Free Rocking Tests on Stone Masonry Wallettes for Evaluation of the Coefficient of Restitution

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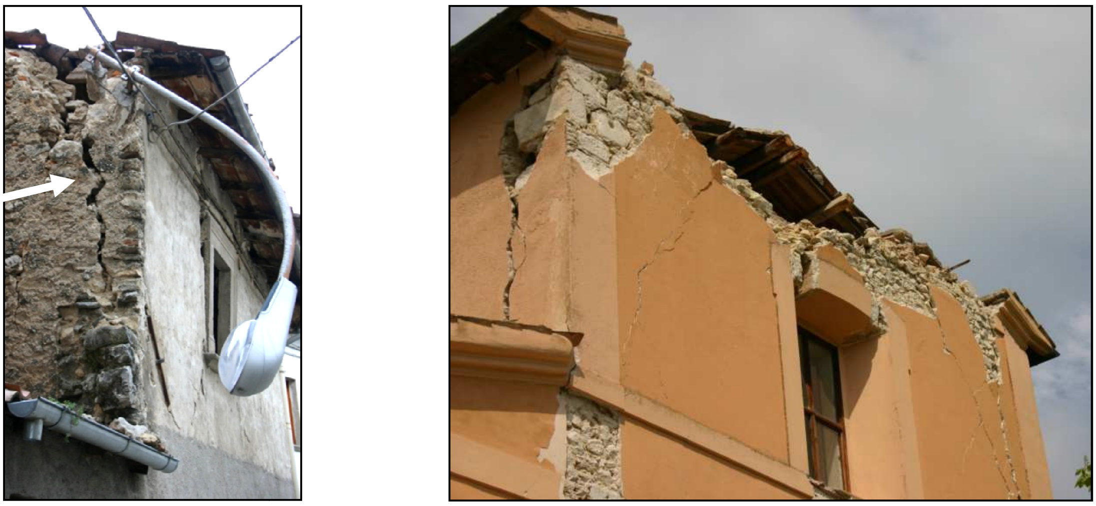
SUMMARY:
The behaviour of masonry façades may be numerically simulated as rigid elements rocking around its base, being the dissipation of energy concentrated at impacts at the base. This energy dissipation is usually represented by the coefficient of restitution factor, a value which correlates the ratio of kinetic energy after and before the impact. For this reason, an experimental test campaign was specifically developed to determine experimentally values of that coefficient for multiple leaf stone masonry walls. This paper reports the test procedure basis, the entire test apparatus involved in these experiments as well as the obtained data. Finally, a comparison with an analytical formulation is presented, including the presentation of the coefficient of restitution experimentally achieved.

Keywords: coefficient of restitution, rocking, stone masonry, out-of-plane, dynamic behaviour.

1. INTRODUCTION

The out-of-plane instability of masonry façades is a major cause of partial and global collapse of existing masonry structures under earthquake actions, also observed in recent events (e.g. Figure 1.1, from L'Aquila 2009 earthquake).
Due to the distributed and high amount of mass present in masonry façades and parapets, these elements are highly vulnerable and prone to overturn/collapse, especially when near-source effects are contained in the earthquake frequency content (e.g. velocity pulses).



(a) **(b)**
Figure 1.1. Out-of-plane damage on a stone masonry façade induced by L'Aquila earthquake: a) overturning mechanism formed; b) partial collapse.

Concerning the simulation of the dynamic behaviour of these elements, they may be considered as rigid elements rocking around the base, as considered by Giuffrè (1993) and more recently by other authors (e.g. (Sorrentino *et al.* 2008; Liberatore *et al.* 2009)).
When simulating the dynamic behaviour, the well-known Housner (1963) model for inverted pendulum structures is commonly used based on the equilibrium of a rocking body (Figure 1.2), represented by Eqn. (1.1), where I_O stands for the rotational inertia, $\ddot{\theta}$ is the angular acceleration corresponding to the block rotation $\theta(t)$, W is the block weight, while R_0 and α_0 are geometrical parameters determined relative to the element mass centre.

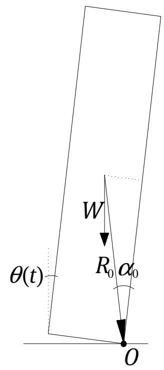


Figure 1.2: Parameters involved on the rocking motion of a rigid body

$$I_O \ddot{\theta}(t) = -W\ R_0 \sin\left[\alpha_0 - \theta(t)\right] \tag{1.1}$$

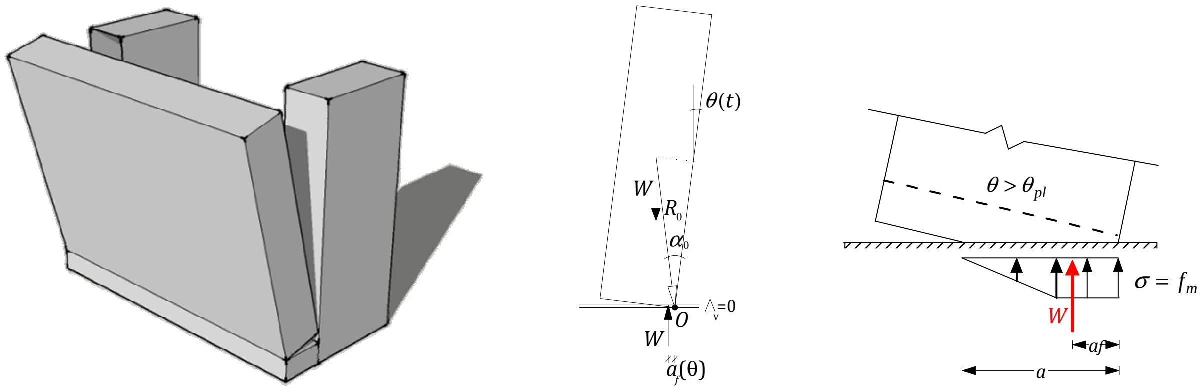
The energy dissipation is only obtained by impacts at the base, which can be represented by the so-called coefficient of restitution (r), dependent of the contact interface as mentioned by ElGawady *et al.* (2011). As reported by Costa (2012), the common definition of this coefficient is given by the ratio of angular velocities before and after the impact (this definition was firstly introduced by Aslam *et al.* (1978)).
For a rectangular element, this restitution coefficient is bounded by a theoretical upper limit ($r_{\max}$), determined by the geometrical parameters of the block and defined by Eqn. (1.2).

$$r_{\max} = 1 - \frac{3}{2}\sin^2 \alpha_0 \tag{1.2}$$

Despite being bounded by a theoretical value, experiences performed by several authors in tests of single blocks made of different materials (e.g. (Priestley *et al.* 1978; Wong *et al.* 1989; Peña *et al.* 2007), among others) have shown a fair disagreement.
In what concerns the evaluation of the restitution coefficient for full scale masonry walls, only the recent work performed by Sorrentino *et al.* (2011) has addressed this problem on a complete wall, by contrast with the previous works where single blocks were tested. Once again, differences were obtained between theoretical and experimental values, leading to a proposed ratio of $r/r_{max} = 0.95$.
In the work presented herein, the coefficient of restitution of a *sacco* stone masonry façade was obtained resorting to the named “equivalent block approach”, as described in the following section.

2. EQUIVALENT BLOCK APPROACH

The behaviour of a block rocking around the base is determined based on the mass and rotational inertia properties, which depend on its geometrical properties. If Eqn. (1.1) is carefully observed, an equivalent structure (where I_O, W, R_0 and α_0 are taken similar to the original rocking block) is likely to reproduce correctly the dynamic behaviour of the block. This may be the case of a stone masonry façade rocking around its base (or first bed joint level), as illustrated in Figure 2.1 a), where an equivalent block with similar properties is adopted to simulate the behaviour of the full façade.



(a) **(b)** **c)**

Figure 2.1: Out-of-plane rocking behaviour of a stone masonry façade: a) schematic representation; b) parameters involved in the rocking motion for a flexible interface; c) assumed complete section partialization and modified rotation axis (for static conditions only)

Since stone masonry walls have limited compressive strength mortar joints, with negligible tensile strength, the position of the rocking point may be shifted inwards, by contrast with Housner's model.
Thus, according to Figure 2.1 b) and Figure 2.1 c), the rocking axis for this type of elements may not be placed at the block edge O, moving to a new position O' due to the actual compressive strength of the mortar joint which shifts inwards the reaction force W located at a distance a_f from the edge. In that case, the equilibrium condition is modified, a procedure also suggested by Giuffrè (1993).
Therefore, neglecting vertical vibrations effects and considering the position of the rotation point constant (O') and at a distance a computed based on mortar's mechanical properties, the dynamic properties of the system simulates the flexibility of the interface by changing only the α_0 value to $\alpha(\theta_0)$.

$$I_{O'} \cdot \ddot{\theta}(t) = -W \cdot R_{O'} \sin\left[\alpha(\theta_0) - \theta(t)\right] \tag{2.1}$$

wherein the term W is independent of the rotation value, while parameters $I_{O'}$, $R_{O'}$ and $\alpha(\theta_0)$ are computed with the initial rotation value θ_0.
By writing $p^2 = WR_{O'}/I_{O'}$, Eqn. (2.1) may be simplified to Eqn. (2.2), referring to the called semi-rigid model.

$$\ddot{\theta}(t) = -p^2 \left\{\sin\left[\alpha(\theta_0) - \theta(t)\right]\right\} \tag{2.2}$$

For a section completely partialized, the approximate position of the reaction force in static conditions is given by Eqn. (2.3).

$$a_f(\theta) = \frac{W}{2 f_m l} \tag{2.3}$$

Therefore, if the value of p is correctly reproduced in an equivalent structure as well as the position of the rotation axis $a = 2\, a_f$ (Eqn. (2.3), considering full section partialization), that equivalent structure is suitable for simulating the behaviour of a masonry façades with similar inertia and geometrical parameters. This is the basis of the equivalent block approach which was adopted to perform the experiments reported in this paper.
The mechanical properties of the mortar interface (lime mortar) were obtained by experimental tests

made according to EN 1015-11 standard (CEN 1999), leading to 0.53 MPa and 1.28 MPa, respectively for flexural and compressive strength.

3. EXPERIMENTAL TESTS

The simulation of the rocking behaviour of a masonry façade was made resorting to a small masonry wallette (representative of the potentially cracked section at the wall base) rigidly connected to a very stiff top mass. This rigid mass was used to avoid the flexible behaviour of the wall and is the equivalent structure adopted to simulate the entire masonry façade sketched in Figure 2.1 a). In this way, it was possible to characterize the coefficient of restitution of sacco stone masonry façades for two-sided rocking under free vibration.
As shown in Table 3.1, good agreement was obtained between the equivalent block and the real structure regarding the dynamic parameters p and α_0, which means that the free rocking response may be adequately reproduced by this approach.

Table 3.1. Properties of the equivalent block and comparison with the original structure

System	Weight [kN]	Height [m]	Rotational inertia [kg m^2]	R_0 [m]	p [rad/s]	α_0 [rad]
Original	48.2	3.0	15426	1.535	2.19	0.213
FR1	51.3	2.9	16107	1.525 (-0.7%)	2.20 (+0.5%)	0.215 (+0.9%)
FR2	52.8	2.9	17652	1.574 (+2.5%)	2.17 (-0.9%)	0.208 (-2.3%)

The complete test apparatus is presented in Figure 3.1, which also includes the scheme adopted for releasing the wall under a predefined rotation level (θ_0).
Four different initial rotation levels θ_0 were given to the specimen in order to evaluate the influence of initial rotations in the final value of the restitution coefficient.
Several repetitions were performed for the same rotation level, as well as some final repetitions at the end of the tests in order to investigate the effects of repetitions on the final results. A total of 36 tests were made as listed in Table 3.2.

(a) (b)

Figure 3.1: Specimen test under free rocking motion: a) test apparatus; b) pullout scheme

The data was acquired resorting to 24 LVDTs (Linear Voltage Displacement Transducers), 5 wire transducers (potentiometers), 8 accelerometers and 2 bi-axial tiltmeters, in accordance to Figure 3.2.
The data acquisition was made with National Instruments (NI) hardware at 4000 Hz sampling frequency, while the acquisition software was developed by the authors within the LabVIEW platform (also form NI).

Table 3.2. Sequence of the experimental tests

Specimen	θ_0/α	Test name	Tests number	Tests	Total number of tests
FR1	0.15	L1	1*, 2*, 3, 4*, 5*	5	18 +3 repetitions at the end
	0.25	L2	1, 2, 3*, 4*, 5	5	
	0.35	L3	1, 2, 3, 4, 5	5	
	0.40	L4	1, 2, 3	3	
	0.35	L3	R	1	
	0.25	L2	R	1	
	0.15	L1	R	1	
FR2	0.35	L3	1, 2, 3	3	9 +3 repetitions + 3
	0.25	L2	1, 2, 3	3	
	0.15	L1	1, 2, 3	3	
	0.35	L3	R	1	
	0.25	L2	R	1	
	0.15	L1	R	1	
	0.40	L4	1, 2, 3	3	

(*) Data not acquired due to acquisition problems

The initial rotation (θ_0) was read using the tiltmeters, while the rotation time histories were obtained by the computation of the displacement monitored with the wire transducers. More information regarding this procedure is presented in Costa (2012).

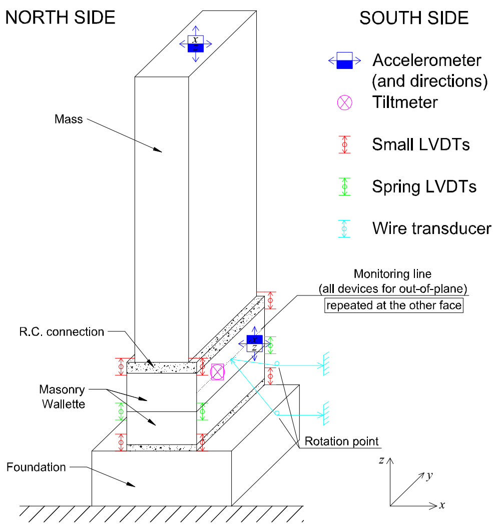
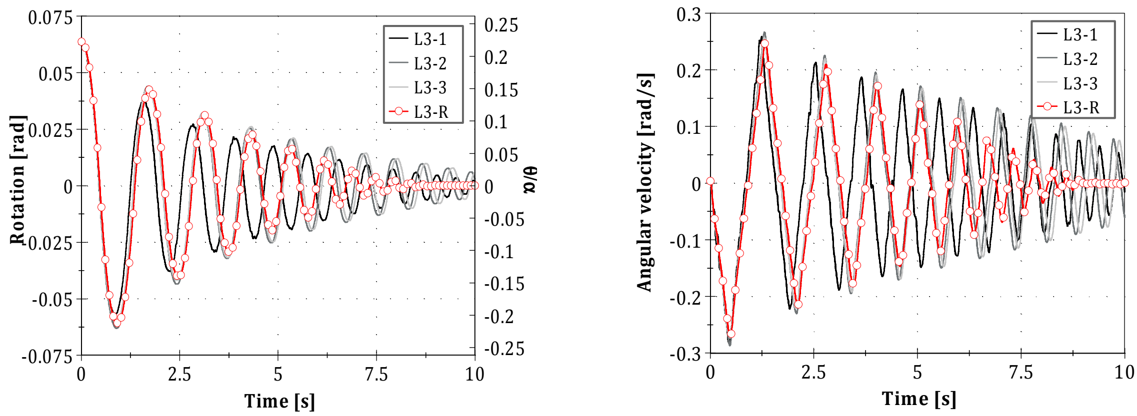


Figure 3.2. Monitoring devices used in the experiments

4. EXPERIMENTAL RESULTS AND DATA INTERPRETATION

Some of the experimental results obtained are presented in this section. From the overall set of experimental results, Figure 4.1 shows those obtained for the test level FR1-L3, where it is possible to observe the repetitions effect in the L3-R. Slight scatter between each test number was obtained due to unintentional different release conditions of the specimens. The angular velocity time histories were obtained by differentiating the rotation time histories.



(a) **(b)**

Figure 4.1. Experimental time histories obtained for specimen 1, θ_0/α = 0.35: a) rotation; b) angular velocity

In order to validate the developed test setup, the experimental results were compared to theoretical values obtained from the dynamic differential equation taking into account an initial rotation value φ_0 = θ_0/α, presented initially by the classic rocking theory (Housner 1963). As described in the cited work, a relation between rocking period and initial rotation is given by Eqn. (4.1), being the results presented in Figure 4.2. The experimental rocking periods (T) were computed through the angular velocity peaks, while the peak rotation values (θ/α) were obtained directly from rotation time histories.

$$T = \frac{4}{p}\cosh^{-1}\left(\frac{1}{1-\varphi_0}\right) \tag{4.1}$$

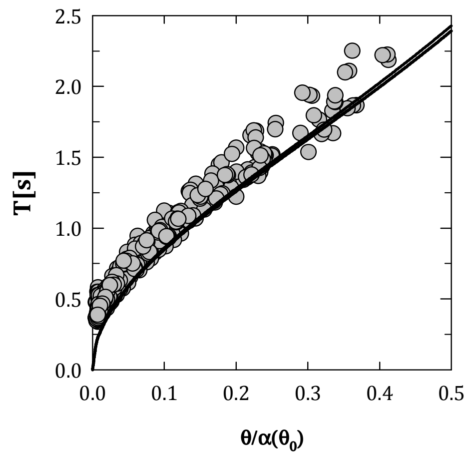


Figure 4.2. Comparison between theoretical curve and experimental values (theoretical curves presented for both specimens FR1 and FR2, which show almost coincident)

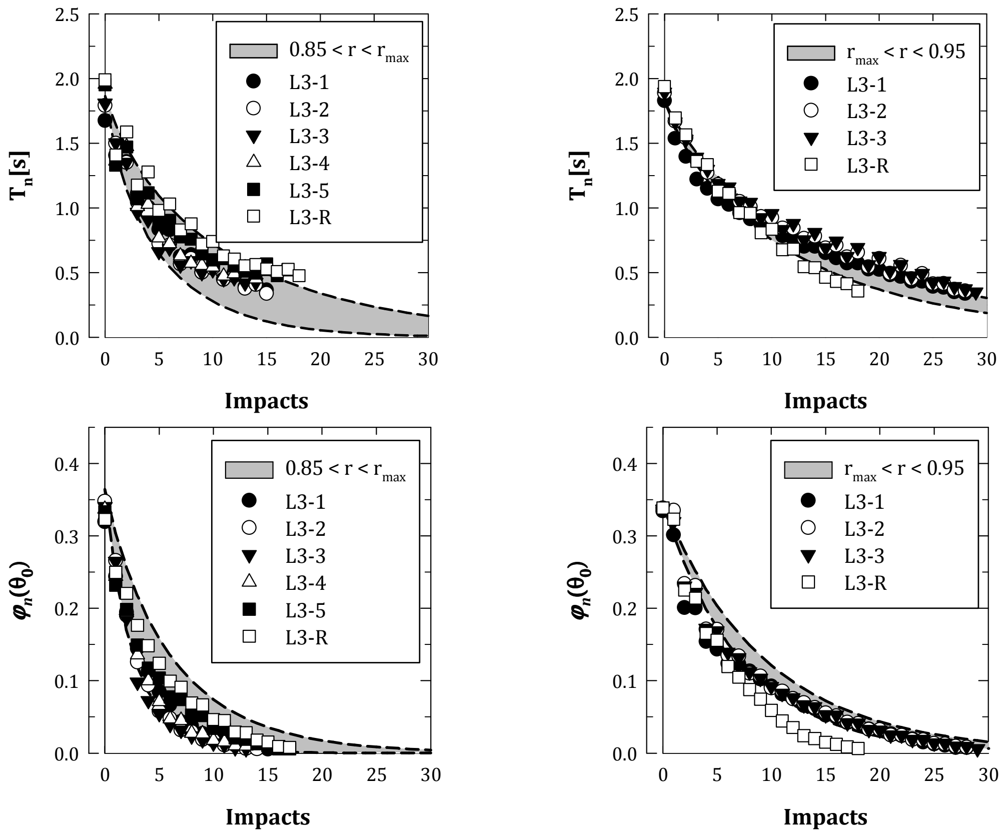
Figure 4.2 validates the use of the test setup for simulating the free rocking behaviour of a masonry façade. Moreover, it should be referred that the theoretical curve of the original structure (considering the characteristics presented in Table 3.1) is almost coincident with the FR1 curve, which also contributes for validating the application of the equivalent block approach.

It should be referred that individual comparisons between experimental and theoretical results for each test level were made in terms of impacts at the base and periods of vibration or rotation angle, given respectively by Eqn. (4.2) and Eqn. (4.3).

$$T_n = \frac{4}{p}\tanh^{-1}\sqrt{r^{2n}\left[1-\left(1-\varphi_0\right)^2\right]} \tag{4.2}$$

$$\varphi_n = 1 - \sqrt{1 - r^{2n}\left[1 - \left(1 - \varphi_0\right)^2\right]} \tag{4.3}$$

An example of this comparison is presented in Figure 4.3 for both specimens, referring to the test stage L3.



(a) (b)

Figure 4.3. Comparison between theoretical and experimental results in terms of periods of vibration and rotation angles (top, Eqn. (4.2) ; bottom, Eqn. (4.3)): a) FR1-L3; b) FR2-L3

The determination of the coefficient of restitution (r) was made adopting two different approaches: *i)* exclusively on pure experimental evidence, using directly the velocity records; *ii)* based on the classic rocking theory and the experimental rotation readings.
For the first mentioned case *i)*, Eqn. (4.4) was used where $\ddot{\theta}_n$ is the peak velocity at impact *n*, while $\ddot{\theta}_{n+2}$ is the peak velocity at impact *n* + 2.

$$r = \sqrt{\frac{\dot{\theta}_{n+2}}{\dot{\theta}_n}} \tag{4.4}$$

Concerning the determination based on the classic rocking theory (case *ii*), also commonly used by several authors in previous works, the restitution coefficient may be obtained directly from Eqn. (4.5), where *n* stands for the *n*-th impact, while φ_0 is the initial rotation (θ_0/α) and φ_n is the rotation at the *n-th* impact (θ_n/α).

$$r = \sqrt[2n]{\frac{1-\left(1-\left|\varphi_n\right|\right)^2}{1-\left(1-\left|\varphi_0\right|\right)^2}} \tag{4.5}$$

Taking into account both computation approaches and the large amount of data available for such calculation, the coefficient of restitution was determined for each test level and for both specimens. The main results are presented in Figure 4.4.

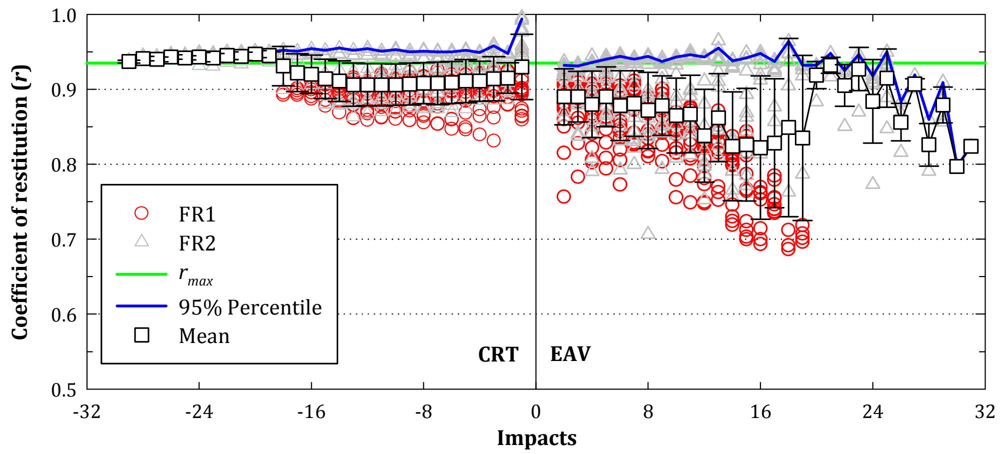


Figure 4.4. Coefficient of restitution determined based on the classic theory (CRT) and experimental evidence (EAV)

Figure 4.4 shows that the values obtained exclusively from the experimental evidence are lower than those computed using the classic theory with measured rotations. Moreover, when compared to the maximum theoretical value (r_{max}), computed using Eqn. (1.2) and the values presented in Table 3.1, similar or even higher restitution coefficient values may be obtained with the classic theory. In order to summarise the most relevant information, Table 4.1 presents a comparison between the values obtained through both approaches and the theoretical maximum value r_{max}.

Table 4.1. Restitution coefficient values obtained in the free rocking tests

Specimen	Classic theory		Experimental evidence		Theoretical maximum
	Average	**r/r_{max}**	**Average**	**r/r_{max}**	**r_{max}**
FR1	0.895	0.96	0.819	0.88	0.931
FR2	0.931	1.00	0.895	0.96	0.935
Average	0.913	0.98	0.857	0.92	0.933

Table 4.1, evidences that an average lower bound of 0.88 was obtained (FR1) from the exclusive experimental evidence, while 1.00 was the average upper limit output computed via the classic theory. For this reason, the proposed value of r/r_{max} = 0.95 as proposed by (Sorrentino *et al.* 2011) seems not advisable for this type of masonry and, therefore, a conservative value of r/r_{max} = 1.0 may be suggested. However, as shown in Figure 4.4, slightly higher values may be obtained, which means that r_{max} may not be seen as the maximum upper bound of the restitution coefficient.

5. CONCLUSIONS

The experimental determination of the restitution coefficient, an important parameter to describe the dynamic behaviour of rigid rocking bodies using Housner's model, was made with a novel test setup. This test setup, based on the so-called "equivalent block approach", was developed and validated against theoretical results, ensuring a proper rocking response of a stone masonry portion at a predetermined bed-joint level.

Furthermore, it was possible to estimate the restitution coefficient both purely based on experimental information and on theoretical computations resorting to experimental results.

In the end, ratios of r/r_{max} = 0.88 and 1.00 were found likely to be seen as lower and upper bounds for the restitution coefficient of *sacco* stone masonry walls.

ACKNOWLEDGEMENTS

This work refers research made with financial contribution of the Portuguese Foundation for Science and Technology (FCT) through the first author PhD grant (SFRH/BD/38138/2007).

Special acknowledgements are due to the technicians of the Laboratory for Earthquake and Structural Engineering (LESE) of the Faculty of Engineering of the University of Porto.

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