Assessment of Soil-Structure Interaction Modeling Strategies for Response History Analysis of Buildings

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SUMMARY:

A complete model of a soil-foundation-structure system for use in response history analysis requires modification of input motions relative to those in the free-field to account for kinematic interaction effects, foundation springs and dashpots to represent foundation-soil impedance, and a structural model. The recently completed ATC-83 project developed consistent guidelines for evaluation of kinematic interaction effects and foundation impedance for realistic conditions. We implement those procedures in seismic response history analyses for two instrumented buildings in California, one a 13-story concrete-moment frame building with two levels of basement and the other a 10-story concrete shear wall core building without embedment. We develop three-dimensional baseline models (MB) of the building and foundation systems (including SSI components) that are calibrated to reproduce observed responses from recorded earthquakes. SSI components considered in the MB model include horizontal and vertical springs and dashpots that represent the horizontal translation and rotational impedance, kinematic ground motion variations from embedment and base slab averaging, and ground motion variations over the embedment depth of basements. We then remove selected components of the MB models one at a time to evaluate their impact on engineering demand parameters (EDPs) such as inter-story drifts, story shear distributions, and floor accelerations. We find that a "bathtub" model that retains all features of the MB approach except for depth-variable motions provides for generally good above-ground superstructure responses, but biased demand assessments in subterranean levels. Other common approaches using a fixed-based representation can produce poor results.

Keywords: Soil-Structure Interaction, Response History Analysis

1. INTRODUCTION

The analysis of building seismic response can utilize various approaches for modeling soil-structure interaction (SSI) at the base of the building, including ignoring SSI effects altogether. While some of these approaches are relatively simple, others require significant effort to capture the linear or nonlinear SSI. What is not clear is whether more complex and time-consuming approaches produce substantially different or more accurate results than simpler approaches. In this paper, we examine this issue while also demonstrating the application of analysis tools that have recently been compiled by the Applied Technology Council (ATC) 83 project team in the NIST/GCR 11-917-14 report (NEHRP CJV, 2012), entitled "Soil-Structure Interaction for Building Structures," referred to in this paper as "NIST report." The analysis tools are applied in the context of substructure analysis of the SSI problem, which requires assumptions of equivalent-linear soil behaviour.

The approach taken in this work is to identify suitable instrumented buildings that have recorded earthquakes, develop complete SSI models for substructure-based analysis of seismic response (referred to as a baseline model), roughly calibrate the structural elements of the baseline model to approximately match recordings, and then remove components of the baseline model for additional analyses to evaluate the impact of those components. We use this process to identify the critical components of an SSI model for use in response history analysis, and develop recommendations based on those results as well as prior results in the literature. Prior studies have taken a similar approach,



but utilized conventional structural software packages (ETABS and SAP) on different buildings (Tileylioglu et al., 2010; Naeim et al., 2008) with slightly different SSI analysis procedures.

2. MODEL TYPES

2.1. Baseline Model (MB)

The analysis of each structure is begun by developing three-dimensional models of the building and foundation system, which are referred to as "baseline models" (MBs). MBs are not intended to be the most accurate models that could be developed (e.g., a direct SSI analysis in a finite element platform could provide improved results), but instead represent the implementation of procedures given in the NIST report. As shown in Figure 1, MBs incorporate SSI in the vertical and horizontal directions, including rocking, with a series of springs and dashpots reflecting site soil properties. Seismic demands imposed on MBs include base translation as well as kinematic loading of basement walls (simulated by displacement histories applied to the ends of horizontal springs attached to basement walls). Using those seismic demands, MBs are calibrated to match the response interpreted from recorded motions.

Figure 1 depicts the MB model, which is implemented using a substructure approach. The springs and dashpots in the figure represent the frequency-dependent foundation impedance for horizontal, vertical, and rocking modes of vibration. The motion u_{FIM} in Figure 1 represents the Foundation Input Motion, which is modified from the free-field motion (u_g) for the effects of kinematic interaction.



Baseline Model (MB)

Figure 1. Schematic illustration of baseline model considered in simulations.

2.2. Simplified and Alternative Models

Once the MBs successfully match the recorded data, components of the seismic demand and soilfoundation interaction elements are replaced, one or more at a time, to match modeling approaches used in practice. The goal of simulations with these simpler models is to assess the changes induced by each simplification on computed engineering demand parameters (EDPs). As illustrated in Figure 2, four simplified models were considered, as follow:

Model 1: Model above-ground portion of structure fixed at ground surface and exclude subterranean and foundation components. Excite base of structure with free-field ground motion u_g . (Flexible structure, rigid basement and soil)

Model 2: Model above-ground and subterranean portions of structure but use no horizontal foundation springs along basement walls and fix the base of the structure against translation or rocking. Excite base of structure with free-field ground motion u_g . (Flexible structure and basement, rigid soil)

Model 3: Subterranean levels are modeled and horizontal and vertical soil springs are included. Horizontal spring at base slab receives input motion (u_g) but horizontal springs at higher elevations are fixed at end with no input. Vertical springs are supported on a fixed base. Applications of Model 3 in engineering practice have been predominantly for pushover analysis, although its use here is for

response history analysis, which is expected to demonstrate poor performance.

Model 4 (bathtub): Subterranean levels are supported by horizontal and vertical soil springs that are fixed at their ends to a rigid "bathtub." The bathtub is excited with the horizontal foundation input motion (u_{FIM}) or free-field motion (u_g) . (Flexible structure, basement and soil)



Figure 2. Schematic illustration of simplified models 1 to 4 considered in simulations.

These simplified approaches were chosen based on consultations between the ATC-83 project team and structural and geotechnical engineering practitioners, with the purpose of documenting the stateof-practice in SSI analysis. The outcome of that process is described in the NIST report. Referring to Figure 2, Models 1 and 2 are used most often, followed by Model 3. Model 4, which was introduced by Naeim et al. (2008) and was evaluated in the present work, has only very recently begun to see applications in practice.

The only differences between Model 4 and MB pertain to the manner in which seismic demand is specified. The implementation of Model 4 with u_{FIM} is identical to MB except that the effect of kinematic loading of basement walls associated with depth-variable displacement histories applied to the ends of horizontal foundation springs is neglected. The implementation of Model 4 with u_g takes this a step further by neglecting kinematic interaction altogether by replacing the recorded motions at the base of the building by equivalent free-field motions applied uniformly to the end of all horizontal foundation springs. Input motions are generally specified in the engineering software as accelerations. An exception is the multi-support excitation runs applied for the MB, in which ground motions are specified as displacement histories that are obtained through integration.

3. BUILDINGS CONSIDERED

Only buildings with seismic instrumentation and available earthquake recordings in California were considered. Desirable attributes of a building in this selection process were as follows: (1) embedded foundations, so that kinematic effects associated with embedment and depth-variable ground motions could be evaluated; (2) two vertical sensors to record rocking at the foundation level; (3) relatively regular structural configurations, so that the results obtained are not peculiar to an atypical building type; and (4) structural configurations and site conditions that would tend to be conducive to significant inertial SSI effects. No single building met all of these criteria. The buildings that were selected are a 13-story building with two-levels of basement in Sherman Oaks, California and a 10-

story building without embedment in Walnut Creek, California. The Sherman Oaks building is a reinforced concrete moment frame structure that was shaken strongly by the 1994 Northridge earthquake and weakly by several other more distant events. The building is founded on grade beams and friction pile foundations supported by alluvial sediments. The Walnut Creek building is a reinforced concrete dual-system structure with perimeter moment frames and a central core of shear walls. The shear walls are founded on a mat foundation with shallow embedment, resting on weathered shale bedrock. The Sherman Oaks building satisfies the criteria (1) and (3) above, while the Walnut Creek building satisfies (2), (3), and (4).

A parameter that is often used to identify conditions where inertial SSI effects are likely to significantly affect structural response is $h/(V_sT)$, where *h* is approximately 2/3 of the building height (distance from base of foundation to centroid of first mode shape), V_s is the time-averaged strain-compatible shear wave velocity of the foundation soil over a representative depth, and *T* is the fixed-base first mode period for the structure. The ratio $h/(V_sT)$ effectively represents a ratio of structure to soil stiffness, and inertial SSI becomes more significant as this parameter increases, typically being important for values of approximately 0.05 to 0.1 or greater (e.g., Veletsos, 1977; Stewart et al., 1999). The $h/(V_sT)$ parameter for each case study is discussed in the following subsections.

3.1. Sherman Oaks Building and Site Description

As shown in Figure 3, the building has 13 stories above the ground surface and a two-level basement. The building is 50 m (164 ft) tall from the ground surface to the roof, with approximately 6.2 m (20.5 ft) of embedment. The height of the first floor is 7.0 m (23 ft), while all other floors above are 3.6 m (11.75 ft). The 1^{st} and 2^{nd} subterranean levels are 3.5 m (11.5 ft) and 2.7 m (9 ft) tall, respectively. The lateral loads are carried by moment-resisting concrete frames that extend from the roof to the foundation, supplemented by perimeter concrete walls at the subterranean levels. Vertical loads are carried by 11.4 cm (4.5 in) concrete slabs supported by the concrete beams and columns in the moment frames. The foundation details can be found in the NIST report. The building was designed in 1964 and was later seismically rehabilitated with friction dampers following the 1994 Northridge Earthquake.

The Sherman Oaks building is designated as California Strong Motion Implementation Program (CSMIP) Station No. 24322. It contains 15 accelerometers at the locations shown in Figure 3a. The accelerometers are located on 5 levels: the 2^{nd} subterranean, ground surface, 2^{nd} , 8^{th} and roof levels and has been instrumented since 1977. Note that there is only one vertical sensor at the foundation level, so base rocking effects cannot be measured. There is also no free-field ground instrument in the vicinity of the site. The horizontal translations recorded at the base of the 2^{nd} subterranean level of the building were used as the foundation input motions (u_{FIM}). The u_{FIM} is the modified u_g response due to base slab averaging and embedment effects. Typically in practice, u_g is known and the u_{FIM} would be calculated for structural analysis from transfer functions. In this case, u_{FIM} was measured and u_g was back-calculated by removing the base slab averaging and embedment effects.

Figures 3b and 3c show geophysical and geotechnical conditions at the site, which are based on from the site and neighboring sites (details in NIST report). The site conditions consist of about 24 m of alluvial sediments overlying sedimentary rock.

The ratio of structure to soil stiffness for the Sherman Oaks building is $h/(V_sT) = 0.07$. Because this ratio is < 0.1, strong inertial SSI effects in the form of period lengthening and foundation damping are not expected. The building was nonetheless analyzed because of other attributes (common building type, regular configuration, embedded foundation, multiple recordings) and the potential for significant kinematic interaction effects on higher mode responses. Moreover, we sought to investigate whether a building for which traditional first-mode SSI metrics indicate no significant effect (i.e., period lengthening near unity, foundation damping near zero), could in fact exhibit potentially significant impacts of SSI on the vertical distribution of EDPs used in structural design (e.g., interstory drifts, story shears, etc.).



Figure 3. Illustration of Sherman Oaks building and site characteristics; a) foundation and east-west oriented building sensor location map, b) soil shear-wave velocity profile, and c) soil material profile description.

3.2. Walnut Creek Building and Site Description

As shown in Figure 4, the Walnut Creek office building has 10 stories above the ground surface and no subterranean levels. The building is 39.2 m (129 ft) tall from the ground surface to the roof. The height of the first floor is 4.9 m (14 ft), while all other floors above are 3.8 m (12.5 ft). Lateral loads are carried by an interior concrete shear wall core that is embedded 3 m (10 ft) and an exterior precast and cast-in-place concrete frame. Vertical loads are carried by 7 cm (2.75 in) lightweight concrete over 7 cm (2.75 in) precast panel slabs supported by a precast, prestressed reinforced concrete beams. The building was designed in 1970.

The Walnut Creek building is designated as CSMIP Station No. 58364. It contains 16 accelerometers at the locations shown in Figure 4a. The accelerometers are located on 4 levels: ground surface, 3rd, 8th and roof levels and has been instrumented since 1979. There are two vertical sensors at the ground level, allowing the base rocking effects to be measured. There is no free-field ground instrument in the vicinity of the site.



Figure 4. Illustration of Walnut Creek building and site characteristics; a) foundation and east-west oriented building sensor location map, b) soil shear-wave velocity profile, and c) soil material profile description.

Figures 4b and 4c show the geophysical and geotechnical conditions at the site (further details in the NIST report). Geologic conditions consist of west dipping layers of sandy clays and silts with variable thickness of 0.6 to 5.5 m (2 to 18 ft) overlying siltstone and sandstone of the Orinda Formation. The

shear wave velocity of rock is based on on-site refraction data, whereas shear wave velocities in soil are inferred based on correlations by Fumal and Tinsley (1985).

The ratio of structure to soil stiffness for this building is $h/(V_sT) = 0.12$, using effective building height, soil shear wave velocity, and period values. Because this ratio is > 0.1, potentially significant inertial SSI effects in the form of period lengthening and foundation damping are expected. Other motivating factors for analyzing this building include the dual-system configuration of the structure and the presence of two vertical instruments at the foundation level that can be used to infer rocking of the core shear wall system.

4. BASELINE MODEL AND COMPARISON TO RECORDINGS

Open System for Earthquake Engineering Simulation (OpenSees) software was used to perform response history analyses of a three-dimensional (3-D) model of the Sherman Oaks and Walnut Creek buildings. OpenSees is an open-source software package that was developed at the University of California, Berkeley for simulations of structural and geotechnical responses to earthquakes (OpenSees 2011). The structural conditions of the buildings are described in the NIST report with reference to their representation in the OpenSees model.

The original OpenSees structural model for the Sherman Oaks building was provided by Erol Kalkan of the CSMIP and the US Geological Survey (personal communication, 2011). The original structural model was improved with inclusion of the subterranean levels and foundation springs and dashpots for the MB, but otherwise the initial structural properties were those from the Kalkan model. The Sherman Oaks building had response history recording data from the 1994 Northridge, 1992 Landers and 1987 Whittier earthquakes.

The Walnut Creek OpenSees model was generated as part of this research. Because of the modest embedment of the Walnut Creek core wall foundation and the lack of subterranean levels, multi-support excitation along embedded portion of the foundation was not considered, causing the baseline model (MB) to match the bathtub model (Model 4) in this case. The implementation of the bathtub model for this structure is illustrated in Figure 5. The other models considered are fixed-base models analogous to Models 1 and 2 from Figure 2 (implementation for Walnut Creek building is shown in Figure 5). To expedite the analyses and the post-processing, the model was reduced to two-dimensional (2-D). The 2-D model includes all the elevations spanning the East-West direction.



Figure 5. Schematic illustration of modeling approaches for the Walnut Creek building.

The development of foundation springs and dashpots for both MBs was based on the analytical approaches presented in the NIST report. The critical input parameters are foundation dimensions, structural period, soil properties, and input motion amplitudes. The soil spring and dashpot modeling details can be found in the NIST report.

Roof displacement and acceleration histories obtained from the calibrated Sherman Oaks MB are compared to recordings in Figures 6a and 6b for the Northridge earthquake. Additional displacement histories for both directions and multiple floor levels are presented in the NIST report. The match in both horizontal directions at the foundation and ground levels is excellent. Elsewhere over the height of the building, the quality of the match is generally better in the longitudinal direction than transverse (e.g. Channel 1 fit at roof level has a better match of phasing and amplitudes than Channel 2). The calibration process began with expanding the original structural model by Kalkan to include the subgrade levels, where shear stiffness of the reinforced concrete walls was of concern. To keep the calibration approach simple, a multiplier on the theoretical shear modulus of the uncracked concrete (G_c) was taken as the calibration parameter, with the goal of matching near-ground response. A multiplier of 0.25 was used (which is a reasonable value to reflect the cracked stiffness properties of the walls), however, the near-ground response was not highly sensitive to this parameter over the range of 0.25-0.40 that was considered. Our next step in the calibration was to seek to match the building period through adjustments of the structural stiffness and mass. The real stiffnesses of a reinforced concrete element is highly variable (Haselton et. al 2011). Therefore the rebar stiffness (as a proxy for overall element flexural stiffness) was modified by 0.7 from the original value, whereas the mass was not modified.



Figure 6. Comparison of baseline model and recorded roof response histories for a) Sherman Oaks building absolute displacement, b) Sherman Oaks building absolute accelerations, c) Walnut Creek building relative displacement and d) Walnut Creek building absolute acceleration.

A comparison of the relative displacement and absolute accelerations of the Walnut Creek MB and the measured data from the Loma Prieta earthquake at the different floors are shown in Figures 6c and 6d. Additional displacement and acceleration histories for multiple floor levels are presented in the NIST report. The plots in this figure indicate that the numerical model is a reasonable representation of the dynamic response of the physical structure. This model used a factor of 0.3 between cracked and uncracked concrete stiffness. The stiffness reduction of 30% was applied to the elastic modulus of the columns and shear wall components (modeled as trusses as described in the NIST report). No further calibration was performed and all other properties represented best estimates develop prior to viewing analysis results and analysis-data comparisons.

The Walnut Creek building has two vertical instruments placed at the ground level that can be used to evaluate base rotation and, hence, the roof lateral displacement due to the base rocking. In a similar manner, the rotation at the base of the numerical model was used to determine the component of the lateral roof displacement due to base rocking. These two displacement histories are compared in Figure 7. The numerical model captures the observed roof lateral displacement due to rocking reasonably well. These rocking-induced displacements account for approximately 10% of the total roof displacement.



Figure 7. Comparison of roof displacement due to base rotation of baseline model and recorded data for the Walnut Creek building.

In addition to the response histories comparisons between MBs and recorded data for each sensor location, the maximum displacements and accelerations were also compared with results given in the NIST report. Based on the favourable comparisons in Figure 6 and many similar plots (not shown here for brevity) we find that the MBs provide a reasonable approximation of the soil-structure system response and to provide a suitable basis for model to model comparisons presented in the next section.

5. MODEL TO MODEL COMPARISONS

Table 1 shows the 1st mode periods for the alternative model configurations for the Sherman Oaks and Walnut Creek structures. As expected, fixed base models have shorter periods than flexible-base models, although the changes are relatively modest, especially for the Sherman Oaks structure. Many of the changes in response that we compute can be related to the change in period coupled with spectral shape near the first mode period.

Model Type	Fundamental Period (sec)	
	Sherman Oaks Bldg.,	Walnut Creek Bldg.,
	Northridge Earthquake	Loma Prieta Earthquake
MB (Baseline Model)	2.72	0.83
1 (fixed @ ground surface)	2.67	0.78
2 (fixed @ base)	2.71	0.78
3 (fixed horizontal spring)	2.65	N/A
4 (bathtub)	2.72	0.83

Table 1 Comparison of 1st Mode Periods for Alternative Model Configurations (for both models)

For the Sherman Oaks building, the results are synthesized from each of the considered alternative practical configurations in Figure 8 along with the MB as profiles of inter-story drift and story shear for transverse response. The Model 3 results are clear outliers for each of the EDP considered. Among the other models, Model 4 is closest to MB, followed by Model 2, and then Model 1. Differences in EDP are generally greater below ground line than above. Response histories provided in the NIST report show that ignoring the subterranean levels (Model 1) results in displacements that are more out of phase (relative to Model 4) than those from Model 2, which is not apparent by the peak response profiles.

A set of EDPs similar to those for the Sherman Oaks building were evaluated for the Walnut Creek building. Figure 9 presents profiles of the peak EDPs of maximum inter-story drift and floor accelerations. There is a general trend towards over-prediction of response by the fixed-base models (Models 1 and 2) relative to MB. Note that Models 1 and 2 are nearly identical in this case (unlike Sherman Oaks), because of the small embedment.



Figure 8. Comparison of peak drift ratios and story shears in transverse direction from all models considered for the Sherman Oaks building.



Figure 9. Comparison of peak drift ratios and story shears in transverse direction from all models considered for the Walnut Creek building.

6. INTERPRETATION AND CONCLUSIONS

Drawing upon the previous findings from Naeim et al (2008), Tileylioglu et al. (2010) and the present work, several consistencies can be observed. First, MB-type models have an encouraging ability to match observed responses with only modest tuning of structural parameters (i.e., damping ratios, element masses or stiffnesses). In particular, the SSI portion of the models appears to be performing well for each of the investigated structures having recordings of foundation rocking (two from prior work and Walnut Creek in the present work). These results suggest that the relatively simple equivalent-linear spring/dashpot approach for SSI modeling presented in the NIST report can provide satisfactory predictions of foundation response for the levels of excitation in these case studies.

Second, there are varying levels of performance for some of the simplified approaches used in practice. The worst-performing approach is Model 3, which should not be used for response history analysis. Model 1 is the next least-desirable option, with the level of difference from the MB being greatest for stiff structures or deeply embedded structures. The errors are especially noticed in under-prediction of building periods, variations in profiles of inter-story drifts, story shears and peak floor accelerations, and phasing errors in response histories. The best-performing models relative to MB are Model 2 and Model 4 (bathtub).

Our third general finding concerns the portions of a structure whose response is significantly affected by the SSI modeling considered in this work. For the buildings considered, above-ground building responses, as measured for example by envelopes such as drift or story shear profiles, were modestly to insignificantly affected by SSI. The largest effects of SSI on above-ground enveloped responses were for the stiffest structures. In contrast, below-ground enveloped responses (e.g. basement wall shear forces and below-ground inter-story drifts) were much more sensitive to SSI. Both the kinematic ground motion description and spring distributions contribute to these effects. This sensitivity was observed for all structures, whether flexible or relatively stiff.

Considering all of the above, it is observed that none of the simplified approaches either currently used or proposed for use in practice (Models 1-4 in Figure 2) can mimic the MB response over the full height of the structure for the range of structural conditions that have been considered to date. For flexible structures with modest embedment, any approach other than Model 3 would likely provide acceptable results above the ground line. More generally, the bathtub approach (Model 4) appears to be the most generally applicable and reliable among the simplifications considered. However, even Model 4 does a poor job below the ground line because the application of depth-variable ground motion or kinematic rocking can significantly affect EDPs for deeply embedded structures (approximately two levels or more). When it is desired to avoid the use of foundation springs, Model 2 has been shown to provide reasonably good results, especially for embedded, flexible structures and EDPs associated with the portion of the structure above the ground line.

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