The Effect of Duration of Waveforms in Earthquake Source Inverse Problems

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SUMMARY:

Earthquake source inversion studies have played an important role in improving our understanding of the nature of earthquake sources. Source inversion studies for determining the spatial and temporal distribution of coseismic slip on relevant earthquake faults have increased dramatically in recent years. There is always a trade-off between the length of the record used in the inverse problem and the amounts of computational effort need to solve it. Thus, there is always a question that how long of strong motion records and broadband waveforms should be considered in analysis. In other words, how long of the after-event recordings or coda waves have effective contribution in constraining and increasing the accuracy of resulting slip values on fault surface. In this article at first, the inverse problem is investigated. Then the effect of the length of the records in inverse solution is also found out.

Keywords: Source Inverse Problems, Duration of waveforms, Kinematics of the rupture process

1. INTRODUCTION

During the past decades, the cities have been developed to a great extent to the hillsides. On the other hand, the foothills and skirts which are generally the intersection of the mountain and hills are simultaneously the trajectory of the faults. However, the seismic activity of the mentioned faults would cause various earthquakes in these regions and due to the presence of important structures in the vicinity of the faults, the characteristic of the ground motion in such near field areas are significant. Indeed, the specific aspects of each fault will efficacious in determination of the near field ground motion characteristics. Though, it is requisite to consider the specific situation of the aforesaid faults in the earthquake hazard evaluation.

In order to take the specific properties of the fault into account while analysing the earthquake hazard of the fault, it is necessary to identify these characteristics. The main issue in fault recognition is the determination of the fault's previous activities or that of similar faults which have experimented seismic activities during the available history for earthquakes. In other words, the fault's characteristics have to be defined by the inverse solution according to the measured information throughout different earthquakes. Providing these facts, the earthquake inverse solution is the first and most essential step while identifying the faults properties and undeniably it is one of the most precious topics in seismology.

Executing the inverse solution and obtaining the slip vector on the fault plane, firstly, the stress drop during the earthquake on different points could be determined. Accordingly, the stress association rate on the fault could be identified. Regarding to the above matter, the prediction of the future earthquake's occurrence time on a specific fault could be feasible (Das & Suhadolc, 1996). Secondly, by virtue of the obtained slip vector, the dislocation of different points of the ground surface in regions with no recorded earthquake could be determined. However, in this way, the recognition of the seismic motion characteristics on the ground surface could be attained during a specific event.

To facilitate the illustration the authors have supposed a number of records due to a given earthquake and they have intended to initiate the inverse solution using these records. Thus, an ascertained length of the records has to be assumed. It is obvious that the greater the length of the signal, the more



massive the computation procedure will be. Meanwhile, if the time of the strong ground motion would be before tl in most of the stations, the consideration of a greater length for the record seems to be ineffectual in the inverse solution results. However, is this acceptable?

2. THE LINEAR INVERSE SOLUTION FORMULA

In faulting process, rupture initiate from hypocenter, then as the propagating rupture sweeps over the fault plane, each point traces an individual slip-time history (slip-velocity function) whose shape and duration depends on the stress conditions and frictional properties on the fault, but also on overall fault size and the position of each point with respect to rupture nucleation. The slip duration (rise time) at each point is usually defined by integrating the slip-velocity function and measuring the time it takes to complete 5 - 95% of the total displacement at each point (Mai, 2008).In kinematic rupture model, rise time, which characterizes the time taken for the offset, is considered to be constant in all point of fault plate. Also several simplified source-time function, such as ramp function, are proposed till now to use instead of these complicated slip functions. In all earthquake forward and inverse solution, the governing equation that relates ground displacement to the motion on the fault is given by a representation theorem (Guatteri, et al., 2004) (Hartzell, et al., 2005) which can explain in following notation:

$$u_n(x,t) = \int d\tau \int_S \Delta u_i(\zeta,\tau) c_{ijkl} \, \mathcal{N}_j \, \mathcal{G}_{nk,l}(x,t;\zeta,\tau) \, ds \tag{2.1}$$

In this equation $u_n(x,t)$ is the nth component of displacement at position x and time t resulting from the slip on fault plane. $\Delta u_i(\zeta,\tau)$ is the ith component of the local discontinuity in displacement across the fault, v is the fault normal vector and c_{ijkl} is the Hook's law parameter. Also, $G_{nk}(x,t;\zeta,\tau)$ is the displacement of position x in nth coordinate direction and in time t due to an impulsive point load applied at position ζ in k th coordinate direction and in time τ . In addition, $G_{nk,l}(x,t;\zeta,\tau)$ is the derivative of green function respect to the spatial variables. Now, we want to rewrite equation 1 in discrete form in space and time, the fault surface is divided in to a set of finite cells (Fig. 2.1). So, for isotropic space we can write:

$$\begin{split} \overline{u}(x_{nn}) &= f(t) \otimes \\ \left\{ \sum_{l=1}^{nmesh \ nmQ^{2}} \left\{ \mu \Big[u_{1}(l) \nu(2) + u_{2}(l) \nu(1) \Big] \times \Big[G_{n2,1}(x_{nn};\xi(k,l)) + G_{n1,2}(x_{nn};\xi(k,l)) \Big] + \\ \mu \Big[u_{1}(l) \nu(3) + u_{3}(l) \nu(1) \Big] \times \Big[G_{n3,1}(x_{nn};\xi(k,l)) + G_{n1,3}(x_{nn};\xi(k,l)) \Big] + \\ \mu \Big[u_{2}(l) \nu(3) + u_{3}(l) \nu(2) \Big] \times \Big[G_{n3,2}(x_{nn};\xi(k,l)) + G_{n2,3}(x_{nn};\xi(k,l)) \Big] + \\ 2 \mu \Big[u_{1}(l) \nu(1) G_{n1,1}(x_{nn};\xi(k,l)) \Big] + \\ 2 \mu \Big[u_{2}(l) \nu(2) G_{n2,2}(x_{nn};\xi(k,l)) \Big] + \\ 2 \mu \Big[u_{3}(l) \nu(3) G_{n3,3}(x_{nn};\xi(k,l)) \Big] \Big\} \end{split}$$

$$(2.2)$$

In this equation $\overline{u}(x_{nn})$ is the displacement signal in *nn* th station. f(t) is source time function and \otimes is illustrated as convolution product. *nmesh* is the number of cells on fault surface and nnQ^2 is the number of Gaussian points in one cell. $u_n(l)$ is the slip of l th cell in *n* th coordinate direction. Finally, $G_{ni,j}(x_{nn};\xi(k,l))$ is the derivative of green function. In this, $G_{ni}(x_{nn};\xi(k,l))$ is the displacement of *nn* th station in nth coordinate direction due to an impulsive point load applied at *k* th Gaussian point in *l* th cell and in *i* th coordinate direction. In this equation $\overline{u}(x_{nn})$, f(t) and $G_{ni,j}(x_{nn};\xi(k,l))$ are strings in time. Unknown parameter in linear inverse solution is a final slip in each cell in longitude and lateral direction. Hence, it is necessary to explain $u_n(l)$ in term of u_d and u_s , in that s and d indicates strike slip and dip slip.



Figure 2.1.Schematic view of fault that shows the discrete model with cells and two directions of slip on fault.

According to figure 2.1, we can write:

$$u_{n}(l) = u_{s}(l)e_{1}(n) + u_{d}(l)e_{2}(n)$$
(2.3)

For inverse solution and using the records of all stations, the least square method is used. For this purpose the following difference between artificial records and actual records in all station should be minimized:

$$E = \sum_{nn=1}^{nst} \sum_{n=1}^{3} \left(\bar{u}_n \left(x_{nn} \right) - u_n \left(x_{nn} \right) \right)^2$$
(2.4)

To minimize the above difference, the partial derivatives of E according to final slip in two directions and in all cells have to set to zero:

$$\frac{\partial E}{\partial u_{s}(l)} = 0$$

$$\frac{\partial E}{\partial u_{d}(l)} = 0$$

$$l = 1, ..., nmesh$$
(2.5)

If the number of time sampling is n_{tf} for source time function and n_{tG} for green function, each of equations in Eqn.2.5, for one cell, contains $(n_{tf} + n_{tG} - 1)$ bonds. Therefore, the total number of equations becomes $2 \times \text{nmesh} \times (n_{tf} + n_{tG} - 1)$, whereas the number of unknown parameters is $2 \times \text{nmesh}$. The Eqn.2.5 can be written in the matrix form:

$$Au_s + Bu_d = C \tag{2.6}$$

$$\begin{bmatrix} A \ B \end{bmatrix}_{(2 \times nmesh \times (n_{tG} + n_{tf} - 1)).(2 \times nmesh)} \begin{bmatrix} u_s \\ u_d \end{bmatrix}_{(2 \times nmesh).1} = \begin{bmatrix} C \end{bmatrix}_{(2 \times nmesh \times (n_{tG} + n_{tf} - 1)).1}$$
(2.7)

As regards the finite number of cells, it seems that there are much more equations than unknowns. Nevertheless it is not true. That is, the equations system 2.7 is very impermanent. In other word, the lines of this equations system are not independent and the coefficient matrix is singular. Overall, the inverse problem in seismology does not have a permanent solution. For having unique and permanent solution, it is necessary to add additional information[(Hartzell & Heaton, 1983), (Das & Kostrov, 1990), (Das, et al., 1996), (Kostrov & Das, 1988)].

3. The EFFECT OF THE CUTTING LENGTH OF THE SIGNAL ON THE INVERSE SOLUTION RESULTS

The previous section has been closed by an appropriate illustration of the inverse solution principles. At the moment, suppose an earthquake and a number of records following that which the inverse solution is initiated by the use of these signals. Therefore, a certain length of each signal should be taken into consideration. Hence, in continuation, the effect of the cutting length of the signal has been investigated. As before said, having a larger signal cutting length, the record's processing time will enlarge and besides, if the time of the strong ground motion in most stations would be before t_1 , assuming a larger length for the records appears to be indifferent while considering the results. Though, in the present study, the sensitivity analysis of the inverse solution due to the cutting length of the record has been investigated. To smooth the noted progress, an example has been demonstrated. A fault with a specific slip has been assumed in this regard and consequently the related dislocation of a station has been calculated using the direct solution. Afterwards, the inverse solution has been presented.

The Imperial fault of the California state due to the 1979 earthquake has been considered in this work. The relevant data base has been offered by Hartzell and Heaton on 1983 in the earthquake data base (SRCMOD) (Mai, 2004) which the same data has been employed here.

The properties of the previously mentioned fault and the modeling and location specifications have been presented in Table 3.1. The slips of the fault's surface have also been denoted in Figure 3.1. The smoothed ramp function (second integration of the step function) is supposed as the time function in the solution.

The under investigation space is a one layer half-space which the one layer soil assumption will assist the calculation of the green function. It is worth mentioning that the relations of the green function of a one layer half-space in the (Johnson, 1974) have been utilized in this study. A presumed station has been supposed for the direct solution and record generation. The location of the station is assumed (60000, 5000, 0). The resulted signal from direct solution is as shown in Figure 3.2. According to these graphs, it is obvious that time of the strong motion is before 33 seconds. Thus, the inverse solution has been performed for the 32, 34, 36, 37 and 40 seconds cutting lengths. The contour plot of the fault's surface slips due to inverse solution with different cutting lengths of signal has been proposed in Figure 3.3-3.7. The concurrent comparison of the results has also been indicated in Figures 3.8 and 3.9.

f four	corners of fault(m)	n1	(-1000,0,0)				
ates o		n2	(-2.5276e4, 3.3543e4,-1000)				
oordin		n3	(-2.5276e4, 3.3543e4,-11500)				
The c		n4	(-11500,0,0)				
		The length of fault	42000m				
		The width of fault	10500m				
		Numbers of elements	4*14				
		The numbers of gauss points in each direction for one cell	4				
		The slip filed of fault in us direction	For each cell is according to fig.3.1				
		Source Time Function	Smoothed ramp function				
		Time step	0.1 sec				
		Rise Time=RT	0.7 sec				
		The Coordinates of hypo center	(-3.9118e3, 5.1911e3,-10500)				
		The velocity of crack propagation	2.5 km/s				

Table 3.1. The properties of Imperial Valley Fault and the simulation parameters

0.001	0.286	0.527	0.908	0.611	0.307	0.54	0.399	0.455	0.331	0.369	0.286	0.095	0.609
0	0	0.366	0.697	1.106	1.063	1.17	1.04	0.931	0.476	0.413	0.27	0.188	0.284
0	0	0.127	0.33	0.995	1.8	1.763	1.402	1.118	0.683	0.16	0.384	0.472	0.244
0	0	0	0.105	0.457	1.493	1.655	1.377	1.359	1.261	0.434	0.496	0.798	0.592

Figure3.1. The slips of the fault's surface in each cell in us direction.



Figure 3.2. The resulted signals from direct solution in assumed station



Figure3.3. The contour plot of the fault's surface slips (m) due to inverse solution with 40 sec cutting length. (Up in us direction, down in ud direction)



Figure3.4. The contour plot of the fault's surface slips (m) due to inverse solution with 37 sec cutting length. (Up in us direction, down in ud direction)



Figure3.5. The contour plot of the fault's surface slips (m) due to inverse solution with 36 sec cutting length. (Up in us direction, down in ud direction)



Figure3.6. The contour plot of the fault's surface slips (m) due to inverse solution with 34 sec cutting length. (Up in us direction, down in ud direction)



Figure3.7. The contour plot of the fault's surface slips (m) due to inverse solution with 32 sec cutting length. (Up in us direction, down in ud direction)



Figure 3.8. The concurrent comparison results (slips of 56 cells on fault in us direction) due to inverse solution with different cutting lengths of signal.



Figure 3.9. The concurrent comparison results (slips of 56 cells on fault in ud direction) due to inverse solution with different cutting lengths of signal.

As it is evident from the contour plots, by the variation of the cutting length in the inverse solution, not only the value of fault's surface slips, but also its pattern will change. Consequently, from these facts it could be concluded that the cutting length of the signal is of great import in inverse solution.

4. CONCLUSION

The effect of the cutting length of the time history signal on the inverse solution results has been investigated through two examples. The most apparent outcome of these analyses is that the cutting time of the record is one of the most significant and efficient factors in the inverse solution issue. The fact becomes principally distinct while using a large number of stations which is the case of most practical approaches. Although, it seems that if the time of the strong ground motion in most stations would be before tl, supposing the greater length for the signal will not affect the inverse solution results, but as it has been revealed in this article, it will. Thus, the important issue is the required time for the propagation of the crack front to all of the fault points and the requisite time for the wave's arrival with minimum velocity to the farthermost station from those which has been included in the inverse solution and a consequently a different slip would be achieved for the fault's surface. In conclusion, it could be claimed that the supposed length of the signal is a significant factor in inverse solution problems.

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