modes. The approximation is introduced in the time domain, then formulated in the frequency domain by CQC. Application of the method to a continuous cantilever beam idealizing a building with shear walls is presented and compared with alternative formulations. The proposed method is able to provide a consistent estimation of the PFA along the entire structure, not only where the PFA is principally influenced by the first few flexible modes, but also where the PFA is mainly related to the rigid response of the structure, for example near the base.

Keywords: continuous structures, CQC, modal analysis, peak floor acceleration, response spectrum analysis.

1. INTRODUCTION

This paper presents a response spectrum analysis method to predict the peak floor acceleration (PFA) in structures under seismic excitation. Estimation of the PFA is required for the design and reliability assessment of acceleration-sensitive non-structural elements and floor diaphragms in buildings. More specifically, the PFA is a key engineering demand parameter in buildings, when floor diaphragms or attached equipment are intended to behave as rigid parts, having natural frequencies much higher than the dominant frequencies of the seismic excitation. Aslani and Miranda (2005) have shown that in non-collapse seismic events, the expected loss resulting from damage to acceleration-sensitive non-structural elements is somewhat higher than that in the structural elements, the biggest contributor being the loss due to drift-sensitive non-structural elements.

The use of PFA in the design of floor diagraphs is illustrated by Rodriguez et al. (2002). In that work, the authors propose a response spectrum method to estimate the PFA, following a numerical campaign on non-linear systems. The PFA at the top of the building is computed using the Squared Root of Sum of Squares (SRSS) modal combination rule. The contribution of the first mode is reduced to account for the effects of ductility and hysteretic behaviour, while the other modes are considered elastic. The profile of the PFA along the building height is approximated by a bi-linear function: Uniform acceleration for the top part of the building, and linear decay from mid-height to the base of the building, where the acceleration equals the Peak Ground Acceleration (PGA). The SRSS rule is also used by Kumari *et al.* (2007), who propose an approximate formula for estimating the PFA. Miranda and Taghavi (2005) also propose a method for estimating the PFA, approximating the first three modes of the building with those obtained by flexural and shear cantilever models. The PFA can also be computed by response history analysis for specified ground motions, possibly using non-linear hysteretic models, as in Medina and Krawinkler (2003).

In this paper, we limit our attention to linear, classically damped structures. The paper benefits from the earlier work of Der Kiureghian and Nakamura (1993), where a method for approximating the quasi-static contributions of truncated higher modes in response spectrum analysis by the CQC (Complete Quadratic Combination) rule is presented. The focus of that work was on response quantities related to relative displacements, such as inter-story drifts and strains or stresses in structural members. Although the paper never mentions floor accelerations, in the last section it reports a numerical example of computing floor pseudo-accelerations. Even though a part of what is presented here was implicitly included in that work, we find it useful to further investigate and shed light on the peculiarity of computing floor accelerations by response spectrum analysis in contrast to computing response quantities related to relative displacements.

2. MODAL COMBINATION FOR TOTAL ACCELERATION

2.1. Discrete system with complete modal information

Consider an N-degrees-of-freedom linear system with classical damping and subjected to a single component of ground motion. The well-known equation of motion is

$$\mathbf{M\ddot{u}} + \mathbf{C\dot{u}} + \mathbf{Ku} = -\mathbf{Mv}\ddot{u}_g(t) \tag{2.1}$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the N×N mass, damping and stiffness matrices, respectively, $\mathbf{u}$ is the N×1 vector of nodal displacements relative to the ground, $\mathbf{v}$ is the $N \times 1$ influence vector connecting the nodal displacements to the base motion, and $\ddot{u}_g(t)$ is the ground acceleration. Using modal decomposition, the solution for the nodal relative displacements can be written as

$$\mathbf{u}(t) = \sum_{i=1}^{N} \boldsymbol{\phi}_i \Gamma_i s_i\,(t) \tag{2.2}$$

where $\boldsymbol{\phi}_i$ denotes the i-th mode shape, $\Gamma_i = \left(\boldsymbol{\phi}_i{}^{\mathrm{T}}\mathbf{Mv}\right)/\left(\boldsymbol{\phi}_i{}^{\mathrm{T}}\mathbf{M}\boldsymbol{\phi}_i\right)$ is the participation factor for the i-th mode, and $s_i(t)$ is solution to the equation:

$$\ddot{s}_i + 2\zeta_i\omega_i\dot{s}_i + \omega_i^2 s_i = -\ddot{u}_g(t) \quad i = 1, \dots, N \tag{2.3}$$

in which ω_i and ζ_i respectively denotes the ith modal frequency and damping ratio. Taking the second derivative of Eqn. 2.2 and adding the contribution of ground acceleration, we obtain the nodal total acceleration vector $\mathbf{\ddot{u}}^t$ in the form

$$\mathbf{\ddot{u}}^t = \sum_{i=1}^{N} \boldsymbol{\phi}_i \Gamma_i \ddot{s}_i + \mathbf{v}\ddot{u}_g(t) \tag{2.4}$$

This equation describes the usual procedure for computing the total acceleration when working in the time domain: we compute the modal relative accelerations, combine them to obtain the nodal relative accelerations, and then add the ground acceleration.

When all modes are included in the analysis, the influence vector $\mathbf{v}$ can be expressed in terms of the modal quantities (Chopra, 1995):

$$\mathbf{v} = \sum_{i=1}^{N} \boldsymbol{\phi}_i \Gamma_i \tag{2.5}$$

Substituting Eqn. 2.5 into Eqn. 2.4 and factoring the two summations, we obtain

$$\mathbf{\ddot{u}}^t(t) = \sum_{i=1}^{N} \boldsymbol{\phi}_i \Gamma_i \ddot{s}_i^t\,(t) \tag{2.6}$$

in which

$$\ddot{s}_i^t = \ddot{s}_i + \ddot{u}_g(t) \tag{2.7}$$

is the modal total acceleration. Eqn. 2.6 suggests an alternative procedure for computing the nodal total accelerations: we compute the modal total accelerations and combine them using the same mapping as that used for nodal relative displacements in Eqn. 2.2. Eqn. 2.6 does not provide any practical advantage relative to Eqn. 2.4 when computing the total acceleration in the time domain. However, as we will show in Section 3, this formulation provides a basis for response spectrum analysis.

2.2. Discrete system with incomplete modal information

In practical modal analysis, usually mode shapes and frequencies are available for only the first n modes, where $n \ll N$. Assuming the truncated modes have frequencies much higher than the predominant frequencies of the input excitation, a natural approximation is to neglect the modal relative accelerations $\ddot{s}_i$ for all truncated modes. Many arguments support this assertion. First, when the natural frequency of a mode is much higher than the frequency content of the seismic excitation, the modal response is almost "static" and the relative acceleration is negligible. More quantitatively, when the modal frequency is much higher than the input frequencies, the term $\omega_i^2 s_i$ on the left-hand side of Eqn. 2.3 dominates the sum in balancing the right hand side. It follows that the relative acceleration term, $\ddot{s}_i$, is negligible in comparison to the ground acceleration. Consequently, the modal total acceleration $\ddot{s}_i^t$ approximately equals the ground acceleration.

Using this approximation in Eqn. 2.6 and also making use of Eqn. 2.5, we obtain

$$\ddot{\mathbf{u}}^t \cong \sum_{i=1}^{n} \boldsymbol{\phi}_i \Gamma_i \ddot{s}_i^t + \mathbf{r}_{(n)} \ddot{u}_g(t) \tag{2.8}$$

where

$$\mathbf{r}_{(n)} = \mathbf{v} - \sum_{i=1}^{n} \boldsymbol{\phi}_i \Gamma_i \tag{2.9}$$

is a residual vector that transfers to the nodal coordinates the fraction of the ground acceleration that is projected onto the rigid modes. Comparing with Eqn. 2.5, it is apparent that $\mathbf{r}_{(N)} = \mathbf{0}$.

2.3. Continuous systems

It is instructive to examine the extension of the above formulation to the case of continuous mass systems. As shown below, such a system seemingly poses a paradox when computing total accelerations at support points of the structure by modal decomposition.

To frame the problem, consider a structure of length L defined by the spatial coordinate $x \in (0, L)$ and mass distribution $\lambda(x)$. For such a structure, the set of N eigenvectors is replaced by an infinite set of eigenfunctions $\phi_i(x)$, $i = 1, \ldots, \infty$, and the influence vector is replaced by an influence function $v(x)$, representing the displacement at x when the support point(s) of the structure is displaced by a unit amount. Furthermore, the modal participation factors become $\Gamma_i = \int_0^L \phi_i(x)\lambda(x)v(x)\,dx \big/ \int_0^L {\phi_i}^2(x)\lambda(x)\,dx$. Mimicking Eqn. 2.5, we express the influence function as a combination of the eigenfunctions in the form

$$v(x) = \sum_{i=1}^{\infty} \phi_i(x)\Gamma_i \tag{2.10}$$

However, this formulation seems to pose a problem at the support point(s) of the structure, where the eigenfunctions $\phi(x)$ must necessarily be zero, while $v(x)$ generally is not zero.

Furthermore, the counterpart of Eqn. 2.6 for the continuous structure reads

$$\ddot{u}^t(x,t) = \sum_{i=1}^{\infty} \phi_i(x)\Gamma_i \ddot{s}_i^t\,(t) \tag{2.11}$$

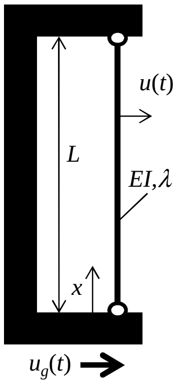

Figure 2.1. Simply supported beam.

This equation seems to suggest zero total acceleration at the support point(s) of the structure, while we know the total acceleration there must equal the acceleration at the support(s). This seemingly paradoxical result is resolved by noting that the sums in Eqns. 2.10 and 2.11 are over an infinite set of modes. The proper results should be obtained as the limits of $v(x)$ and $\ddot{u}^t(x,t)$ when x goes to the coordinate of the support, say $x = 0$, which formally read $\lim_{x\to 0}\lim_{N\to\infty}\sum_{i=1}^{N}\phi_i(x)\Gamma_i$ and $\lim_{x\to 0}\lim_{N\to\infty}\sum_{i=1}^{N}\phi_i(x)\Gamma_i\ddot{s}_i^t\,(t)$. We illustrate this idea through a simple example.

Consider a simply supported beam of length L placed vertical inside a C-shape rigid body, which is subjected to horizontal ground motion $u_g(t)$, as shown in Fig. 2.1. Assume the beam has constant properties along its length and let λ denote the mass per unit length and EI the bending stiffness. For this structure, the influence function is $v(x) = 1, x \in (0, L)$, which has a non-zero value at the support points $x = 0$ and $x = L$. It is well known (Chopra, 1995) that the i-th mass-normalized eigen-function for the simply supported beam is $\phi_i(x) = \sqrt{2/(\lambda L)}\sin(i\pi x/L)$. Using this result and the unitary influence coefficient, the modal participation factors are obtained as $\Gamma_i = 2\sqrt{2\lambda L}/(i\pi)$ for odd i and $\Gamma_i = 0$ for even i. It follows that

$$\sum_{i=1}^{\infty}\phi_i(x)\Gamma_i = \frac{4}{\pi}\sum_{\substack{i=1\\ \text{odd } i}}^{\infty}\frac{1}{i}\sin\left(\frac{i\pi}{L}x\right) \tag{2.12}$$

The above series is identical to the Fourier series expansion of a unitary function over the interval $(0, L)$. This proves the validity of Eqn. 2.10 at all points, including the support points as $x \to 0$ or L.

The counterpart of Eqn. 2.8 for the continuous system is

$$\ddot{u}^t(x) \cong \sum_{i=1}^{n}\phi_i(x)\Gamma_i\ddot{s}_i + r_n(x)\ddot{u}_g\,(t) \tag{2.13}$$

where

$$r_n(x) = v(x) - \sum_{i=1}^{n}\phi_i(x)\Gamma_i \tag{2.14}$$

It can be seen that at the boundary the sum over the finite number of modes goes to zero and Eqn. 2.13 yields $\lim_{x\to 0}\ddot{u}^t\,(x) = \ddot{u}_g(t)$, which is the correct solution.

3. MODAL COMBINATION BY THE CQC RULE

CQC is a well-known method for estimating the maximum of a dynamic response by response spectrum analysis. The reader is referred to Der Kiureghian (1981) and Der Kiureghian and Nakamura (1993) for a detailed introduction and justification of the method. For the acceleration at the kth floor of the structure, $\ddot{u}_k^t$, including all the modes, the CQC rule reads

$$\mathrm{E}\left[\max\left|\ddot{u}_k^t(t)\right|\right] = \left[\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ki}a_{kj}\rho_{ij}A(\omega_i,\zeta_i)A(\omega_j,\zeta_j)\right]^{1/2} \tag{3.1}$$

where $a_{ki} = \phi_{ki}\Gamma_i$, in which ϕ_{ki} denotes the kth element in the eigenvector $\boldsymbol{\phi}_i$, $A(\omega_i,\zeta_i)$ is the ordinate of the total acceleration response spectrum for the ith modal frequency and damping ratio, and ρ_{ij} is the cross-modal correlation coefficient between modes i and j for the total acceleration response. The correlation coefficient is computed as

$$\rho_{ij} = \frac{\lambda_{0,ij}}{\sqrt{\lambda_{0,ii}\lambda_{0,jj}}} \tag{3.2}$$

where $\lambda_{0,ij}$ is the cross-modal spectral moment defined by

$$\lambda_{0,ij} = \mathrm{Re}\left[\int_0^\infty H_i(\omega)H_j(-\omega)G_{\ddot{u}_g\ddot{u}_g}(\omega)\mathrm{d}\omega\right] \tag{3.3}$$

in which $G_{\ddot{u}_g\ddot{u}_g}(\omega)$ is the power spectral density (PSD) of the ground acceleration, and

$$H_i(\omega) = \frac{\omega_i^2 + 2i\zeta_i\omega_i\omega}{\omega_i^2 - \omega^2 + 2i\zeta_i\omega_i\omega} \tag{3.4}$$

is the frequency response function (FRF) of mode i for the total acceleration response.

The approximation introduced in Eqn. 2.8 replaces the modal acceleration for all truncated modes with the ground acceleration. In the framework of CQC, this implies the following relations:

$$\forall i > n: A(\omega_i,\zeta_i) = PGA; \quad \forall i,j > n: \rho_{ij} = 1; \quad \forall i \le n, j > n: \rho_{ij} = \rho_{ig} \tag{3.5}$$

In the above, ρ_{ig} is the cross-correlation coefficient between the ith modal response and the ground acceleration. With this approximation, Eqn. 3.1 reads:

$$\begin{aligned}\mathrm{E}\left[\max\left|\ddot{u}_k^t(t)\right|\right] \cong \Big[&\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ki}a_{kj}\rho_{ij}A(\omega_i,\zeta_i)A(\omega_j,\zeta_j) + \\ &+2\sum_{i=1}^{n} a_{ki}r_{nk}\,\rho_{ig}A(\omega_i,\zeta_i)PGA + r_{nk}^2 PGA^2\Big]^{1/2}\end{aligned} \tag{3.6}$$

where and r_{nk} is the kth element in the residual vector $\mathbf{r}_{(n)}$. To compute the cross-correlation coefficient ρ_{ig}, it is sufficient to note that the ground acceleration is identical to the response of a mode with infinitely large frequency. Since $\lim_{\omega_i\to\infty} H_i(\omega) = 1$, ρ_{ig} is computed according to Eqn. 3.2 with the corresponding $\lambda_{0,ig}$ and $\lambda_{0,gg}$ values obtained from Eqn. 3.3 with $H_j(-\omega)$ and $H_i(\omega)H_j(-\omega)$ replaced by 1, respectively. These computations require the definition of the PSD $G_{\ddot{u}_g\ddot{u}_g}(\omega)$ compatible with the specified response spectrum. Here, we have used a modified version of the formulation proposed by Der Kiureghian and Neuenhofer (1992), which is more accurate in the high frequency range.

The CQC formulations in Eqns. 3.1 and 3.6 are in terms of the total acceleration response spectrum, which in practice is seldom available. Response spectra in design codes are usually specified in terms of the pseudo acceleration. However, it is well known (Chopra, 1995) that the pseudo-acceleration spectrum is a good approximation of the total acceleration spectrum for a wide range of frequency and damping values. The two spectra are identical for un-damped systems and nearly identical for low-damped systems. The difference between the two is only significant for high damping values and long periods. For such cases, correction factors such as those suggested by Sadek *et al.* (2000) may be used. These observations suggest that $A(\omega_i,\zeta_i)$ in the above CQC rules can be replaced with the pseudo-acceleration spectrum, which we denote as $A'(\omega_i,\zeta_i)$.

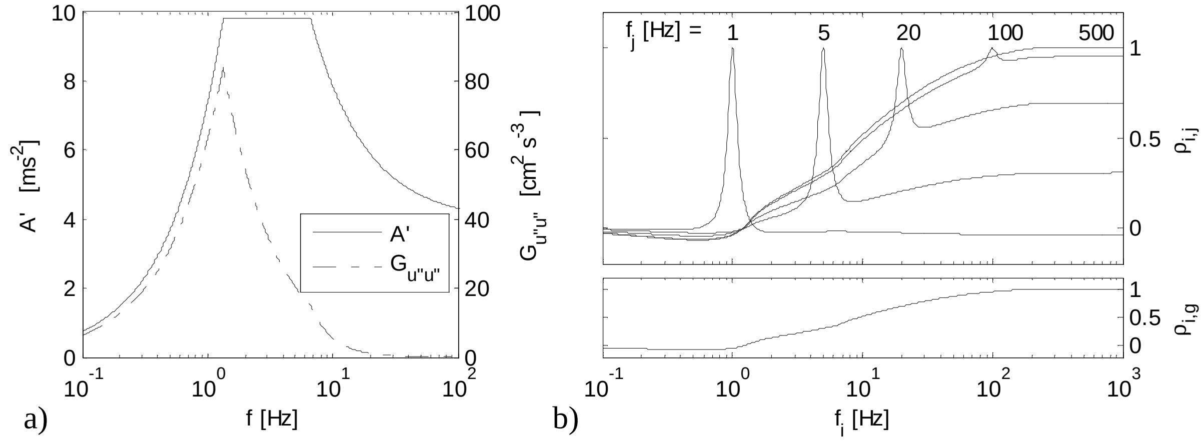

Figure 3.1. (a) Pseudo-acceleration response spectrum and consistent PSD, (b top) modal cross-correlation coefficients, (b-bottom) cross-correlation coefficient between modal response and ground acceleration.

As an example, Fig. 3.1(a) shows an ASCE (2006) pseudo-acceleration response spectrum for 5% damping with the corresponding consistent PSD. Example cross-correlation coefficients are shown in Fig. 3.1(b). The upper graph reports the coefficients ρ_{ij} for $f_j = \omega_j/2\pi = 1$, 2, 20, 100 and 500Hz and $f_j = \omega_j/2\pi$ ranging from 0.1Hz to 1kHz. This graph shows that up to about 10Hz, which is near the upper limit of the frequency content of the seismic excitation, coefficients ρ_{ij} decay when frequencies f_i and f_j are well separated. However, for higher modal frequencies, the correlation coefficient is high even when the frequency are well separated. This observation supports the approximation in Eqn. 3.6 and, indirectly, provides an argument supporting Eqn. 2.8. The lower graph in Fig. 3.1(b) reports the cross-correlation coefficient ρ_{ig} as a function of the modal frequency f_i. We again observe that the correlation coefficient between a modal response and the ground acceleration is approaches unity for high modal frequencies, support the "rigid approximation" proposed above.

4. NUMERICAL APPLICATION TO A CONTINUOUS CANTILEVER

Consider a cantilever beam fixed at the base and subjected to ground excitation modeled by the response spectrum described in Section 3. As shown in Fig. 4.1(a), the x coordinate indicates the position along the cantilever, which has uniform mass distribution λ, and bending stiffness EI. The parameters are selected such that the first natural period of the structure is 3s.

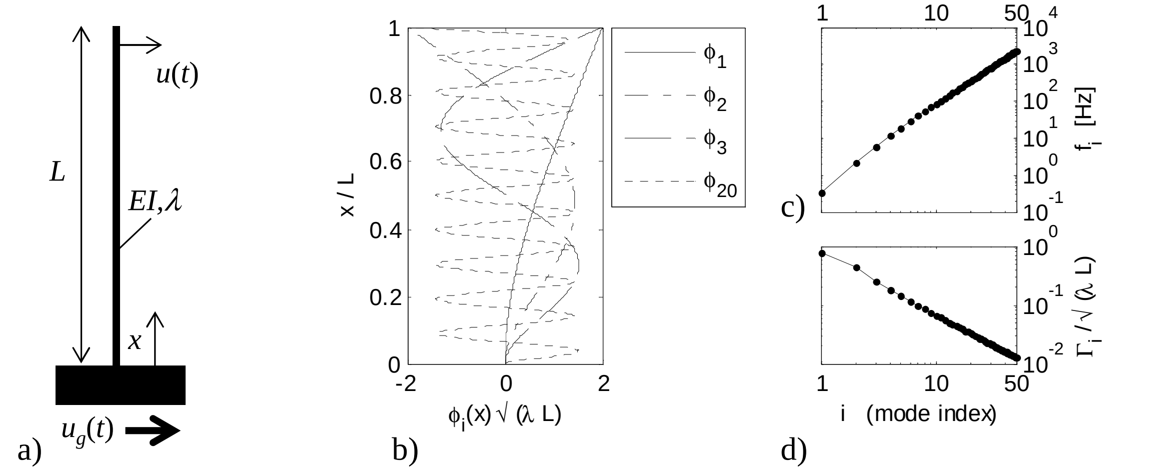

Figure 4.1. (a) Example cantilever beam; (b) mass-normalized mode shapes; (c) modal frequencies; (d) modal participation factors.

Formulas for the natural frequencies and mode shapes of the cantilever beam are provided in, e.g., Chopra (1995). We include 50 modes in our analysis. Four mass-normalized mode shapes are shown in Fig. 4.1(b), the first 50 frequencies in Fig. 4.1(c) and the corresponding participation factors in Fig. 4.1(d). As expected, in the lower part of the structure, the lower modes have negligible contributions, while the 20th mode has significant contribution. The residual contribution r_n of the rigid modes is plotted in Fig. 4.2(a) for selected number of modes n. If we include just one mode in the analysis ($n = 1$), the acceleration in the lower half of the cantilever is dominated by the rigid contribution, while this contribution is reduced as more modes are included. For $n = 50$, the rigid contribution is significant only in the close proximity of the base.

The correlation coefficient between modes i and j is depicted in a contour plot in Fig. 4.2(b). We note that the correlation between two modes with high indices is close to one. This is consistent with Fig. 3.1(b), but it is also due to the fact that the ratio f_{i+1}/f_i approaches unity as $i \to \infty$.

The graphs in Figs. 4.3 report the estimated PFA for an increasing number of modes included in the analysis, from $n = 1$ to $n = 50$. Three points on the cantilever have been selected: one at the top of the structure ($x = L$), and two close to the base (at $x = 0.15L$ and $x = 0.08L$). The proposed approach described by Eqn. 3.6 is referred to as CQC with rigid contribution ("CQC with RC"). The results are plotted as the ratio of the prediction using n modes to the result of CQC with rigid contribution using 50 modes, which is assumed as the "correct" result. In the same figures, we show the estimated PFA if the rigid contributions of truncated modes were to be neglected ("CQC w/o RC"). These results are obtained by including only the double-sum term in Eqn. 3.6. The third line in the graphs refers to the use of SRSS, including the rigid contribution ("SRSS with RC"). The key assumption of SRSS is that the responses of each pair of modes are uncorrelated, and that the response of each mode is uncorrelated with the ground excitation. Hence, the SRSS rule is equivalent to the CQC rule if we set all the cross-correlation coefficients equal to zero. This yields

$$\mathrm{E}_{SRSS}\left[\max\left|\ddot{u}_k^t(t)\right|\right] = \left[\sum_{i=1}^{n}\left(a_{ki}A(\omega_i,\zeta_i)\right)^2 + r_{nk}^2 PGA^2\right]^{1/2} \tag{4.1}$$

Of course, for the application to the cantilever beam, $a_{ki} = a_i(x)$ and $r_{nk} = r_n(x)$. By including the rigid contribution of higher modes in the above SRSS formulation, we are essentially assuming that modal responses are uncorrelated for all modes up to mode n, and perfectly correlated for all truncated modes. Thus, the SRSS rule employs two extreme approximations of the cross-modal correlations: zero for the modes included in the analysis and 1 for the truncated modes. Admittedly, the SRSS rule is much simpler than the CQC, as the former does not require computation of correlation coefficients. However, it is apparent from Fig. 4.3 that, even for the simple structure examined, the approximation of zero correlation among high modes is untenable.

It is well known that the reliability of the SRSS approximation in response spectrum analysis of structures depends on the specific application. As shown by Chopra (1995), when we consider quantities related to the relative displacement, SRSS may be adequate in some cases of practical relevance. This is particularly the case when the response is influenced only by the first few modes, and the actual correlation among them is small. In such cases, the SRSS rule gives results that are close to those of the CQC rule. However, in the analysis of the total acceleration response, the context is different. As explained in Section 2.3, we expect the response close to the base of the structure to be driven by the higher modes, which are usually strongly correlated among themselves. In the SRSS approach, the modes are treated as either uncorrelated or as fully correlated, depending on the selection of n. Therefore, the prediction based on Eqn. 4.1 strongly depends on the number of modes used in the analysis. An insight into that issue is given by the following argument: consider a structure whose natural frequencies, even the lowest ones, are much higher than the frequency components of the ground excitation. Obviously, such a structure behaves approximately as a rigid body, and the PFA equals the PGA at every location in the structure. This means that the rigid approximation is adequate even when we include no modes in the analysis, i.e., when $n = 0$. In that case, the predictions based on the CQC and SRSS rules are identical and correct. However, when we explicitly include some modes ($n > 0$), the predictions disagree. Note that, for the rigid structure, the actual correlation coefficients are unitary for each pair of modes and for each mode and the ground excitation, while the spectral acceleration is equal to the PGA for each mode. Consequently, the CQC prediction is not affected by n. On the other hand, the correlation structure assumed by the SRSS depends on n. Consider the limit when n goes to infinity and all modes are included in the analysis. In that case, the rigid contribution vanishes, so that Eqn. 4.1 is reduced to the PGA multiplied by square root of the sum of squares of the coefficients a_{ki}. Fig. 4.4(a) reports the outcome of this sum, along the profile of the cantilever. The outcome has been computed numerically, investigating the convergence of the series up to a few thousand modes. It appears that the series converges to 0 at the base of the cantilever, and to 2 at its top. Thus, when all modes are included, SRSS overestimates the acceleration at the top by a factor two, and underestimates the acceleration near the base by an arbitrarily large factor (depending on selected value of x). The remarkable conclusion is that, the less the number of modes included in the analysis, the better the SRSS prediction, the reason being that the actual correlation structure is captured much better by the rigid contribution than by the sum in Eqn. 4.1.

The first graph in Fig. 4.3 shows that, for the top acceleration, the rigid contribution plays no significant role when at least two modes are included in the analysis. Hence, the results of CQC with and without the rigid contribution are similar. SRSS also produces reasonable results, with a 5% overestimation when a large number of modes is included in the analysis. This overestimation is related to the "rigid body" argument described above. The overestimation at the top is smaller because the correlations among the lower modes are much smaller than one. When we consider a location closer to the base of the structure, the differences between the three approaches is more evident. The results of CQC including the rigid contribution are "stable", i.e., they show small variation with n. This indicates that the main contribution comes from modes that are sufficiently rigid, such that the approximation proposed in Sections 2-3 is acceptable. When only a single mode is included in the analysis, the error is less than 20% for the acceleration response at $x = 0.15L$ and less than 10% for that at $x = 0.08L$. As expected, the quality of the approximation improves as more modes are included. However, one cannot guarantee that the error will monotonically decrease with increasing number of modes included in the analysis. When we do not include the rigid contribution in the CQC formulation, the approximation close to the base can be extremely poor, unless we include a large number of modes, with the required number depending on how close we are to the base. The estimation by SRSS with the rigid contribution shows a trend near the base that we have anticipated in the previous paragraph. With a small number of modes included, the approximation is acceptable; however, the approximation deteriorates when we include an increasing number of modes. In fact, we expect it to predict a zero acceleration at the base when we include all the infinite modes.

Similar considerations can be drawn from Figs. 4.4(b), which show the profile of the estimated acceleration including $n = 1$, 3 or 50 modes in the analysis. When we include the rigid contribution, both CQC and SRSS estimate the PGA at the base, independently of n, while the prediction is zero without the rigid contribution. For $n = 3$, the agreement between CQC and SRSS is acceptable. However, as we include more modes, SRSS progressively estimates smaller accelerations close to the base, down to zero, while the result based on the CQC without the rigid approximation improves with increasing number of modes.

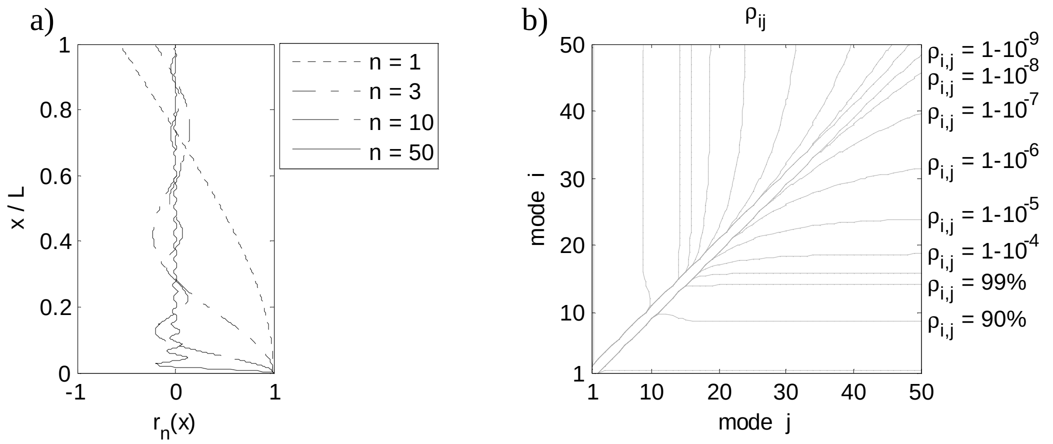

Figure 4.2. (a) Residual contribution of rigid modes for varying number of modes; (b) cross-modal correlation coefficients.

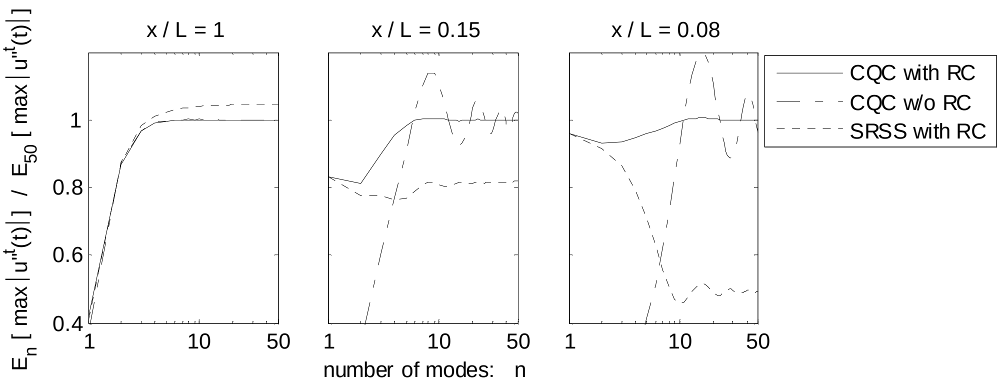

Figure 4.3. Ratio of estimated PFA versus number of modes included at three locations in the structure.

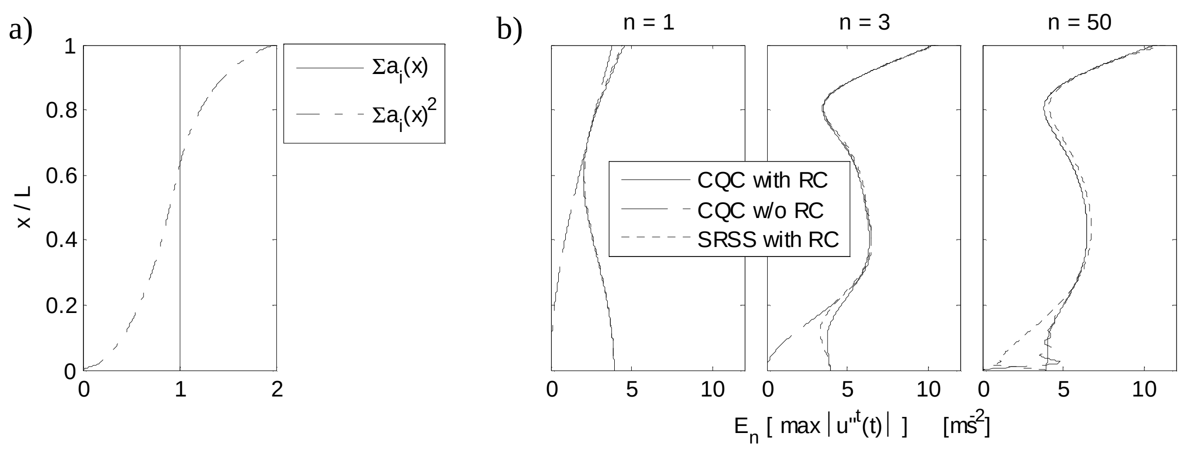

Figure 4.4. (a) Limit of modal contributions by CQC and SRSS for a rigid structure; (b) profile of estimated PFA for varying number of modes.

5. CONCLUSION

A formulation is presented for including the contribution of truncated modes in the CQC modal combination rule for the peak floor acceleration. This approximation is equivalent to the "static correction" (Der Kiureghian and Nakamura, 1993; Chopra, 1995) used in estimating quantities related to relative displacement. While the "static correction" requires that the user compute the static response of the structure under a distributed load, the "rigid" approximation we propose for acceleration does not require any computations involving the response of the structure: only the flexibility of lower modes explicitly included in the analysis is used. This happens because the acceleration in rigid modes is identical to the ground acceleration. Furthermore, while the "static correction" usually plays a minor role in estimating displacement-related responses, the rigid contribution is crucial for correct estimation of total accelerations.

Through a simple example, it is shown that the commonly used SRSS rule may lead to incorrect estimation of acceleration responses, particularly near the support points of the structure. Furthermore, this method is associated with the undesirable behavior of possibly producing less accurate results with increased number of modes included in the analysis.

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