

# STFT Based Real-time Control with Variable Stiffness and Damping of Smart Tuned Mass Damper for Seismic Protection



**C.Sun**

*Department of Civil and Environmental Engineering, Rice University, 6100 Main Street, Houston, TX 77005, United States*

**S.Nagarajaiah**

*Department of Civil and Environmental Engineering and Department of Mechanical Engineering and Material Science, Rice University, 6100 Main Street, Houston, TX 77005, United States*

## SUMMARY

A short time Fourier transformation (STFT) based real-time control algorithm for semiactive tuned mass damper (STMD) is proposed for seismic protection. STFT is implemented to obtain the time-frequency distribution of the structural response, where the dominant frequency within each time segment is extracted. Then the stiffness and damping of the STMD are tuned based on the dominant frequency in the new control algorithm. In addition to the STMD, a multiple TMD (MTMD) consisting of an STMD and a TMD in parallel is studied. Three sets of recorded seismic ground motions are utilized as excitation to evaluate the performance of the STMD and the MTMD. Results show that the proposed STMD and MTMD are more effective than a conventional TMD in reducing the peak responses. Response spectra evaluation also indicates the effectiveness of the STMD and MTMD over a broad period range as compared with the case with and without a TMD.

*Keywords: Smart tuned mass damper, Real-time tuning, Variable stiffness and damping, Seismic protection*

## 1. INTRODUCTION

Tuned mass dampers (TMDs) have been widely used in civil structures to attenuate wind or earthquake induced vibrations. Since Watts (1883) originated the application of a TMD, much effort has been made into the investigation and design of TMDs. Ormondroyd (Hartog 1928) theoretically investigated the TMD in case of harmonic excitations and later the optimal design formula was presented by Den Hartog (1956).

In order to appraise the effectiveness of TMDs for seismic protection, a series of researches were implemented. On the one hand, Gupta and Chandrasekaran (1969) investigated the reduction effect of TMDs with elastic-plastic properties for a SDOF structure subjected to the S21W component of the Taft acceleration. Their findings indicated that TMDs are not as effective in reducing structural responses under seismic excitations as they are in reducing responses under harmonic excitations. Kaynia et al. (1981) evaluated the effectiveness of TMDs for reducing fundamental mode response by using an ensemble of 48 earthquake accelerograms. They found that TMDs are less effective than expected. Sladek and Klingner (1983) analyzed a TMD designed using Den Hartog (1956) formula. The TMD was placed on the top floor of a 25-storey building subjected to the S00E component of Elcentro accelerogram, Imperial Valley earthquake. Similarly, the authors concluded that the TMD is ineffective.

On the other hand, Wirsching and Yao (1973) studied the first mode response of a five- and ten-storey building with 2% damping ratio and subjected to a non-stationary ground acceleration. They tuned the TMD's frequency to the fundamental frequency of the structure, and used a 20% damping ratio. Considerable reduction of response was observed. In addition, Wirsching and Campbell (1973) optimized the selection of TMD parameters and he obtained effective reduction effect. Afterwards, Villaverde (1985, 1994, Koyama 1993, Martin 1995) investigated the effect of TMD parameters for seismic application. Their results indicated that the TMD should be in resonance with the main

structure and a design formula for the damping ratio of the TMD was proposed (Villaverde 1985) for effective seismic response reduction. Based on the work of Villaverde et al. (1985, 1993, 1994, 1995), Sadek et al. (1997) proposed a modified formula for the frequency ratio and damping ratio of the TMD.

In contrast with passive TMDs in seismic protection, semi-active TMDs (STMDs), which use variable stiffness or damping devices, have been proposed and investigated. Hrovat et al. (1983) controlled wind-induced vibrations of tall buildings using an STMD with variable damping. Their study showed that an STMD is better than a TMD in reducing the structural responses, including the displacements and accelerations. Abe (1996a, 1996b) developed an optimal control algorithm for STMD in reducing transient responses and then utilized it for seismic protection of civil structures (1996b). The authors found that the STMD can obtain higher reduction effect than the TMD.

Recently, Nagarajaiah (2000) has developed a new semi-active continuously and independently variable stiffness (SAIVS), based on which variable stiffness STMD has been proposed and analyzed in reducing structural responses under stationary and non-stationary excitations (Varadarajan and Nagarajaiah 2004, Nagarajaiah and Varadarajan 2005, Nagarajaiah and Sonmez 2007, Nagarajaiah 2009). Their findings indicated that the SAIVS-TMD's stiffness can be smoothly and reliably retuned, thereby effectively attenuating the responses of the main structures. Short time Fourier transform (STFT) is the most widely used method in analyzing non-stationary signals (Cohen 1995). Nagarajaiah et al. (2005, 2009) tuned the stiffness of an STMD using a STFT based control algorithm to control wind induced vibration of a tall building and obtained comparable reduction effect to that of an active TMD. Nagarajaiah and Narasimhan (2007) proposed semi active variable damping device, similar to SAIVS device.

In the present study, the SAIVS-TMD controlled by an STFT based control algorithm for seismic protection is numerically analyzed. The structural response is tracked and analyzed using STFT continuously, based on which the stiffness and damping of the SAIVS-TMD is retuned in real time. In addition to the variable stiffness (Nagarajaiah and Varadarajan 2005), the damping ratio is also variable in the control algorithm proposed in the present study. Besides, an MTMD consisting of an STMD and a TMD in parallel is also studied. Recorded seismic ground motions are used to evaluate the effectiveness of the STMD and the MTMD. Results show that the STMD and the MTMD have comparable effectiveness on the reduction of displacement spectra. When compared to the TMD, the STMD and the MTMD have better reduction. In addition, the STMD and MTMD can reduce the acceleration spectra over a broad range of period.

## **2. DYNAMICAL MODEL**

The configuration of the primary structure with the STMD, and the TMD, and the relevant equations of motion are presented in this section.

### **2.1 Configuration of the model**

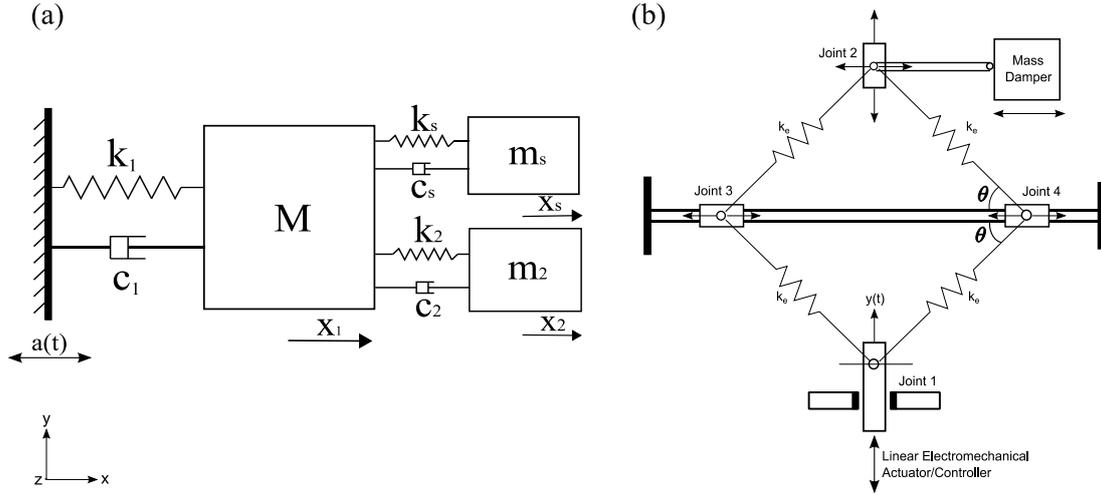
As shown in Fig. 1, the primary structure is simplified as a single degree-of-freedom system excited by earthquake ground acceleration  $a(t)$ . The semi-active variable stiffness and damping is adopted for controlling the STMD.

### **2.2 Equations of motion**

The equations of motion of the model are obtained as:

$$\begin{aligned}
M\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 + c_s(\dot{x}_1 - \dot{x}_s) + k_s(x_1 - x_s) + \\
c_2(\dot{x}_1 - \dot{x}_2) + k_2(x_1 - x_2) &= -Ma(t) \\
m_s\ddot{x}_s + c_s(\dot{x}_s - \dot{x}_1) + k_s(x_s - x_1) &= 0 \\
m_2\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) &= 0
\end{aligned} \tag{2.1}$$

where  $M$ , and  $m_s$  denote the mass of the primary structure and the STMD; parameters  $c_1$ , and  $c_s$  represent the damping coefficient of the primary structure and the STMD; parameters  $k_1$ , and  $k_s$  represent the stiffness of the primary structure and the STMD. Variables  $x_1$ , and  $x_s$  represent the displacements of the primary structure and the STMD; variable  $a(t)$  is the acceleration time history of an earthquake.



**Figure 1:** (a) Configuration of the primary structure with STMD and TMD and (b) STMD based on Semi-Active and Independently Variable Stiffness (SAIVS) device.

To express Eqn. 2.1 using the design parameters, following equations are defined:

$$\omega_1 = \sqrt{k_1/M}, \quad \omega_s = \sqrt{k_s/m_s}, \quad \omega_2 = \sqrt{k_2/m_2} \tag{2.2a}$$

$$f_1 = \omega_1/2\pi, \quad f_s = \omega_s/2\pi, \quad f_2 = \omega_2/2\pi \tag{2.2b}$$

$$\zeta_1 = c_1/2M\omega_1, \quad \zeta_s = c_s/2m_s\omega_s, \quad \zeta_2 = c_2/2m_2\omega_2 \tag{2.2c}$$

$$\varepsilon_s = m_s/M, \quad \varepsilon_2 = m_2/M, \quad \Omega_s = f_s/f_1, \quad \Omega_2 = f_2/f_1 \tag{2.2d}$$

where  $\omega_1(f_1)$ ,  $\omega_s(f_s)$  and  $\omega_2(f_2)$  denote the circular (natural) frequencies of the primary structure, the STMD, and the TMD, respectively; parameters  $\zeta_1$ ,  $\zeta_s$  and  $\zeta_2$  represent the damping ratios of the primary structure, the STMD, and the TMD; parameters  $\varepsilon_s$ , and  $\varepsilon_2$  are the mass ratios of the STMD and the TMD. Variables  $\Omega_s$ , and  $\Omega_2$  denote the frequency ratios of the STMD and the TMD.

Substitution of Eqn. 2.2 into Eqn. 2.1 yields:

$$\begin{aligned}
\ddot{x}_1 &= -2\zeta_1\omega_1\dot{x}_1 - \omega_1^2x_1 - 2\varepsilon_s\zeta_s\omega_s(\dot{x}_1 - \dot{x}_s) - \varepsilon_s\omega_s^2(x_1 - x_s) - \\
&\quad 2\varepsilon_2\zeta_2\omega_2(\dot{x}_1 - \dot{x}_2) - \varepsilon_2\omega_2^2(x_1 - x_2) - a(t) \\
\ddot{x}_s &= -2\zeta_s\omega_s(\dot{x}_s - \dot{x}_1) - \omega_s^2(x_s - x_1) \\
\ddot{x}_2 &= -2\zeta_2\omega_2(\dot{x}_2 - \dot{x}_1) - \omega_2^2(x_2 - x_1)
\end{aligned} \tag{2.3}$$

### 3. SHORT TIME FOURIER TRANSFORMATION (STFT) ALGORITHM

The basic idea of STFT is to divide a time signal into a number of time segments and to analyze each of the segments using Fourier transform to determine the frequencies existing in it. The spectrum is obtained for each different time and the totality of the spectra is the distribution of the signal in time-frequency domain.

Consider a signal  $x(\tau)$ , multiplying the signal by a window function  $h(\tau - t)$  yields:

$$\hat{x}(\tau) = x(\tau)h(\tau - t) \quad (3.1)$$

where  $\hat{x}(\tau)$  is a weighted signal;  $t$  is the fixed time and  $\tau$  is the running time. Fourier transformation to  $\hat{x}(\tau)$  gives the spectrum  $S_t(\omega)$  at the fixed time  $t$ :

$$S_t(\omega) = \frac{1}{\sqrt{2\pi}} \int e^{-j\omega\tau} \hat{x}(\tau) d\tau = \frac{1}{\sqrt{2\pi}} \int e^{-j\omega\tau} x(\tau)h(\tau - t) d\tau \quad (3.2)$$

The power spectral density at time  $t$  is

$$P(t, \omega) = |S_t(\omega)|^2 = S_t(\omega) \overline{S_t(\omega)} \quad (3.3)$$

In the present study, a moving Hamming window is used to multiply the original non-stationary signal in each of the time segments. After zero padding, FFT is implemented to produce  $S_t(\omega)$  and the corresponding power spectral density is obtained by  $S_t(\omega) \cdot \overline{S_t(\omega)}$ , where  $\overline{S_t(\omega)}$  is the conjugate of  $S_t(\omega)$ . At the fixed time  $t$ , the dominant frequency  $\omega_d$  is the value maximizing the power spectral density  $P(t, \omega)$ . A log scaling is used to represent the energy density where the unit is dB.

### 4. TUNING ALGORITHM

In this section, the tuning algorithm based on STFT is presented and the selection of the parameters in the control algorithm is described.

#### 4.1 STFT based control algorithm

Tuning of the SAIVS-TMD is implemented based on the feedback provided by the STFT control algorithm. The displacement of the primary structure is sensed and the power spectrum is obtained (Eqn. 3.1 ~ Eqn. 3.3) continuously. Then the current frequency of the STMD is returned to be the dominant frequency of the spectrum. The STFT based control algorithm is shown in Fig. 2. and described as follows. In the beginning at  $t = 0$ , the following initial parameters are adopted: frequency ratio  $\Omega_s = 1$  and the damping ratio  $\zeta_s = 0.01$ .

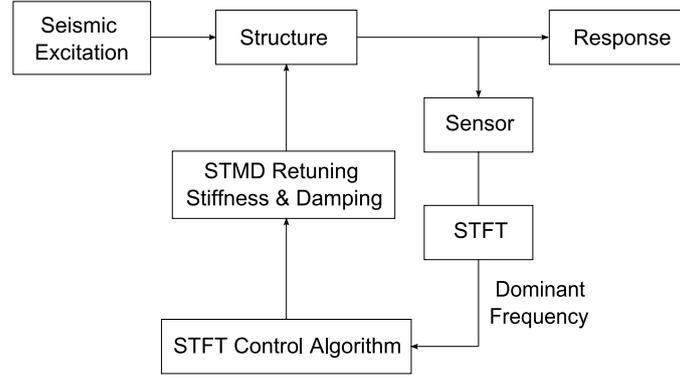
(1). A moving window of  $n$  time steps of signal at time  $t_i$  is convolved with a Hamming window  $h$  (Eqn. 3.1 ~ Eqn. 3.2) to compute the spectrum and then the power spectral density (Eqn. 3.3) is computed resulting in a vector  $P_i$  of size  $m \times 1$ .

(2). The frequency at which the element in  $P_i$  is maximum  $f_{id}$  is selected as the dominant frequency  $f_{id}$  at time  $t_i$ .

(3). If  $|f_{id} - f_1| < \delta$ , the frequency of the STMD  $f_s$  is tuned such that  $f_s = f_1$  ( $\Omega_s = 1$ ) and the damping ratio to be  $\zeta_s = 0.1$ , where  $f_1$  is the frequency of the primary structure in Hz,  $\delta$  is a parameter which is set to 0.05 in this paper; if  $|f_{id} - f_1| \geq \delta$ , the frequency of the STMD is tuned to be  $f_{id}$  and the damping ratio is to be  $\zeta_s = 0.01$ . A smoothing function  $g = \frac{g_1 + g_2}{2} + \frac{g_1 - g_2}{2} \cos(\frac{\pi}{\Delta T} t)$  is used to smoothly vary the stiffness or the damping from  $g_1$  to  $g_2$ , where  $\Delta T$  is the duration of

transition;  $\Delta T = 0.5s$  in the present study.

It is observed in the present study that a small damping ratio of the STMD helps to reduce the peak response of the primary structure under seismic excitation. But this small damping will produce beat vibration (Tsai 1993), thereby making the structure suffer from relatively long lasting oscillations with considerable amplitude as shown in Fig.3. In this case, it is proposed in the present study not only to retune the stiffness but also the damping of the STMD based on the feedback dominant frequency as described in step (3). This will be discussed in detail in Section 4.2.



**Figure 2** STFT based control algorithm

The frequency ratio and damping ratio of the TMD in the MTMD is computed using Eqn.4.1 & Eqn. 4.2 (Sadek *et al.* 1997)

$$\Omega = \frac{1}{1 + \varepsilon} \left[ 1 - \zeta_1 \sqrt{\frac{\varepsilon}{1 + \varepsilon}} \right] \quad (4.1)$$

$$\zeta = \frac{\zeta_1}{1 + \varepsilon} + \sqrt{\frac{\varepsilon}{1 + \varepsilon}} \quad (4.2)$$

where  $\zeta_1$  is the damping ratio of the primary structure;  $\varepsilon$  is the mass ratio, and  $\Omega$ , and  $\zeta$  are the optimal frequency ratio and damping ratio of the TMD needed for seismic protection.

## 4.2 Numerical example

This section numerically shows the effectiveness of using variable damping ratio for the STMD to reduce the structural response in time domain. Mass ratio of the STMD is set as  $\varepsilon_s = 0.03$ ; the mass ratios of the MTMD are  $\varepsilon_s = 0.02$ , and  $\varepsilon_2 = 0.01$ . To evaluate the effectiveness of the STMD and MTMD, a conventional passive linear TMD with a mass ratio of  $\varepsilon_2 = 0.03$  is used for comparison. Based on Eqn. 4.1 & Eqn. 4.2, the frequency ratio and the damping ratio of the TMD in the MTMD are computed to be  $\Omega_2 = 0.988$ , and  $\zeta_2 = 0.120$ . Similarly, for the TMD,  $\Omega_2 = 0.968$ , and  $\zeta_2 = 0.188$ . Period of the primary structure is chosen as  $T_n = 1.5s$ .

Fig. 3 illustrates the time history of the primary structure excited by Kobe (KJM) earthquake, the fault parallel component whose peak ground acceleration is 0.566g in three cases: fixed damping ratio  $\zeta_s = 0.01$ , or  $\zeta_s = 0.10$ , or variable damping ratio. It is observed in Fig. 3 that a fixed damping ratio  $\zeta_s = 0.01$  helps to reduce the peak response more effectively but produces beat vibration in the later part. Damping ratio  $\zeta_s = 0.10$  acts effectively after  $t = 15s$ , but is less effective for the reduction of peak response. In contrast, a variable damping ratio of the STMD not only improves the peak response reduction but also suppresses the beat vibration in the later part effectively. Details on the reduction can be found from Table 4.1 showing the peak reduction of each of the three cases (the value in parentheses is reduction in percentage). In Table 4.1, the reductions are identical in  $\zeta_s = 0.01$  and

variable  $\zeta_s$  cases; however, the variable  $\zeta_s$  reduces the free vibration in the later part more effectively because  $\zeta_s = 0.10$ .

**Table 4.1** Comparison of peak response reduction in different cases of  $\zeta_s$  (the minus sign “-” means reduction).

	$\zeta_s = 0.10$	$\zeta_s = 0.01$	Variable $\zeta_s$
No TMD	0.5457	0.5457	0.5457
TMD	0.4342(-20.4%)	0.4342(-20.4%)	0.4342(-20.4%)
STMD	0.4175(-23.5%)	0.3801(-30.3%)	0.3801(-30.3%)
MTMD	0.4209(-22.9%)	0.3847(-29.5%)	0.3874(-29.5%)

## 5. RESULTS AND DISCUSSIONS

In this section, the effectiveness of STMD and MTMD in seismic protection is evaluated and compared to the cases with TMD and no TMD for response spectra. Three recorded seismic data are used to evaluate the effectiveness of the STMD and MTMD, namely the Kobe earthquake (JMA) the fault parallel component (peak ground acceleration  $P_{GA} = 0.821g$ ), the Northridge earthquake (Newhall) the fault normal Newhall component (peak ground acceleration  $P_{GA} = 0.566g$ ), and the Imperial Valley earthquake (El Centro) the fault parallel component (peak ground acceleration  $P_{GA} = 0.313g$ ).

### 5.1 STFT spectrogram of the response

The STFT power spectrograms of the displacement of the primary structure under the three selected accelerograms are obtained of which only the one in the case of Kobe earthquake (JMA) is shown in Fig. 4(a) and its contour is shown in Fig. 4(b). Thick solid line in the contour represents the peak ridge which is the dominant frequency of the response.

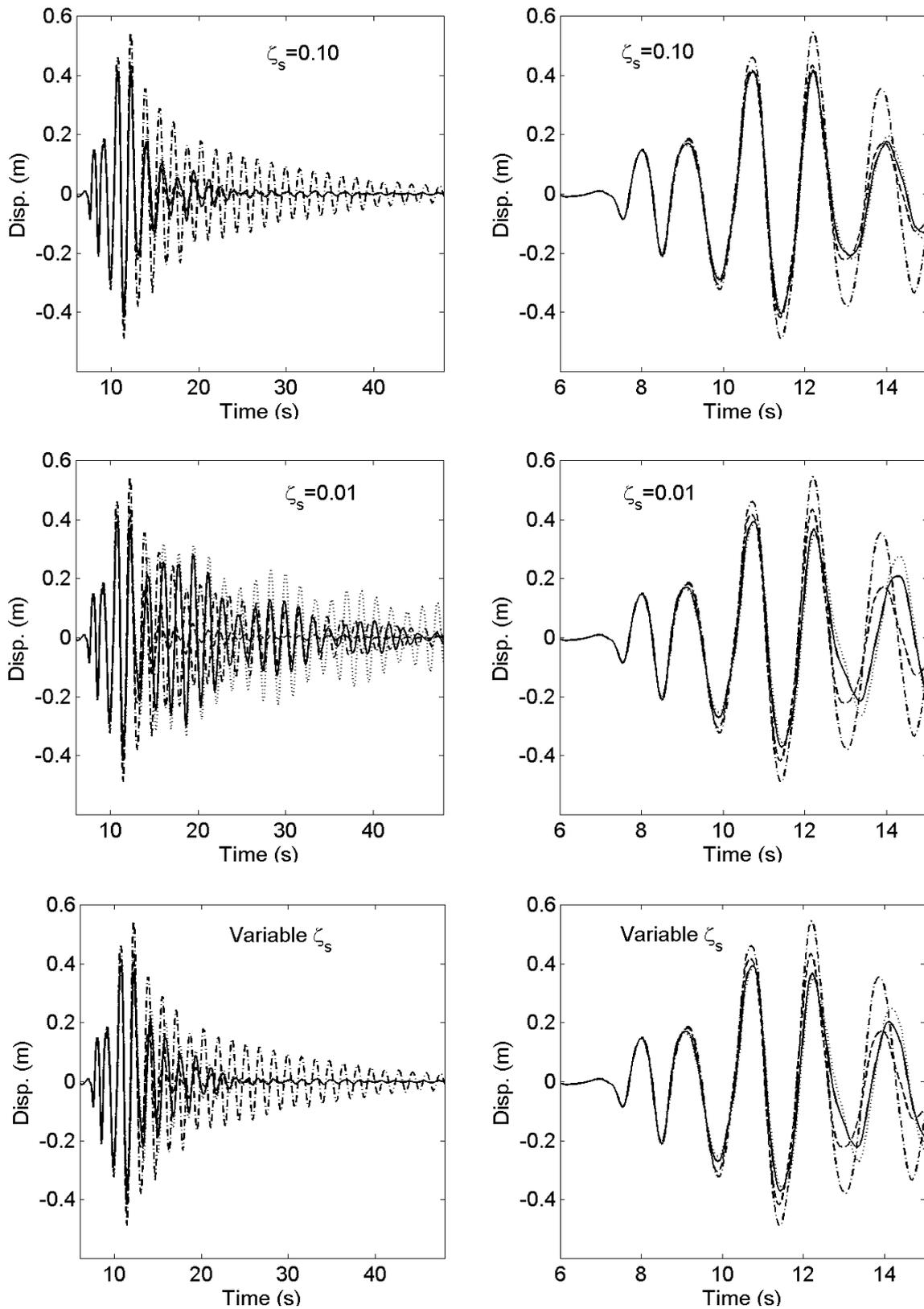
For each of the window, a set of response data within 4 seconds are taken and are zero-padded to a length of 1024. Based on the dominant frequency demonstrated in Fig. 4, frequency and damping ratio of the STMD is varied in the control algorithm. The results are presented in Fig. 5 where at each variation of frequency or damping ratio, the real time tuning value is delayed 0.5 seconds due to the use of a smoothing function introduced in Section 4.1. This smoothing function helps to mitigate the jump of the damping force or spring force, thereby decreasing the acceleration of the primary structure when the stiffness or damping is varied.

### 5.2 Spectra Response

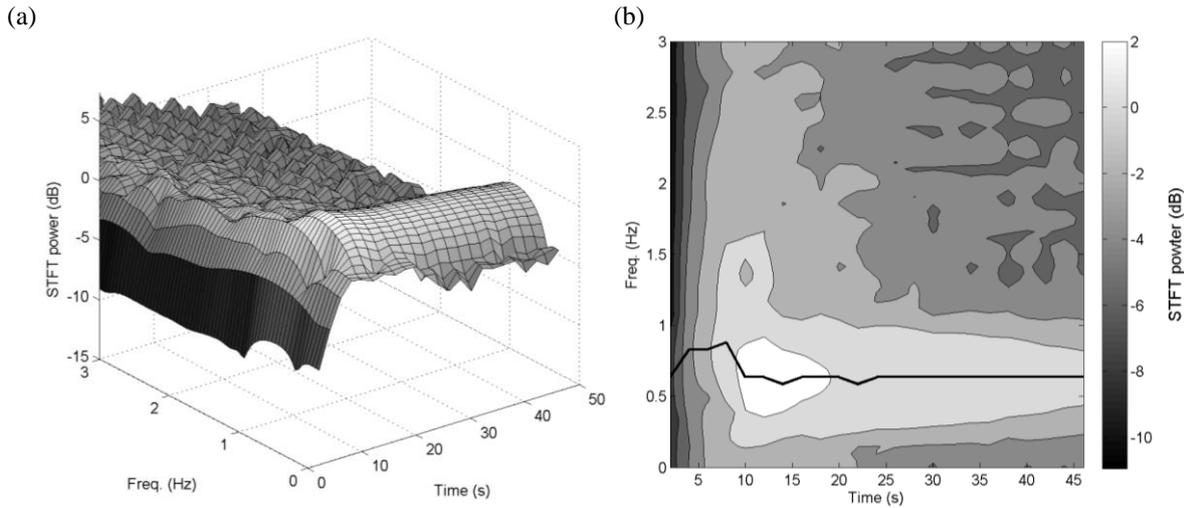
Fig. 6 shows the displacement and acceleration spectra of the primary structure excited by the three earthquakes. The response spectra are computed in the range of  $T_n \in [0.1 \quad 5]$  as shown in Fig 6.

It is evident in Fig. 6(a), Fig. 6(b) and Fig. 6(c) that the STMD or the MTMD, controlled by the STFT based tuning algorithm, can improve the reduction of the displacement spectra as compared to the case of a TMD. The peak spectra reduction due to the STMD or the MTMD is nearly 25% to 35% as compared to the case with no TMD as shown in Table 5.1. STMD and MTMD can reduce the spectral response further when compared to a TMD.

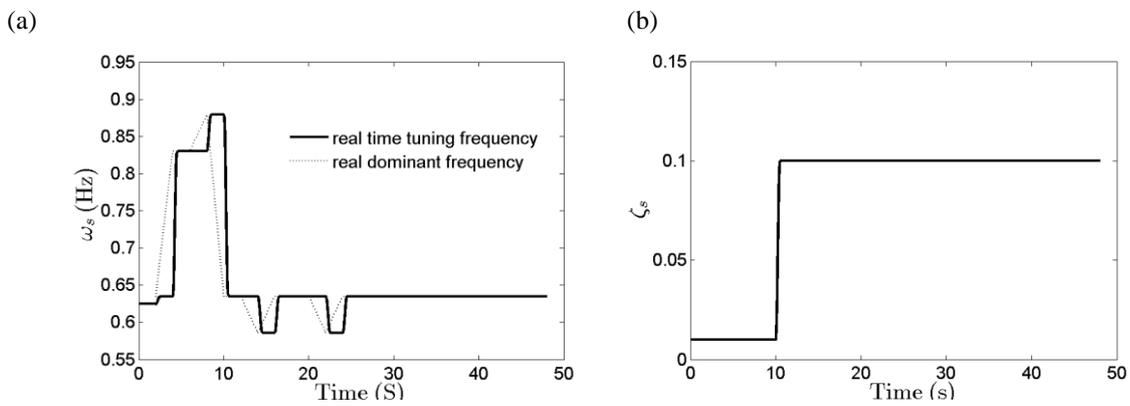
The use of STMD and MTMD produces reduction for acceleration spectra over a broad period range for Kobe (JMA) and Newhall ground motions; however, for Elcentro earthquake the reductions of acceleration spectra occur only for structures with period  $T_n \geq 0.7s$  as it is a far field earthquake with dominant period of 0.8 seconds.



**Figure 3.** Time history of displacement excited by Kobe (JMA) in different cases of  $\zeta_s$ . Left column of the plot shows the entire time history and the right column shows a response from  $t=6s$  to  $t=15s$  to demonstrate the details. Dashdot line denotes the case of no TMD; dashed line denotes TMD; dotted line denotes STMD, and solid line denotes MTMD.



**Figure 4.** (a) STFT power spectrogram corresponding to the Kobe earthquake (JMA); (b) contour of the power spectrogram. Thick solid line in the contour denotes the dominant frequency of the spectrogram.



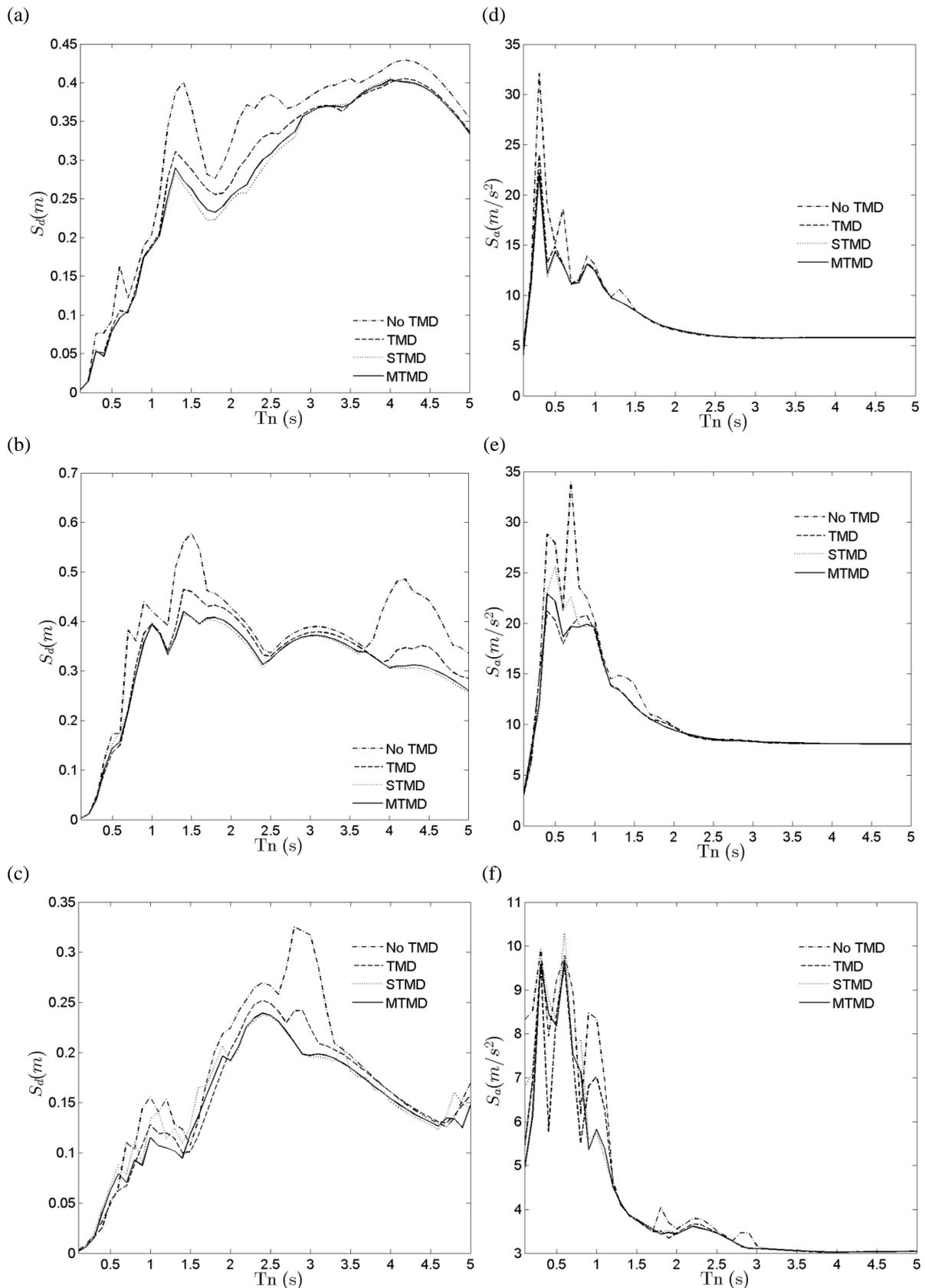
**Figure 5.** (a) time history of the real time tuning frequency of STMD and (b) time history of the real time tuning damping ratio.

## 6. CONCLUDING REMARKS

A new variable stiffness and damping control algorithm based on STFT has been developed and evaluated in the present study. By tracking the structural displacement and processing it using the control algorithm developed, real time retuning of the frequency and damping ratio of the STMD is implemented. It is shown that both the STMD and the MTMD are effective in reducing the displacement and acceleration spectra over a broad period range, when compared to the case without TMD and with TMD. Of the STMD and the MTMD, the MTMD may be more preferable since it provides a reduction effect comparable to that of the STMD but it requires less power due to a smaller STMD mass ratio.

**Table 5.1** Comparison of reduction effects among w/o TMD, TMD, STMD, and MTMD. (The minus sign “-” means reduction and positive sign “+” means increase;  $S_d$  is in *meters* and  $S_a$  is in *meters/sec<sup>2</sup>*)

	Northridge (Newhall 90)		Imperial Valley (El centro 90)		Kobe (JMA)	
	Peak $S_d$	Peak $S_a$	Peak $S_d$	Peak $S_a$	Peak $S_d$	Peak $S_a$
No TMD	0.401	32.12	0.325	9.688	0.578	33.99
TMD	0.310(-22.7%)	24.02(-25.2%)	0.242(-25.5%)	9.506(-2%)	0.464(-19.7%)	19.67(-42.1%)
STMD	0.267(-33.4%)	22.01(-31.5%)	0.210(-35.4%)	10.30(2%)	0.419(-27.5%)	22.54(-33.7%)
MTMD	0.273(-31.9%)	21.63(-32.7%)	0.210(-35.4%)	9.784(-1%)	0.414(-28.4%)	19.67(-42.1%)



**Figure 6.** Response spectra of the primary structure. Plots (a), (b) and (c) are the displacement spectra of the primary structure excited by the 1995 Kobe earthquake JMA parallel component, the 1994 Northridge earthquake (Newhall) and the Imperial Valley earthquake (Elcentro), respectively; (d), (e) and (f) are the acceleration spectra corresponding to each of the earthquakes.

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