# A comparison of the performances of various ground–motion intensity measures

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#### SUMMARY:

Ground-motion Intensity Measures (IM) are parameters that synthetically describe the intensity of groundmotions; with particular reference on their effects on structures. IMs play an important role in performance based seismic design and in ground-motion selection procedures, because they are the link between seismic hazard and seismic demand analysis. Many researchers have proposed IMs and investigated their performances, in particular, in terms of efficiency, sufficiency and scaling robustness. Unfortunately most of these analyses are based on limited case studies that make use of small sets of ground motions. The present paper aims at providing a comprehensive comparison of the efficiency and sufficiency of the most important IMs proposed in the literature. This study makes use of a large dataset of accelerograms (1700 records) and employs cloud-analysis and stepwise regression to investigate their efficiency of sufficiency with reference to various inelastic SDOF and MODF structures.

Keywords: Intensity measure, ground-motion, accelerogram, nonlinear analysis, seismic demand analysis.

#### **1. INTRODUCTION**

Probabilistic performance based design of structures allows to evaluate risk related to possible earthquake and therefore represents a powerful tool for both assessing existing structures and design new ones. One of the objectives of this type on analysis in the estimate of the annual exceedance ratio of given levels of structural response  $\lambda_{EDP}$ . This latter frequency may be estimated combining the results of Probabilistic Seismic Demand Analysis (PSDA) with information on the site under consideration. According to this approach the measure of the structural response of interest indicated by a parameter named Engineering Demand parameter (EDP), such as interstorey drift, chord rotation, etc. The magnitude of the EDP during ground motions may be estimated using non-linear dynamic analysis for specific intensities of the ground motions, i.e. for specific values of the ground-motion intensity measure (IM). The seismic hazard of the site under consideration, i.e. the annual exceedance ration of different levels of IM,  $\lambda_{IM}$ , is normally evaluated through PSHA. It is worth noting that the IM links seismic hazard, related to the site, with structural response. The IM used in PSDA if for structural engineers a quantification of the features of the ground motion that are significant for determining structural response and for seismologists the parameter used to describe seismic hazard.

As an example one of the most commonly used IM is the pseudo spectra acceleration at the natural frequency of structures  $Sa(T_1)$ , for 5% damping. This parameter is widely used because hazards maps in its terms are very common and because it is the most efficient IM for elastic SDOF systems. Nevertheless, because of the scarce information that this parameter provides on the spectral shape, many researchers have shown that the structural response that an engineer obtains by scaling ground motions at  $Sa(T_1)$  may depended on ground motion selection parameters and in particular on magnitude, source to site distance, and epsilon (Baker and Cornell 2006; Luco and Bazzurro 2007). Similar issues have been observed for other IMs as PGA and PGV. For these reasons research has been focused on the definition of the optimal properties that a IM should possess and on the definition of procedures for assessing these latter properties (Baker and Cornell 2004; Baker and Cornell 2005;

Intensity measure	Name	Reference
$PGA = \max\left(\left a(t)\right \right)$	Peak ground acceleration	-
$a_{sq} = E_a = \int_0^{t_f} a^2(t) dt = a_{rs}^2$	Squared acceleration	_
$P_{a} = \frac{1}{t_{2} - t_{1}} \int_{t_{1}}^{t_{2}} a^{2}(t) dt$	Earthquake power index	Housner (1975)
$a_{rms} = \sqrt{P_a}$	Root mean square acceleration	Housner & Jennings (1964)
$I_{A} = \cos^{-1}\beta / \sqrt{1 - \beta^2} \int_0^{t_f} a^2(t) dt$	Arias Intensity.	Arias (1970)
$a_{rs} = \sqrt{a_{sq}}$	Root square acceleration	Housner (1970)
$I_{C} = a_{rms}^{1.5} \left( t_{95} - t_{05} \right)^{0.5}$	Characteristic Intensity	Park et al. (1985)
$I_a = a_{\max} \left( t_{95} - t_{05} \right)^{1/3}$	_	Riddell & Garcia (2001)
$A_{95} = E_x = 0.05E_a$	_	Sarma & Yang (1987)
$I_{ap} = PGA \cdot Dur_{0.5 \cdot PGA}^{0.5}$	-	Aptikaev (1982)
$P_D = \frac{I_A}{v_0^2}$	Potential destructiveness	Araya & Saragoni (1980)
$Sa(T_1)$	Spectral acceleration at T <sub>1</sub>	Shome, Cornell et al. (1998)
$\overline{EPA} = \frac{1}{2.5} \frac{1}{0.4} \int_{0.1}^{0.5} Sa(5\%, T) dT$	Effective peak acceleration	FEMA (1994)
$Sa(T_{1} - > T_{\mu}) = \frac{1}{T_{\mu} - T_{1}} \int_{T_{1}}^{T_{\mu}} Sa(5\%, T) dT$	-	Kurama & Farrow (2003)
$Sa_{avg}(T_1,\ldots,T_n) = \left(\prod_{i=1}^n Sa(T_i)\right)^{1/n}$	-	Baker & Cornell (2006)
$I_{N_p} = Sa(T_1) \left[ \frac{Sa_{avg}(T_1, \dots, T_n)}{Sa(T_1)} \right]^{\alpha}$	-	Bojórquez & Iervolino (2011)
$S_a(T_1) \left[ \frac{S_a(cT_1)}{S_a(T_1)} \right]^{\alpha}, c = 2, \alpha = \frac{1}{2}$	-	Cordova et al. (2001)
$I_{M,D} = \alpha Sa(T_1)^{\beta_1} R_{Sa}^{\beta_2} e^{\beta_3 t_{SM}}$	_	Mehanny & Deierlein (2000)
$IM_{SE} = Sa(T_1)^{1-\alpha} Sa(\sqrt{R}T_1)^{\alpha}$	_	Mehanny (2009)
$IM_{CR} = Sa(T_1)^{1-\alpha} Sa(\sqrt[3]{R}T_1)^{\alpha}$	-	Mehanny (2009)
$\left[Sa(T_1),Sa(T_2)\right]$	-	Shome (1999)
$\left[Sa(T_1),Sa(T_2)/Sa(T_1)\right]$	_	Baker & Cornell (2004)
$\left[Sa(T_1),\varepsilon\right]$	_	Baker & Cornell (2005)

Table 3.1. Acceleration based intensity measures considered in the present study.

Baker and Cornell 2006; Luco and Cornell 2007; Tothong and Luco 2007).

The present paper illustrates some results of a vast comparison among the efficiency and sufficiency of the most important IMs that can be found in the literature. The study was carried out considering both SDOF and MODF non-linear systems and allowed to identify the IMs most suited for describing the response of the various systems.

Intensity measure	Name	Reference
$PGV = \max\left(\left v\left(t\right)\right \right)$	Peak ground velocity	-
$v_{sq} = E_v = \int_0^{t_f} v^2(t) dt = v_{rs}^2$	Squared velocity	-
$P_{v} = \frac{1}{t_{2} - t_{1}} \int_{t_{1}}^{t_{2}} v^{2}(t) dt$	_	_
$v_{rms} = \sqrt{P_v}$	Root mean square acceleration	_
$v_{rs} = \sqrt{E_v}$	Root square velocity	_
$I_F = PGV (t_{95} - t_{05})^{0.25}$	-	Fajfar et al. (1990)
$I_{v} = PGV^{2/3} (t_{95} - t_{05})^{1/3}; t_{d} = t_{95} - t_{05}$	_	Riddell & Garcia (2001)
$MIV = \max(IV)$	Maximum incremental velocity.	Bertero et al. (1976)
$Sv(T_1)$	Spectral velocity	-
$\overline{EPV} = Sv(5\%, 1s)$	Effective peak velocity	FEMA (1994)
$SI_{H}(\beta,T_{1},T_{2}) = \int_{T_{1}}^{T_{2}} Sv(\beta,T) dT$	Housner's spectral intensity	Housner (1952)
$SI_{HC} = SI_{H}(\beta, 0.1, 1.0);$	-	Hidalgo & Clough (1974)
$SI_{M} = \frac{1}{T}SI_{H}(T, 2T)$	_	Matsumura (1992)
$SI_{MR} = \frac{1}{T - T_y} SI_H(T, T_h)$	_	Martínez-Rueda (1998)
$SI_{KK} = SI_H \left(\beta, T_1 - t_1, T_1 + t_2\right)$	_	Kappos & Kyriakakis (2000)

Table 3.2. Velocity based intensity measures considered in the present study.

#### 2. APPROACHES TO PSDA

The main output of PSDA is the exceedance ratio of a given value y of EDP conditional to an IM value, i.e. P[EDP > y | IM = im]. Combining this function with information on seismic hazard the annual exceedance frequency of various EDV values is easily computed as:

$$\lambda_{EDP}(y) = \int_{im} P[EDP > y | IM = im] \cdot | d\lambda_{IM}(im) |$$
(2.1)

As Jalayer (2003) and Baker (2005) have shown the function P[EDP > y | IM = im] may be obtained through different approaches that will be briefly discussed in the following.

#### 2.1. Cloud analysis

According to this type of analysis the structure under consideration is analysed considering a set of unscaled ground motions, each one in general will therefore have a different IM value. In the following regression analysis is used on the couples EDP-IM in order to estimate the mean value and the standard deviation of EDP given IM. In the literature the most common regression technique considers a liner relationship between the natural logarithms of the variables under consideration (Cornell, Jalayer et al. 2002), i.e.:

$$\ln(EDP) = a_1 + a_2 \ln(IM) + e$$
(2.2)

where  $a_1$  and  $a_2$  are unknown regression parameters and e is a standard error variable (Cornell, Jalayer

Intensity measure	Name	Reference
$PGD = \max( d(t) )$	Peak ground	-
$\Gamma O D = \max( u(t) )$	displacement	
$P_{d} = \frac{1}{t_{2} - t_{1}} \int_{t_{1}}^{t_{2}} d^{2}(t) dt = \frac{1}{t_{95} - t_{05}} \int_{t_{95}}^{t_{05}} d^{2}(t) dt = d_{sq}^{2}$	-	_
$d_{sq} = E_d = \int_0^{t_f} d^2(t) dt$	_	-
$d_{rs} = \sqrt{E_d}$	-	_
$I_{d} = d_{\max} t_{d}^{1/3}; t_{d} = t_{95} - t_{05}$	_	Riddell & Garcia (2001)
$Sd(T_1)$	_	_

**Table 3.3.** Displacement based intensity measures considered in the present study.

Table 3.4. Hybrid intensity measures considered in the present study.

Intensity measure	Name	Reference
$I_{z} = \left(\int_{0}^{Dt} a(t)^{2} dt\right) / \left(PGA \cdot PGV\right)$	_	Cosenza & Manfredi (1998)
$SI_{a} = \frac{1}{0.157} SI_{H} (\beta, 0.028, 0.185)  T \in [0.118, 0.500] s$	_	Nau & Hall (1984)
$SI_{v} = \frac{1}{1.715} SI_{H} (\beta, 0.285, 2.000)  T \in [0.500, 5.000] s$	_	Nau & Hall (1984)
$SI_{d} = \frac{1}{8.333} SI_{H} (\beta, 0.4.167, 12.500)  T \in [5.000, 14.085] s$	_	Nau & Hall (1984)

et al. 2002). The function P[EDP > y | IM = im] is simply obtained from Eq. (2.2) by substituting the IM value of interest.

#### 2.2. Stripe Analysis

When structural analysis is aimed ad characterizing the EDP distribution for one singe value of IM, the ground-motions considered can be scaled at the target IM value. At this point the data obtained are described through a statistical distribution, most commonly a lognormal distribution:

$$P[EDP > y | IM = im] = 1 - G_{EDP|IM}(y | im) = 1 - \Phi\left(\frac{\ln y - \hat{\mu}}{\hat{\beta}}\right)$$
(2.3)

where  $\hat{\mu}$  and  $\hat{\beta}$  are the estimates of the distribution parameters obtained from data. This procedure can be repeated for various values of IM. Sometimes empirical distributions are used without the need of defining parametric distributions.

## 2.3. Capacity analysis

With this approach the distribution P[EDP > y | IM = im] is not directly estimated but results of incremental dynamic analyses are used to define the distribution of the values of IM that lead to the a specific EDP level, i.e. the distribution,

$$F_{IM_{CAP}|EDP}(im | y) = P(IM_{CAP} < im | EDP = y)$$

$$(2.4)$$

where  $IM_{CAP}$  is a random variable indicating the distribution of the IM values leading to a give level y of EDP.

# **3. GROUND-MOTION INTENSITY MEASURES**

Many authors have proposed IMs in order to find synthetic indicators for the intensity of ground motions. IMs may be defined in many different way, the most simple being define on parameters of ground motions record such as acceleration, velocity, displacement, duration, etc. More advanced definition make use of response spectra and therefore take into account structural properties such as the natural period. Finally advanced IMs have been defined considering the response of non-linear systems (Tothong and Luco 2007). The present study was focused on the analysis of efficiency and sufficiency of the IMs listed in Tables 3.1-3.4. Most of the IMs proposed in the literature were considered, not considering only advanced non-linear IMs because those are very structure-specific measures.

# 3.1. Properties of the intensity measures

In order to achieve good accuracy in the evaluation on structural response through PSDA it is important to use IMs with specific properties, the most important being efficiency, sufficiency and scaling robustness.

# 3.1.1. Efficiency

A IM is defined efficient if it allows to obtain a reduce variability in the structural response for a given IM vale,  $\sigma_{\ln EDP|IM}$ . This reduced variability is desirable because, the standard error on the sample mean of the natural logarithm of EDPs, for a specific IM values, is proportional to  $\sigma_{\ln EDP|IM}$ , and the sample mean of  $\ln(EDP|IM)$  is the first order information normally used to evaluate the first term of Eq (2.1). Furthermore, since standard error is in ratio to the square root of the number of analyses carried out a reduction of  $\sigma_{\ln EDP|IM}$  allows to reduce the number of analyses required to achieve a given accuracy on the estimate of EDP|IM.

## 3.1.2. Sufficiency

An intensity measure, IM, is sufficient when the conditional probability distribution  $P[EDP > y | IM = im] = 1 - G_{EDP|IM}(y | im)$  is independent form other parameters that are involved in the assessment of seismic hazard, such as magnitude, distance and epsilon (Baker and Cornell 2006). This feature is desirable because it follows that any set of ground motion chosen for the analysis will produce the same  $G_{EDP|IM}$ . On the contrary if a IM is not sufficient the estimate of  $G_{EDP|IM}$  will also depend on the aforementioned parameters.

## 3.1.3. Scaling robustness

Another feature that a good IM should posses is scaling robustness, i.e. when accelerograms are scaled to a target IM values the results in terms of EDP should be unbiased compared to results of unscaled ground motions. In other words structural response should be independent from the scaling factor.

## 3.1.4. Hazard computability

An IM possessing all the aforementioned properties will likely be extremely structure specific. Theoretically the best IM would be the EDP itself. Obviously, that would mean computing the hazard in terms of structural response, and therefore performing a large number of nonlinear analyses (Baker, Cornell et al. 2005; Tothong and Luco 2007; Buratti, Stafford et al. 2011). For this reason an additional requirement of an IM is computability of seismic hazard.

## 4. COMPARRISON OF THE PROPERTIES

In the present study efficiency and sufficiency in terms of magnitude and distance of the IMs listed in Tables 3.1-3.4 were compared. This section describes the adopted ground-motion dataset and the SDOF and the MDOF systems considered. In any case the EDP considered was the interstorey drift.

## 4.1. Ground-motion dataset

The reference ground-motions dataset used in the present work contains 1704 accelerograms selected



Figure 4.1. SDOF system with T = 0.25 s: stepwise regression for IM = PGA (left) and correlation between the residual of the regression for IM = PGD and moment magnitude (right).

among those contained in the Next Generation of Attenuation dataset. The selection criteria adopted to select the ground motions were the following: *i*) the moment magnitude,  $M_w$ , is known, *ii*) the Joyner-Boore distance,  $R_{JB}$ , is known, *iii*) the shear wave velocity in the uppermost 30 m is known *iv*) the maximum usable period is greater than 3 s. Furthermore recordings from the Chi-Chi earthquake were excluded in order to avoid bias in the data since their number is as large as ne number of all the other recordings. For each system considered a nonlinear dynamic analysis was carried out with each of the accelerograms here described.

### 4.2. Structures considered

#### 4.2.1. Single Degree of Freedom (SDOF) systems

SDOF systems with periods T = 0.1, 0.25, 0.5, 0.75, 1.0, 1.5 e 2.0 s were considered. The Takeda hysteretic model was adopted and Opensees was used to carry out the analyses (Buratti 2011).

## 4.2.2. Multi Degree of Freedom (MDOF) systems

Three plane RC frames were considered in the present study. The number of storeys of each frame is respectively, two, four and six. The natural periods of these structures are T = 0.46 s for the two storey frame, T = 0.69 s for the four storey structure, and T = 0.84 s for the six storey frame. Further details can be found in (Buratti 2011). Non-liner models of these frames were implemented in Opensees considering fibre-based elements with material nonlinearities confined to the element ends over specified plastic hinge lengths (Scott and Fenves 2006).

## 4.3. Non-linear analysis method

In the present study cloud analysis was used in order to avoid any scaling of the ground motions, scaling in fact would require the IMs considered to be robust but this latter property was not investigated. The EDP considered is the interstorey drift, probably the most widely used EDP in the literature. It was evaluated for all the systems described in Section 4.2 and for all the accelerograms introduced in Section 4.1.

## 4.4. Regression analysis

Maximum interstorey drifts obtained from the analyses were used to investigate sufficiency and efficiency of the IM considered. In the present work, in order the reduce the model dependency of the results, the regression model normally adopted in the literature (Cornell, Jalayer et al. 2002) was extended considering a  $3^{rd}$  degree polynomial model in terms of  $\ln(IM)$ :

$$\log(EDP) = a_1 + a_2 \log(IM) + a_3 \log(IM)^2 + a_4 \log(IM)^3 + e$$
(4.1)

High efficiency Low efficiency				High efficiency Low efficiency											
IM	Eff.	Suff.	Suff.	IM	Eff.	Suff.	Suff.	IM	Eff.	Suff.	Suff.	IM	Eff.	Suff.	Suff.
	σ	$M_w$	$R_{JB}$		σ	$M_w$	R <sub>JB</sub>		σ	$M_w$	$R_{JB}$		σ	$M_w$	$R_{JB}$
$I_{Np}^{\dagger}$	0.121	0.01	0.028	$v_{rs}$	0.439	0.53	0.692	$IM_{SR}^{\dagger}$	0.148	0.127	0.008	$I_c$	0.363	0.536	0.222
EPA	0.130	0.005	0.021	$d_{Trif}$	0.446	0.18	0.374	$IM_c$	0.160	0.229	0.055	$I_a$	0.371	0.520	0.176
$IM_{CR}^{\dagger\dagger}$	0.131	0.09	0.081	Pd	0.530	0.47	0.751	$v_{sq}$	0.160	0.238	0.081	Pa	0.388	0.524	0.181
$IM_c$	0.134	0.14	0.131	$d_{rms}$	0.530	0.47	0.751	$v_{rs}$	0.160	0.238	0.081	$a_{rms}$	0.388	0.524	0.181
$SIh_{kk}$	0.136	0.056	0.008	pgd	0.530	0.46	0.763	Sa <sub>avg</sub>	0.161	0.221	0.019	pga	0.390	0.517	0.151
Sa <sub>avg</sub>	0.138	0.13	0.147	$I_z$	0.557	0.18	0.697	$SI_M$	0.164	0.231	0.022	EPA	0.395	0.503	0.133
Sa	0.143	0.05	0.006	$I_d$	0.569	0.41	0.789	pgd	0.181	0.204	0.449	SI <sub>a</sub>	0.419	0.486	0.107
$IM_{SR}^{\dagger\dagger\dagger\dagger}$	0.147	0.18	0.185	$d_{sq}$	0.574	0.41	0.787	$IM_{CR}$	0.182	0.307	0.082	$I_z$	0.424	0.407	0.209
$SI_M$	0.154	0.15	0.182	$d_{rs}$	0.574	0.41	0.787	Pd	0.186	0.177	0.363	$d_{Trif}$	0.482	0.371	0.240
$^{\dagger}\alpha = 0.4; ^{\dagger\dagger}R = 4; ^{\dagger\dagger\dagger}R = 6$					$^{\dagger}R = 6$	; $^{\dagger\dagger}R =$	4								

**Table 5.1.** Results of the analyses con the SDOF systems with T = 0.25 s. (left) and T = 2.0 s (right)

where  $a_1$  to  $a_4$  are regression parameters and e is a standard error term. Adding terms to a regression model may in general not have positive effects only. In fact if there is no theoretical basis for selecting the form of the regression model one may use redundant terms that reduce the accuracy of the regression model and do not make possible to identify the most significant effects. Conclusions of sufficiency and efficiency will therefore be affected. In order to avoid these issues a stepwise regression procedure was used. It is a systematic procedure for adding or removing terms from a multi-linear regression model based on their statistical significance (F-test). Stepwise regression requires to define a trial model (Eq. (4.1) in the present work) the fitness to the data of which is compared to the one of models obtained by adding or removing terms form the initial model. Figure 4.1 shows, as an example, one of the regression models fitted with respect to the drift of the SDOF with T = 0.25 s and assuming IM = PGA. It is clear from the figure that the inclusion of a non-linear term in log(IM) enhances the regression model. If a linear model were used larger residuals would be obtained and conclusions on the efficiency of the IM would also be affected. The goodness of fit of all the regressions performed was checked using normality tests on the residuals and quantile–quantile plots.

## 4.5. Efficiency analysis

An IM is efficient, when for a given IM values the variability in structural response is small. An efficiency indicator is the dispersion of the residuals of the regression models described previously. A measure of this dispersion is the standard deviation of the standard error term e, in Eq (4.1).

#### 4.6. Sufficiency analysis

The sufficiency of the IM may be evaluated, in the framework adopted, by verifying whether the residuals obtained from the regression carried out as described in Section 4.4 show any dependency on other ground-motion parameters as: magnitude, distance and epsilon. If no dependency is found then it is possible to assume that they do not affect structural response for a give IM values. In other words it means that with respect to the EDP considered the selected IM provides a sufficient description of the features of the ground-motion affecting structural response. As an example, the right panel of Figure 4.1, shows a plot of the residuals of the regression carried out for the drift of the SDOF system with T = 0.25 s and IM = PGD. Sufficiency with respect to magnitude is clearly lacking in this case. In the present study the sufficiency in terms of M and R only was considered.

## **5. RESULTS OF THE ANALYSIS**

This section describes the main results obtained. For the sake of brevity only the most and the least efficient IMs will be reported for the some of the systems considered.

High effi	ciency		Low efficiency								
IM	Eff.	Suff. Suff.		IM	Eff.	Suff.	Suff.				
	σ	$M_w$	$R_{JB}$		σ	$M_w$	R <sub>JB</sub>				
$I_{Np}^{\dagger}$	0.08	0.15	0.10	SIa	0.31	0.25	0.058				
Sd	0.09	0.03	0.00	pgd	0.35	0.40	0.635				
$SI_M^{\dagger\dagger}$	0.10	0.15	0.15	Pd	0.35	0.39	0.610				
SIh <sub>KK</sub>	0.10	0.05	0.03	$d_{rms}$	0.35	0.39	0.610				
Sa <sub>avg</sub> <sup>††</sup>	0.10	0.14	0.10	$I_d$	0.38	0.36	0.685				
EPV	0.11	0.14	0.11	$d_{Sq}$	0.38	0.35	0.676				
Sv	0.11	0.14	0.11	$d_{Rs}$	0.38	0.35	0.676				
$SI_v$	0.12	0.05	0.00	$d_{Trif}$	0.40	0.21	0.271				
$IM_{CR}^{\dagger\dagger\dagger}$	0.12	0.04	0.05	$I_z$	0.41	0.03	0.493				
$^{\dagger}\alpha = 0.4;$	$^{\dagger}\alpha = 0.4$ ; T = 0.84 s; $^{\dagger\dagger\dagger}R = 4$ , T = 0.84 s										

Table 5.2. Results of the analyses on the six storey RC frame.

## 5.1. Analysis of the SDOF systems

The left panel of Table 5.1 reports the IMs with the largest and the smallest efficiencies for the SDOF systems with periods T = 0.25 s and T = 2.0 s. The correlation coefficient between the regression residuals and moment magnitude and Joyner-Boore distance are also reported. These latter are indicator of the sufficiency. From the literature it is known that the most efficient IMs for short period structures are those base on acceleration. This conclusion is confirmed by the results in Table 5.1. It is interesting to notice that the most efficient IMs are not those based on a single spectral ordinate but those based on combinations of more than one spectral value in the proximity of the natural period of the system considered. These IMs, in fact, take into account the period elongation due to stiffness reduction when non-linear deformations occur. The most efficient IM,  $I_{NP}$ , contains also information of the shape of the response spectrum (Bojórquez and Iervolino 2011). The least efficient IMs in this case are those based on displacements and velocities. It is also interesting to notice that high efficiency measure have in general also good sufficiency while low efficiency ones have poor sufficiency as well. The results in the right panel of Table 5.1 are referred to the SDOF system with T = 2.0 s. In this case the IMs based on acceleration or on a single spectral acceleration show little efficiency while those based on multiple spectral accelerations still perform well. In this case, as known in the literature, measure based on displacements and velocities have higher efficiencies with respect to the case previously discussed. Most of the efficient measures show also good sufficiency.

## 5.2. Analysis of the MDOF systems

This section describes the results concerning the MDOF structures described in Section 4.2.1. Also in this case for the sake of brevity it has not been possible to provide the whole set of results. Only the maximum interstorey drift for the ground-storey of the six-storey frame will be discussed because this is the storey that showed the largest inelastic deformations. Table 5.2 reports the IMs with the largest and the smallest efficiency as well as theirs sufficiency with respect to  $M_w$  and  $R_{JB}$ . The most efficient measures are also in this case those based on multiple spectral acceleration. The IM with the best performance is  $I_{NP}$ . Good efficiency was also obtained with respect to velocity and spectral displacements evaluated at the natural period of the structure. The least efficient measures are those based on displacements. In terms of sufficiency the same conclusions previously drawn are valid.

## 5.3. Definition of vector intensity measures

In order to increase the efficiency of traditional IMs, researchers have proposed to define vector IMs (Baker and Cornell 2004). In the present study the results of the analyses discussed in Sections 5.1 and 5.2 were used to define couples of IMs best suited to form vector IMs, (IM<sub>1</sub>, IM<sub>2</sub>). In particular the residuals of the step-wise regression performed for any IM where compared to all the remaining IMs in order to identify possible correlations. In fact, if a strong correlation is found between the residuals of the regression for IM<sub>1</sub> and a second intensity measure IM<sub>2</sub>, then adding IM<sub>2</sub> to the regression model would lead a better prediction of the structural response because IM<sub>2</sub> describes features of the ground motions that IM<sub>1</sub> does not describe. The vector measure (IM<sub>1</sub>, IM<sub>2</sub>) would be more efficient than IM<sub>1</sub>

**Table 5.3.** SDOF system with T = 2.0 s: correlation between the residuals of the stepwise regression for IM =  $I_{NP}$  and some of the IMs considered in the present study.

$d_{Sq}$	$I_d$	$P_d$	pgd	$v_{sq}$	$v_{rs}$	$d_{Trif}$	$I_v$	IM <sub>SR</sub>
0.466	0.450	0.431	0.421	0.205	0.205	0.186	0.180	0.158

alone. As an example Table 5.3 lists the highest correlations found among the residuals of the regression performed considering the response of the SDOF system with T = 2.0 s and the intensify measure  $I_{NP}$  with all the other IMs considered in the present study. The efficiency of the so obtained vector measures was evaluated using the same procedure described in Section 4.5, extending the model in Eq. (4.1) by including polynomial terms related to the independent variable IM<sub>2</sub>. Stepwise regression was used to remove non–significant terms. For example, for the SDOF system with T = 2.0 s the standard deviation of the error term for the vector IM ( $I_{NP}$ ,  $d_{Sq}$ ) is 0.126 while for the scalar IM  $I_{NP}$  is 0.196.

## **6. CONCLUSIONS**

The present study investigated the efficiency and the sufficiency with respect to  $M_w$  and  $R_{JB}$  of a large number of IMs proposed in the literature. These properties were evaluated considering various SDOF and MODF nonlinear systems. "Cloud" analysis was used adopting 1704 unscaled accelerograms. The results of the analyses were post-processed using stepwise regression in order to define the optimal regression model for each IM considered. The results of the analysis allowed to classify the IMs in terms of efficiency and sufficiency for the various systems considered. Furthermore it was define a procedure for searching candidates for forming vector intensity measures. These latter have higher efficiency that scalar measure but normally increase the effort required for computing seismic hazards.

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