Vertical Spatial Variability of Geotechnical and Geophysical Parameters -Application on a slope stability

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SUMMARY
In this paper, it is proposed to carry out a study where soil parameters are modelled as statistical forms called random fields. The parameters of these random fields are determined using a regression analysis based on sample functions obtained from in-situ geotechnical and geophysical measurements. While the geotechnical in-situ investigation consisted of conducting Standard Penetration Tests (SPT) at 1.5 m intervals, the geophysical tests consisted of estimating the shear wave velocity (V_s) from passive array noise measurements. Using measured SPT and V_s profiles, the variability of these parameters with depth is statistically analyzed with the random field theory, in order to determine its influence on the design process.
Consequently, the calculation for factor of safety against slope failure is carried out where the angle of internal friction is assigned to the sandy soil of the slope as a random field on one hand and as an average value on the other hand; Moreover, a dynamic probabilistic and deterministic analysis is carried on to estimate the dynamic amplification factor of a seismic wave at the top of the slope.

Keywords: Spatial variability, Soil parameters, slope stability, Safety factor, dynamic amplification.

1. INTRODUCTION

Soils present heterogeneity at various scales of description, thus their properties may change relevant to location and depth; consequently, the use of probability theory in geotechnical data analysis may be of major advantage since it allows modeling the randomness and variability of the soil medium. The soil parameter uncertainties and variability can be modeled as a random variable or as a random field. The random variables are defined by a probability density function and a correlation can exist between two random variables. On the other hand, when modeling the spatial variability of the soil properties as a random field, each field will be represented by a probability density function and an autocorrelation function. Recently, several authors have investigated the application of the random field theory in geotechnical engineering (Popescu et al. 1997, Cho 2007, Youssef Abdel Massih et al., 2009, Jaksa et al. 1997, Griffiths et al. 2009, …). However, the statistical parameters of the properties used by these authors were not based on real data measurement, but they were taken based on some literature review.

It follows then that the modeling of the spatial variability of soil in geotechnical engineering design was made feasible through the use of probability theory and precisely through the notion of random fields. In addition, the representation of soil parameters as random fields requires the quantification and determination of the random field characteristics and these are the probability density and the autocorrelation functions that most fit the sample data. In this regard, Radu Popescu (1995) has followed a methodology to estimate the statistics of the spatial variability of soil parameters which were obtained using in-situ tests, mainly cone penetration tests (CPT) and standard penetration tests (SPT) showing that random fields representing most of the soil parameters have a probability density function that follows a lognormal distribution and their autocorrelation function is best fitted by multi-parameter autocorrelation structures such as the cosine decaying and the exponential decaying functions, which showed more goodness of fit values than the exponential square function usually used in geotechnical engineering.

This paper will present a study where soil parameters (the friction angle and the shear modulus) are modeled as random fields. The statistical parameters of these random fields are determined using a regression analysis based on sample functions obtained from in-situ geotechnical (SPT) and geophysical (Shear wave velocity) measurements. Then, a static and dynamic slope stability analysis is carried out to calculate the probability function of the static slope safety factor and to estimate the dynamic amplification at the top of the slope.

2. STATISTICAL DATA ANALYSIS

2.1. Site data

The soil data that will be used in what follows of analysis and calculation is based on the geotechnical investigation carried out at lot 4748, Achrafieh, Beirut whose location is shown in Figure 1.

Figure1. Lot 4748, Achrafieh, Beirut

Both geotechnical and geophysical investigations have been conducted at the site; the geotechnical investigation consisted of drilling nineteen boreholes, fourteen drilled to 50 m depth and five drilled to a 25 m depth (See Figure 2 for the borehole location). The site subsurface conditions were found to be consisting of a layer of dense to very dense gravely sand, underlain by a layer of silty sand. Standard penetration tests were conducted on granular soil encountered in order to estimate its penetration resistance. Geotechnical data provided from this investigation consisted of fifteen SPT-N_{60} profiles; however, these profiles that usually give the N value at 1.5 meter intervals were not totally complete either due to the presence of refusal values or the test was not conducted at a certain depth, leading that only four complete profiles were used in the analysis shown in Figure 2 below. On the other hand, the geophysical investigation consisted of measuring the shear wave velocity at different depths using passive array noise measurements.

For design purposes, the SPT-N values (SPT-N is not an intrinsic property of granular soils) were used to evaluate a real design parameter like angle of internal friction (φ') using the relation of Wolff (1989) given below in equation 1:

$$\varphi' = 27.1 + 0.3N_{60} - 0.00054N_{60}^2. \quad (1)$$

Consequently, the values of φ' calculated from the existing SPT profiles will be corresponding to the angle of internal friction of the soil underlying the site area and since in our case a sandy soil is encountered, the value of cohesion is usually adopted to the range 0<c<1kPa.

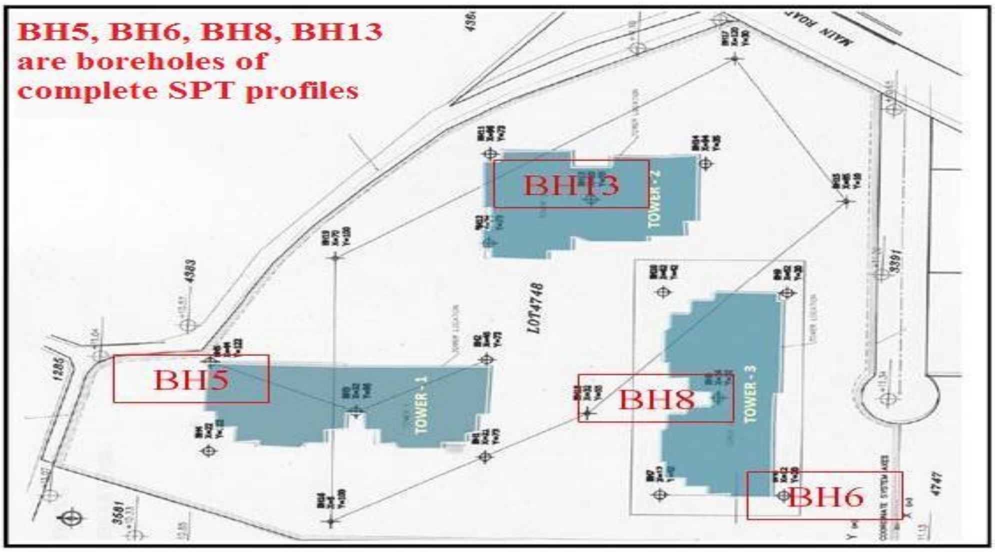

Figure2. Boreholes from which complete SPT profiles are obtained

2.2. Statistical data analysis

2.2.1 Overview of random field theory

The illustration of random field and probability basics is beyond the scope of this paper; however, a brief definition could be provided in this context. A random field is described as a family of random variables (elements of the random field), indexed or identified according to a certain parameter (for example time or space) and having the notation form H(x, θ), where $x \in R^n$ represents the indexing parameter and θ represents the set of outcome values that the field may take. Random fields are characterized using their probability density and autocorrelation functions. This characterization yields to the definition of different types of fields, for example Gaussian (normal) random fields, lognormal fields, etc…

2.2.2 Determination of the statistical parameters

The statistical analysis of a random field, representing a soil property, is achieved through the determination of the mean, standard deviation, probability density function and the autocorrelation function of this field. In fact, the autocorrelation functions are a useful indicator of dependencies as a function of distance in time or space, and they can be used to assess the distance required between sample points for the values to be effectively uncorrelated.

The probability density function (pdf) of a sample data is estimated using least square method through fitting the sample probability density function (pdf) to an existing pdf type that gives least values of error on its parameters; this procedure is simply done using the distribution fitting tool in MATLAB. On the other hand, the autocorrelation coefficient function estimation follows the same procedure as for the determination of the pdf type. The sample autocorrelation function is first evaluated in the vertical direction (with respect to depth) using equation 2 (Popescu 2005):

$$\rho(r\Delta z) = \frac{1}{\mathrm{variance}_u \cdot (N-r)} \sum_{i=1}^{N-r} [u(i\Delta z) - \mathrm{mean}_u][u((i+r)\Delta z) - \mathrm{mean}_u] \qquad (2)$$

Where rΔz is the lag distance, r=1…m is the lag number, m is the maximum lag number and u is the values of the random field at different lag distances. The values of this function represent the autocorrelation between the field values for different lags in depth. For example, $\rho_{\varphi'\varphi'}(1.5)$ is the autocorrelation coefficient (degree of relationship) between φ' values for a lag distance of 1.5 meters (r=1).

Once the sample autocorrelation function is evaluated, a fitting procedure (using a MATLAB program) is followed to determine the autocorrelation function, from existing models shown in table 1, which best fits the sample autocorrelation function. This fitting process, using the least square method, gives two important outputs, the coefficient of determination (R^2) which provides a measure of how well new values are likely to be predicted by the fitting autocorrelation function and the parameters of the fitting autocorrelation function used to determine the autocorrelation distance of the field in the vertical direction. The autocorrelation distance is the distance beyond which two discretized values of the random field are uncorrelated. However, it has to be noticed that if we suppose that the autocorrelation distance for the random field representing (φ') of a certain type of soil is (θ) then the sampling distance (s) from this soil should be $s< \theta/2$ (cf. Popescu 1995) in order to get accurate data for statistical analysis.

Table1. Common one dimensional autocorrelation structures

Correlation structure	**Correlation function**	**Correlation distance**
Squared exponential	$e^{-(\frac{\xi}{a})^2}$	$a\sqrt{\pi}$
Cosine decaying	$e^{-b\xi}\cos a\xi$	$\frac{2b}{b^2+a^2}$
Triangular	$\begin{cases} 1-\frac{\lvert\xi\rvert}{a}, & \lvert\xi\rvert \le a \\ 0, & \text{otherwise} \end{cases}$	a
Exponential	$e^{-a\lvert\xi\rvert}, (a>0)$	$\frac{2}{a}$

2.3. Results for Site data analysis

2.3.1. Standard penetration test-N_{60}

Using data from all available profiles, it was observed that the lognormal probability density function best fits the SPT data probability distribution. In addition, the autocorrelation coefficient function of (N_{60}) sample data in vertical direction was found to be best fitted to the cosine decaying autocorrelation function, where the value of R^2 was the highest for all used profiles as shown in table 2 below.

Table2. Results of fitted autocorrelation function for the SPT-N_{60} data

Function type	**R^2**	**Function parameters**
Exponential square	0.7932	a=1.598
Cosine decaying	0.8657	a=0.3109, b=0.2267

2.3.2. Angle of internal friction (φ')

Four 50 meter borehole profiles were used to calculate the value of (φ') with 1.5 meter interval from the SPT profile. The probability distribution of sample data was best fitted to the lognormal probability distribution. The errors on the statistical mean and standard deviation parameters are 0.0041 and 0.0029 respectively.

The autocorrelation coefficient function of (φ') sample data in vertical direction was found to be best fitted to the cosine decaying autocorrelation function, where the value of R^2 was the highest for all used profiles. Figure 3 below shows the fitted autocorrelation function with the sample autocorrelation functions of borehole profiles and table 3 shows the output results of fitting for two well known autocorrelation functions in geotechnical engineering.

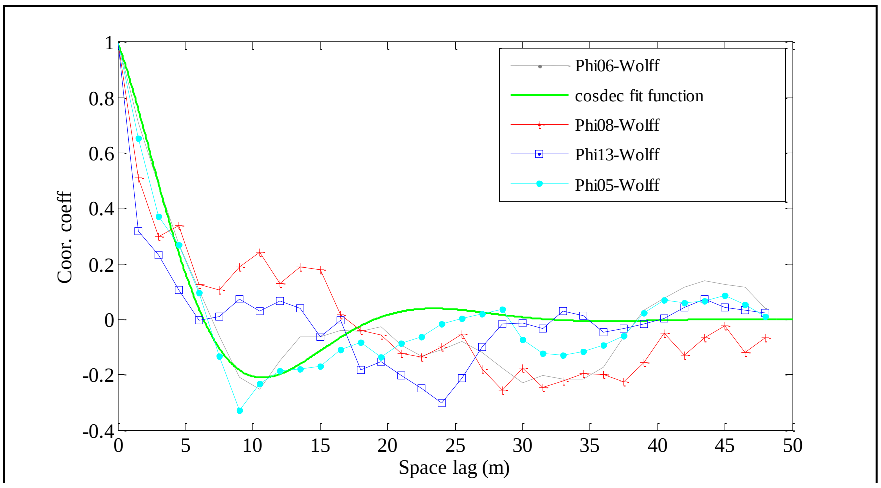

Figure3. Fitted autocorrelation function for the friction angle data

Table3. Results of fitted autocorrelation function for the friction angle data

Function type	R2	Function parameters
Exponential square	0.776	a=3.162
Cosine decaying	0.9042	a=-0.2469, b=0.133

2.3.3. Shear wave velocity Vs relation with SPT-N_{60}

In this part of the research, we will be concerned about developing a new relation between Vs and SPT for the present site (Lot 4748 Achrafieh). Four borehole profiles were used from the geotechnical investigation and these profiles had been chosen based on the fact that the boreholes are located in the area where the geophysical investigation was carried out. For the geophysical investigation, 686 Vs profiles were provided from the iterations of the test results on site and in each profile different values of Vs were observed and at different depth intervals as shown in Figure 4 where no profile showed a constant depth interval between measurements. The SPT-N_{60} profiles are obtained for a 50 meter depth and at 1.5 meter intervals, therefore an approach was used to get a similar profile for Vs through using all data from all Vs profiles and then filtering all these values for a depth less than or equal to 50 m and then adjusting the resulted profile by taking the average value at intervals of 1.5 meters

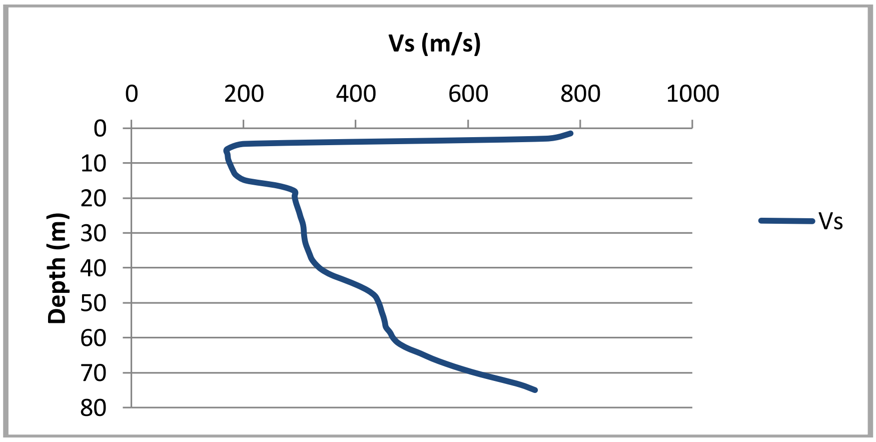

Figure4. Shear wave velocity profile from ambient noise test

Finally, we have one SPT profile whose values at each depth are calculated from averaging the four SPT values at the same depth from all the profiles, and one Vs profile calculated as mentioned above. Then using a simple regression analysis for the existing data base, a relation was developed between SPT and Vs for the as shown in Figure 5 below.

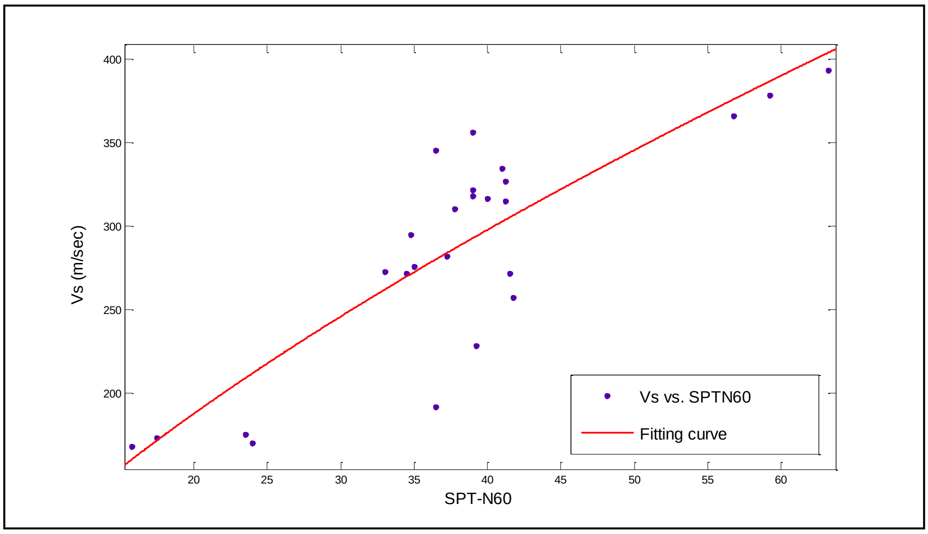

Figure5. Correlation between Vs and N_{60} for sandy soil

The best relation with a coefficient of correlation (r = 0.834) between Vs and N_{60} for the present site is shown in equation 3 below:

$$Vs = 25.44N_{60}^{0.66} \tag{3}$$

2.3.4. Shear modulus (G)

The shear modulus (G) of soil is calculated from the shear wave velocity (V_s) using equation 4:

$$G = \rho V_s^2 \tag{4}$$

Where, ρ is the unit mass of the soil in (Kg/m^3), V_s in (m/sec) and G in (N/m^2).

Using data from all available profiles, it was observed that the lognormal probability density function best fits the (G) data probability distribution. In addition, the autocorrelation coefficient function of (G) sample data in vertical direction was found to be best fitted to the cosine decaying autocorrelation function, as shown in table 4 below.

Table4. Results of fitted autocorrelation function for the shear modulus data

Function type	**R2**	**Function parameters**
Exponential square	0.694	a=6.7
Cosine decaying	0.954	a=-0.1125, b=0.0406

3. STATISTICAL MODELLING OF SUBSURFACE CONDITIONS

Any soil medium set under study is defined through its design parameters; consequently, introducing this soil material into numerical static or dynamic calculation is done through assigning values to its mechanical or dynamic properties. In classical deterministic calculations, an average value is assigned to each parameter without taking into account any kind of uncertainty (spatial variability of soil in our case) on these parameters. Conversely, in statistical calculations different values are assigned to the same parameter at different locations of the soil mass, and this process was made possible through the definition of random fields to represent soil parameters. Several methods have been proposed to carry out this task, such as the spatial average method and the midpoint method; however, the most efficient approaches used for discretization of random fields are the series expansion methods, such as the Karhunen-Loève expansion and the Expansion Optimal Linear Estimation or the EOLE method (Sudret and Der Kiureghian, 2000).

3.1. Random field discretization (EOLE method)

The EOLE method was first proposed by Li and Der Kiureghian (1993); it is based on the pointwise regression of the original Gaussian random field with respect to selected values of the field, and a compaction of the data by spectral analysis. The theoretical description of the method is beyond the scope of this paper; however, for the sake of clarity in application and results, some terms should be defined and for more details reader should refer to Sudret and Der Kiureghian (2000).

Discretization: Process of changing the random field from an infinite set of random variables to a discrete statistical feature.

Realization: Process of assigning a value for each random variable at a single step.

Order of expansion (m): During discretization, an autocorrelation matrix is created having N Eigen values to calculate a series representing the discretized random field, m is the minimum number of the Eigen values required to have an acceptable mean variance error.

Mean variance error: the value of the error between the discretized and the original random field.

3.2. Application and results

Two properties of the same soil medium were modeled as a random field: the angle of internal friction for the static analysis (calculation of factor of safety against slope failure), and the shear modulus for the dynamic analysis (calculation of the amplification factor of a seismic wave applied at the bottom of the slope).

3.2.1. Angle of internal friction (φ')

One multidimensional, univariate, lognormal random field was used to represent the angle of internal friction of a sandy soil medium underlying the site area of lot 4748, Achrafieh, Beirut. The field statistical properties are shown in table 5 below.

Table5. Statistical properties of the random field

Property	Description
PDF	Lognormal
Mean (Degrees)	36.68
Standard deviation	3.14

As a first approach, the field was supposed to be isotropic, that is having the same autocorrelation distance in both vertical and horizontal directions. Results from discretizing the random field using two autocorrelation functions are shown in table 6.

Table6. Comparison between the autocorrelation functions used in discretization

Discretization features	Exponential square	Cosine decaying
Order of expansion	62	150
Mean variance error	0.105	0.1067

3.2.2. Shear modulus (G)

Using same analysis as above, tables 7 and 8 show the statistical properties and the discretization features respectively for the random field representing the shear modulus of sandy soil medium.

Table7. Statistical properties of the random field

Property	Description
PDF	Lognormal
Mean (MPa)	153.9
Standard deviation	66.4

Table8. Comparison between the autocorrelation functions used in discretization

Discretization features	Exponential square	Cosine decaying
Order of expansion	62	200
Mean variance error	1.796e-05	0.026

4. APPLICATION OF THE STATISTICAL MODEL IN GEOTECHNICAL ENGINEERING

The soil model used for both static and dynamic analysis is illustrated in Figure 6 below, the soil properties shown in the figure are used in the case of deterministic static and dynamic calculation. For the case of probabilistic static and dynamic calculation, the angle of internal friction and the shear modulus were modeled as a random field.

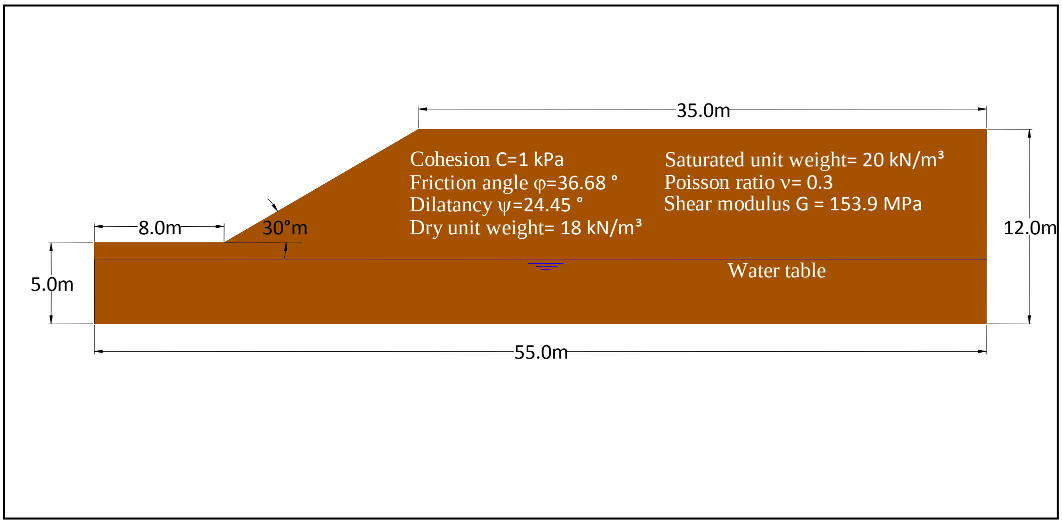

Figure6. Slope geometry, soil properties and site conditions (water table at 8 m below NGL)

4.1. Static analysis

The factor of safety (FOS) was evaluated using two approaches, one deterministic and the other is probabilistic. A single FOS value equal to 1.61 was obtained in the deterministic calculation, while table 9 below shows the statistical results of probabilistic calculation for 500 realizations using cosine decaying autocorrelation function. It was found that the Lognormal distribution best fits the FOS. (cf. Figure 7)

Table9. Statistical results of the response (FOS) calculated with the cosine decaying correlation function

Statistical property	Value
Mean	1.6042
Standard deviation	0.1046
Best fitted PDF	Lognormal

It was obvious from the above results of the factor of safety calculation that the probabilistic analysis shows a mean value slightly lower than the deterministic analysis, where the FOS in the deterministic case was equal to 1.61 and its mean value was found equal to 1.6042 in the probabilistic calculation. The slightly lower mean value in probabilistic calculation is normally due to the fact that lower values of the angle of internal friction are assigned to some elements in the soil model during the realization process. Additional important aspect that could be observed from the probabilistic calculation is the probability of failure (P_f) which could be used as an indication about the safety of certain design or structure. The probability of failure is defined as P_f=P ($FOS \leq FOS_{critical}$) and can be obtained from the cumulative distribution function (CDF) of the safety factor by reading the corresponding ordinate of the chosen target $FOS_{critical}$. For example the probability of having a safety factor less than 1.5 for this slope was found equal to 16%.

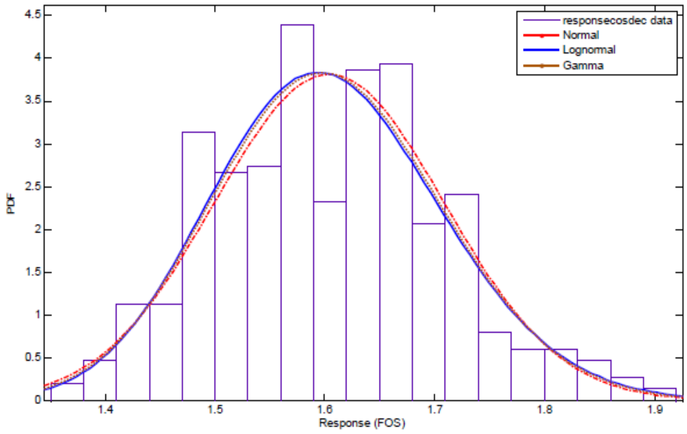

Figure7. FOS histogram of the 500 realizations and the fitted probability density functions

4.2. Dynamic analysis

The dynamic amplification, maximum output acceleration at the top of the slope divided by the maximum input acceleration applied at the bottom of the model, was evaluated using both the deterministic and probabilistic approach. In the deterministic case, all parameters were assigned a single average value and a signal was applied at the bottom of the slope, then a dynamic calculation was carried out to get the output acceleration at the top of the slope.

On the other hand, the probabilistic calculation was based on assigning the same values for all parameters as for the deterministic case except for the shear modulus that was defined as a random field. In this analysis, the shear modulus have been chosen as random field since it was found that the dynamic calculation is much more sensitive to the variation of G (consequently, Vs) than the other soil parameters. Two autocorrelation functions were used for discretizing the random field, shown in tables 7 and 8, and for each type of autocorrelation function 50 realizations were carried out. Then a dynamic calculation, similar to that executed in the deterministic case, was performed for each realization. Figure 8 below illustrate the results obtained from the deterministic calculation showing the amplification at the top of the slope. Moreover, Table 10 shows the amplification ratio results for deterministic and probabilistic analysis.

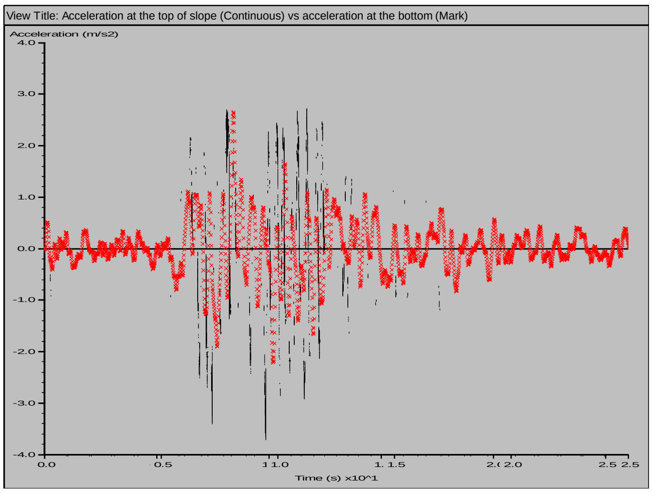

Figure8. Deterministic calculation amplification

Table10. Results of amplification factor estimation

Method	Amplification factor (AF)- value	AF-stdev	AF-pdf
Deterministic	1.4	-	-
Probabilistic (Cosine decaying)	6.82 (mean)	15.5	Lognormal
Probabilistic (Square exponential)	10.97 (mean)	18.25	Lognormal

As one can see, the probabilistic results are found more critical then the deterministic one since greater mean amplification factors are obtained. This may be due to the fact that when modeling the shear modulus as a random field we may encounter in the realization some zones of weak shear modulus values which highly amplify the acceleration of the wave. Also, one can notice that the use of the square exponential autocorrelation structure is more conservative than the use of the cosine decaying.

It has to be noticed that the properties' degradation during dynamic calculation is not taken into account and is beyond the scope of this paper.

5. CONCLUSION

The geotechnical design is usually based on a deterministic calculation procedure where the parameters of soil are assigned single average values; however, subsurface conditions show that the soil parameter values are variable from one location to another all over the investigated domain, and this special property of soil required a particular study and analysis to be conducted on soil parameters before being assigned to a particular design model.

In summary, the probabilistic analysis on soil parameters and their spatial variability was based on data provided from lot 4748 located in Achrafieh, Beirut. The data consisted of SPT- N_{60} profiles obtained from several soil borings carried out on site, and they were used to get similar data profiles but for the angle of internal friction of sandy soil. In addition to geotechnical in situ tests, geophysical shear wave velocity (Vs) tests were carried out on the same site using the ambient noise test method which is a simple and economical method. So, there was an orientation to develop a correlation between SPT- N_{60} and Vs, as this could be of major advantage if similar soil was encountered in a neighborhood site area.

The statistical analysis of SPT- N_{60}, φ', and shear modulus G data has shown that their probability density fits well with lognormal probability density function and their sample correlation function is most fitted to the cosine decaying autocorrelation function. Following the statistical analysis, a probabilistic calculation was carried out to estimate the factor of safety of a natural slope of sandy soil (static analysis) and the amplification factor (dynamic analysis). Results of probabilistic static calculations showed that the difference was slight between deterministic FOS value and probabilistic FOS value. Moreover, results of the probabilistic dynamic calculations showed quiet different results, this may be due to the fact that when modeling the shear modulus as a random field we may encounter in the realization some zones of weak shear modulus values which highly amplify the acceleration of the wave.

REFERENCES

Cho, S. (2007). Effects of spatial variability of soil properties on slope stability. *Eng. Geology* 92, 97–109

Griffiths DV, Huang J and Fenton GA (2009) Influence of spatial variability on slope reliability using 2-d random fields. *J Geotech Geoenviron*, vol.135, no.10, pp.1367-1378.

Jaksa, M., Brooker, P. and Kaggwa, W. (1997). Modeling the spatial variability of the undrained shear strength of clay soils using geostatstics. *Geostatistics Wollongong* 96, Volume 2, 1284-1295.

Popescu, R. (1995). Stochastic variability of soil properties, data analysis, digital simulation, effects on system behavior, degree of doctor of philosophy.

Popescu, R., Prevost,J.H & Decodatis,G. (1997). Effects of spatial variability on soil liquefaction: some design recommendations. *Geotechnique* 47, 1019-1036.

Sudret B, Der Kiureghian A. (2000). Stochastic finite elements and reliability: a state-of-the-art report, Department of Civil and Environmental Engineering, Institute of structural Engineering, Mechanics and Materials, University of California, Berkelay

Youssef Abdel Massih, D., Harb, J. and Soubra, A.-H. (2009). Reliability analysis of Lebanese slopes subjected to seismic loads. *2nd international conference on Computational Methods in Structural Dynamics and Earthquake Engineering*, Island of Rhodes, Greece, 22-24 June 2009, 18p.