# What Are Recorded In A Strong-Motion Record? 

H.C. Chiu<br>Institute of Earth Sciences, Academia Sinica, Taipei, Taiwan

## F.J. Wu

Central Weather Bureau, Taiwan


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H.C. Huang<br>Institute of Earthquake, National Chung-Chen University, Taiwan


#### Abstract

SUMMARY: Accelerographs were designed mainly for measuring three-component ground translational accelerations. Their sensor (accelerometer) can also detect rotational motion and its induced effects. These effects include centrifugal acceleration, gravity (ground tilt) effect and effects of rotation frame (Chiu, 2012), in most cases, are much smaller than that of translational motion and have been ignored in most analyses of strong-motion data. However, the rotation motions might have a significant growth as the ground motions increasing (Lee et al., 2009; Takeo, 2009) and more observations of near-fault and extreme large ground motions suggest that these effects might be underestimated. In this study, we calculate these effects numerically using a set of rotation rate-strong motion velocity data recorded in a magnitude 6.9 earthquake and examine their effects on the strong-motion waveforms. Although the rotation angles, centrifugal acceleration and gravity effects are small, but they might significantly modify the acceleration, velocity or the displacement waveforms.


Keywords: Translational motions; Rotational motions; Strong motion;

## 1. INTRODUCTION

An ideal accelerometer is a damped oscillator with a high nature frequency (e.g. 200 Hz in Kinemetrics' Episensor) and a damping value about $70 \%$ of critical damping. This type of accelerometer gives a very flat frequency response below the nature frequency and the ground acceleration can be a good approximation to the displacement of the mass of the accelerometer multiplied by an amplification factor. Since this type of sensor measure the ground acceleration, it also can detect the induced acceleration due to rotational motions. The rotational motions related accelerations include the centrifugal acceleration and the change of gravity on three-component accelerometers. In most cases, the translational motions are much larger than that of all other signals and the latter can be ignored. This approximation is no more valid as ground motions or ground deformations become large which are very common in near-fault records. Although three-component translational motions plus the collocated three-component rotational motions can fully describe ground motions, the quantitative analysis of these effects becomes possible only recently. The recent progress in rotation sensor makes it possible to have sufficient accuracy to measure both ground acceleration and rotational motion data. With these collocated ground acceleration and rotational motions, we are able to estimate the effects of rotational motion on the strong-motion data. In order to have a better understanding of these signals in a strong-motion record, time domain analyses of these effects are necessary. In this study, we solve the attitude equation and equation of motion in the rotation frame numerically by using a set of 6-component ground motion data. Results give the three-component Euler's angles, centrifugal accelerations and gravity effects in time domain. With this set of information, we can further calculate the translational motions in the reference frame and estimate the effects of rotation frame on the translational motions.

## 2. DATA AND DATA PROCESSING

The data selected in this study was recorded at the seismographic station HWLB during the December 19, 2009 earthquake. This earthquake, registered 6.9 in the local magnitude with focal depth of 44 km , is the largest event during an experimental deployment from May 2008 to now (November 2011). The epicenter locates in the south of HWLB and the epicenter distance is about 21 km .

The installation at HWLB includes a set of collocated R1 rotation sensor and VSE-355G3 broadband velocity sensor with a 6-channel Quanterra 24-bit Q330 data logger. There is another independent strong-motion accelerograph installed in the same location and its station code is HWA019. HWA019 belongs to an island-wide strong-motion network in the Taiwan Strong Motion Installation Program.

Our analyses require both three-component rotation rate and three-component acceleration waveforms in common timing system as inputs. Therefore, we need convert these broadband velocity waveforms to acceleration first. However, we need to check this conversion with the independent observation at HWA019. Comparisons of waveforms and Fourier amplitude spectra between the derived acceleration at HWLB and the observation at HWA019 are shown to have high correlations. The correlation coefficients between the derived acceleration and the observation at HWA019 in the north-south up-down and east-west components are $0.9990,0.9986$ and 0.9991 respectively. The derived acceleration and observed rotation rate after baseline correction are shown in Fig. 1 and all these waveforms have been applied a band-pass filter from 0.1 Hz to 20 Hz . The maximum rotation rate of two horizontal components are about $2 \times 10^{-3}$ while the vertical component is about $5 \times 10^{-4}$. The peak ground acceleration is about $186 \mathrm{~cm} / \mathrm{s} / \mathrm{s}$ in the horizontal component and about $52 \mathrm{~cm} / \mathrm{s} / \mathrm{s}$ in the vertical component.


Figure 1. Three-component rotation rate (left) and ground acceleration (right) used in this study. From top to the bottom are east, north and up directions. The acceleration is derived from the broadband velocity seismogram by numerical differentiation

## 3. METHOD

The method is the same as that used by Chiu et al. (2012). The brief introduction of this method is given in the following discussion.

The explicit form of the equation of motion in the rotation frame is

$$
\begin{equation*}
\ddot{U}^{R}(t)=A^{R}(t)-\dot{\Theta}^{R}(t) \times \dot{U}^{R}(t)-G^{R}(t) \tag{1}
\end{equation*}
$$

In equation (1) and the following discussions, the symbols in capital letters represent a vector or a matrix and the superscript " R " of a parameter denotes that the parameter is measured in the rotation frame. This rotation frame is relative to reference frame which is fix before seismic loading. In equation (1), acceleration $\ddot{U}^{R}$ and velocity $\dot{U}^{R}$ of the ground are two unknown to be solved, $A^{R}$ and $\dot{\Theta}^{R}$ are observed acceleration and rotation rate in the rotation frame. The second and the third terms on the right of equation are effects due to the present of the rotation motions; the second term is the induced centrifugal acceleration which is equivalent to the cross product of the rotation rate and velocity vector while the third term $G^{R}$ is the effective gravity on three components. Equation (1) can be rewritten as an ordinary differential equation of $\dot{U}^{R}$

$$
\begin{equation*}
\frac{d \dot{U}^{R}(t)}{d t}=A^{R}(t)-\dot{\Theta}^{R}(t) \times \dot{U}^{R}(t)-G^{R}(t) \tag{2}
\end{equation*}
$$

Before calculating $G^{R}$, we need to evaluate Euler's angles $\Psi=\left(\begin{array}{lll}\alpha & \beta & \gamma\end{array}\right)$ which define the transformation between the reference frame and rotational frame. To calculate the Euler's angles as time functions, we need to solve the attitude equation (Lin et al., 2010).

$$
\left(\begin{array}{c}
\dot{\alpha}  \tag{3}\\
\dot{\beta} \\
\dot{\gamma}
\end{array}\right)=\left(\begin{array}{ccc}
1 & s_{1} \tan \beta & c_{1} \tan \beta \\
0 & c_{1} & -s_{1} \\
0 & s_{1} \sec \beta & c_{1} \sec \beta
\end{array}\right)\left(\begin{array}{c}
\dot{\theta}_{x} \\
\dot{\theta}_{y} \\
\dot{\theta}_{z}
\end{array}\right)
$$

Equation (3) is another ordinary differential equation of $\alpha, \beta$ and $\gamma$. Solving equation (3) gives the time function of $\alpha(\mathrm{t}), \beta(\mathrm{t})$ and $\gamma(\mathrm{t})$. Once we have Euler's angles at hand, we can apply them to calculate $G^{R}$ and to transform the ground motions from rotation frame to that in the reference frame.

The gravity effect $G^{R}$ in equation (2) is the change of gravity before and after the present of rotation motions, therefore

$$
G^{R}=G-T_{3} T_{2} T_{1} G=\left(\begin{array}{l}
0  \tag{4}\\
0 \\
g
\end{array}\right)-\left(\begin{array}{ccc}
c_{3} & -s_{3} & 0 \\
s_{3} & c_{3} & 0 \\
0 & 0 & 1
\end{array}\right)\left(\begin{array}{ccc}
c_{2} & 0 & s_{2} \\
0 & 1 & 0 \\
-s_{2} & 0 & c_{2}
\end{array}\right)\left(\begin{array}{ccc}
1 & 0 & 0 \\
0 & c_{1} & -s_{1} \\
0 & s_{1} & c_{1}
\end{array}\right)\left(\begin{array}{l}
0 \\
0 \\
g
\end{array}\right)=\left(\begin{array}{c}
-g s_{2} \\
g c_{2} s_{1} \\
-g\left(1-c_{1} c_{2}\right)
\end{array}\right)
$$

where $s_{1} \equiv \sin \alpha, c_{1}=\cos \alpha, s_{2}=\sin \beta$, etc.

Substituting (4) and two known variables $A^{R}$ and $\dot{\Theta}^{R}$ to (2) gives $\dot{U}^{R}$. The pure translational motion in the rotation frame $\dot{U}^{R}$ can be transformed to reference frame using the same transformation matrix as that in equation (4) and result in the translational velocity in the reference frame $\dot{U}$. The translational acceleration and displacement in the reference frame can be derived from $\dot{U}$ by numerically differentiation and integration.

## 4. RESULTS AND DISCUSSION

Solving equation (3) gives the time functions of Euler's angels $\alpha(\mathrm{t}), \beta(\mathrm{t})$ and $\gamma(\mathrm{t})$. Plots of these time functions are given in Fig. 2. These time histories differ from the integration of the rotation rate functions because the latter in the rotational frame constantly changes its orientation during the seismic loading. The maximum Euler's angle, as shown in Fig. 2, is about 0.008 degrees which indicates that the rotational motions are moving within a very small range.

The induced centrifugal acceleration due to rotational motions are shown in Fig. 3, the centrifugal acceleration of the vertical component has a larger value and appears to have an asymmetric time history. The peak centrifugal acceleration is about $0.03 \mathrm{~cm} / \mathrm{s} / \mathrm{s} /$ which is much larger than the conventional estimates (e.g., Graizer,, 2009). Following Graizer's approach and keeping the same length of the spring at 20 cm , the induced centrifugal acceleration is less than $0.000135 \mathrm{~cm} / \mathrm{s} / \mathrm{s}$ as the highest angular velocity in this case is $0.0026 \mathrm{rad} / \mathrm{sec}$. This estimation is much smaller than our result, $0.03 \mathrm{~cm} / \mathrm{s} / \mathrm{s}$. In fact, the VSE-355G3 belongs to the mass-on-spring type sensor which has much shorter length of the spring and will cause even smaller centrifugal acceleration. The discrepancy implies that the center of rotation motion is not at the fixed point of the spring in the seismometer.

The estimated effects of gravity using equation (4) are shown in Fig. 4. The time histories of two horizontal components are similar to the ground-motion waveform and both have a similar peak value of about $0.15 \mathrm{~cm} / \mathrm{s} / \mathrm{s}$. It is worth noticing that the effects of gravity in the vertical component are much smaller than those of the two horizontal components and to be negative. The negative value is expected because the tilt of the instrument always causes a reduction on the effect in the vertical direction. A smaller effect for the vertical component is also expected. For a small ground tilt, the horizontal component is proportional to the tilt angle while the vertical component will be proportional to the square of the tilt angle.


Figure 2. Three-component Euler's angles in unit of degree. The maximum is about 0.01 degree.


Figure 3. Three-component centrifugal acceleration due to rotation motions.


Figure 4. The gravity effects on the three axis of the rotation frame.
Solving the ordinary differential equation in (2) gives the three-component velocity waveforms in the rotation frame. The acceleration waveforms in the rotation frame can be obtained by substituting the velocity back to Equation (1) and the displacement waveforms can be obtained by taking numerical integration of the velocity time histories.

Both the maximum centrifugal acceleration and effect of gravity are small ( $0.03 \mathrm{~cm} / \mathrm{s} / \mathrm{s}$ and $0.14 \mathrm{~cm} / \mathrm{s} / \mathrm{s}$ ). This result is expected because our data are not near-field and wave motions are expected to be nearly plane waves.

The effects due to the rotation frame can be examined by comparing the waveforms in both frames. The acceleration waveforms in the reference and the rotation frame are shown in the first and second columns in Fig. 5 and their difference is given in the third column. The maximum difference is about $16 \mathrm{~cm} / \mathrm{s} / \mathrm{s}$ (about 10\%). This difference has lower frequency content than that of the original signal and is much larger than the combination of the centrifugal acceleration and gravity effects. It seems that this large difference in waveforms to be related to the phase difference between two waveforms, but we are still unable to identify the cause of this large difference which warrants further investigation. The velocity waveforms in the reference and the rotation frame are shown in the first and second columns in Fig. 6 and their differences are given in the third column. The difference is about $0.0012 \mathrm{~cm} / \mathrm{s}$ (about $0.01 \%$ ).

## 5. CONCLUSIONS

This study develops a numerical algorithm for estimating the effects of rotational motion on the strong-motion data using a set of six-component ground motions. Due to the limitation of rotation data, the estimation in this study is limited in a frequency band from 0.1 to 20 Hz . However, the algorithm itself doesn't have such limitation.

Effects of rotation motions on translational motions include centrifugal acceleration, gravity effects and the effects of the rotation frame. Results show that all these effects have the same order of contribution to the strong-motion data. Since all these analyses are in time domain, our results provide some detailed features of these effects.

The centrifugal acceleration calculated in this study is $0.03 \mathrm{~cm} / \mathrm{s} / \mathrm{s} /$ which is much larger than 0.000135 $\mathrm{cm} / \mathrm{s} / \mathrm{s}$ calculated base on Graizier’s estimation (Graizer, 2009). This discrepancy implies that the center of the rotation motion might be not at the fixed point of the pendulum. What cause this discrepancy warrants further studies.

The maximum difference between the corrected and uncorrected acceleration waveforms is about $16 \mathrm{~cm} / \mathrm{s} / \mathrm{s}$ (about $10 \%$ ). This difference has higher low-frequency content than that of the original signal and is much larger than the combination of the centrifugal acceleration and gravity effects. The cause of this large difference is not known and it needs further investigation.


Figure 5. The three-component accelerations on the reference frame are in the first column while the corresponding components on the rotation frame are in the second column. The differences between these two types of accelerations are shown in the third column.


Figure 6. The three-component velocities on the reference frame are in the first column while the corresponding components on the rotation frame are in the second column. The differences between these two types of accelerations are shown in the third column.

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