The Complete Multiple Support SRSS modal Combination Rule

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SUMMARY:
In this paper a modal combination rule for seismic analysis of multiple supported structures is presented. This rule allows expressing the structural response as the sum of uncoupled SDOF modal responses yet accounting fully for modal response correlation and spatial variation of seismic ground motion. This formulation is an extension of the complete Square-Root-of-Sum-of-Squares (c-SRSS) modal combination rule originally formulated for structures subjected to a single ground motion.

It is shown that spectral moments can be rigorously expressed as the sum of uncoupled modal responses fully accounting for the contributions of the variances and cross-covariances between them. The great advantage of this rule is that we can estimate the maximum structural response without approximations that neglect the modal response covariances or assuming certain type of ground motions.

Keywords: modal combination rule, multiple supports, random vibration

1. DYNAMIC RESPONSE VARIANCE

Consider a multi-support (MS) linear structural system with n response degrees of freedom subjected to m support earthquake ground motions. Assume that the ground motions at the supports of the structure are zero-mean jointly stationary random processes and that the duration of the ground motions is long enough for the response to reach stationarity. Let ω_i , ξ_i denote the modal frequencies and critical damping ratios of the structure, and $u_k(t)$, $k = 1..m$ denote the ground displacement at the k^{th} support. A generic response of interest, $Z(t)$, e.g. internal forces in a member, displacements at a node, or stress at a point, in general can be expressed as the sum of a pseudo static and a dynamic component.

$$Z(t) = \sum_{k=1}^{m} a_k u_k(t) + \sum_{k=1}^{m}\sum_{i=1}^{n} c_{k,i} y_{k,i}(t) \qquad 1.1$$

where $y_{k,i}$ is the SDOF displacement response of a modal oscillator with natural frequency ω_i and critical damping ratio ξ_i subjected to the ground acceleration $\ddot{u}_k(t)$; a_k and $c_{k,i}$ are the effective-influence coefficients and effective modal participation factors, respectively, which depend on the mass and stiffness properties of the structure only (Der Kiureghian and Neuenhofer, 1992). From the double sum in the right hand side of (1), we obtain for the spectral density function of the dynamic responses Z_d ,

$$S_{Z_d Z_d}(\omega) = \sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} c_{k,i} c_{l,j} \operatorname{Re}_{i,j} S_{\ddot{u}_k \ddot{u}_l}(\omega) + \imath \sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} c_{k,i} c_{l,j} \operatorname{Im}_{i,j} S_{\ddot{u}_k \ddot{u}_l}(\omega) \qquad 1.2$$

In which $S_{\ddot{u}_k\ddot{u}_l}$ is the ground acceleration cross-spectral density, $\mathrm{Re}_{i,j} = \mathrm{Re}\lfloor H_i(\omega)H_j(-\omega)\rfloor$ and $\mathrm{Im}_{i,j} = \mathrm{Im}\lfloor H_i(\omega)H_j(-\omega)\rfloor$., and $H_i(\omega)$ is the complex transfer function of the SDOF modal oscillators,

$$H_i(\omega) = \frac{1}{\omega_i^2 - \omega^2 + 2\imath\xi_i\omega_i\omega}; \quad \imath = \sqrt{-1} \qquad 1.3$$

In general, the cross-spectral density $S_{\ddot{u}_k\ddot{u}_l}$ is a complex function of frequency, $S_{\ddot{u}_k\ddot{u}_l} = C_{k,l} + \imath Q_{k,l}$ where the co spectrum $C_{k,l}$ and the quadrature spectrum $Q_{k,l}$ are even and odd functions of frequency, respectively. Taking into account that $S_{\ddot{u}_k\ddot{u}_l} = S_{\ddot{u}_l\ddot{u}_k}*$ and $\lfloor H_i(\omega)H_j(-\omega)\rfloor = \lfloor H_j(\omega)H_i(-\omega)\rfloor*$ (where *denotes complex conjugate), Ec. 1.2 becomes:

$$S_{Z_dZ_d}(\omega) = \sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ki}c_{lj}\,\mathrm{Re}_{ij}(\omega)C_{kl}(\omega) - \sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ki}c_{lj}\,\mathrm{Im}_{ij}(\omega)Q_{kl}(\omega) \qquad 1.4$$

Using partial fractions decomposition Heredia-Zavoni (2011) has expressed $\mathrm{Re}_{i,j}(\omega)$ in terms of the square norms of $H_i(\omega)$ and $H_j(\omega)$ for deriving the complete SRSS modal combination rule,

$$\mathrm{Re}_{ij} = A_{ij}\left|H_i(\omega)\right|^2 + B_{ij}\frac{\omega^2}{\omega_i^2}\left|H_i(\omega)\right|^2 + D_{ij}\left|H_j(\omega)\right|^2 + E_{ij}\frac{\omega^2}{\omega_j^2}\left|H_j(\omega)\right|^2 \qquad 1.5$$

Factors A_{ij}, B_{ij}, D_{ij}, and E_{ij} only depend on the modal frequencies ω_i and the critical damping ratios ξ_i. In terms of ratio $q = \frac{\omega_i}{\omega_j}$ they are given by,

$$E_{ij} = \frac{[4\xi_j q(\xi_i - \xi_j q) + q^2 - 1](1-q^4) - 2q^2(1-q^2)[q^2(1-2\xi_j^2) - (1-2\xi_i^2)]}{4q^2[(1-2\xi_i^2)q^2 - (1-2\xi_j^2)][(1-2\xi_j^2)q^2 - (1-2\xi_i^2)] - (1-q^4)^2} \qquad 1.6$$

$$B_{ij} = -q^2 E_{ij} \qquad 1.7$$

$$D_{ij} = \frac{4\xi_i\xi_j q - q^2 - 1 + 2q^2(1-2\xi_j^2) - (q^4-1)E_{ij}}{2q^2[q^2(1-2\xi_j^2) - (1-2\xi_i^2)]} \qquad 1.8$$

$$A_{ij} = q^2 - q^4 D_{ij} \qquad 1.9$$

and satisfy $A_{ij} = D_{ji}$, $B_{ij} = E_{ji}$; for $q = 1$ $A_{ij} = 0.5, B_{ij} = 0$. For the imaginary part $\mathrm{Im}_{ij} = \mathrm{Im}\lfloor H_i(\omega)H_j(-\omega)\rfloor$ we can use the partial fraction expansion in Heredia-Zavoni and Vanmarcke, 1994):

$$\mathrm{Im}_{ij} = A'_{ij}\frac{\omega}{\omega_i}\left|H_i(\omega)\right|^2 + B'_{ij}\frac{\omega^3}{\omega_i^3}\left|H_i(\omega)\right|^2 + D'_{ij}\frac{\omega}{\omega_i}\left|H_j(\omega)\right|^2 + E'_{ij}\frac{\omega^3}{\omega_j^3}\left|H_j(\omega)\right|^2 \qquad 1.10$$

Factors $A'_{ij} + B'_{ij}$, D'_{ij} and E'_{ij} are derived here in terms of ratio $q = \dfrac{\omega_i}{\omega_j}$,

$$E'_{ij} = \frac{(1-q^4)[2\xi_i q - 2\xi_j - 2(2\xi_j q^2 - 2\xi_i q)(2\xi_j^{\,2} - 1)]}{-(1-q^4)^2 - [2q^2(2\xi_i^2 - 1) - 2(2\xi_j^{\,2} - 1)][2q^2(2\xi_i^2 - 1) - 2q^4(2\xi_j^{\,2} - 1)]}$$

$$+\frac{-(-2\xi_j q^2 + 2\xi_i q)(2q^2(2\xi_i^2 - 1) - 2q^4(2\xi_j^{\,2} - 1))}{-(1-q^4)^2 - [2q^2(2\xi_i^2 - 1) - 2(2\xi_j^{\,2} - 1)][2q^2(2\xi_i^2 - 1) - 2q^4(2\xi_j^{\,2} - 1)]} \qquad 1.11$$

$$B'_{ij} = -q^3 E_{ij} \qquad 1.12$$

$$D'_{ij} = \frac{2\xi_i q - 2\xi_j q^2 - E_{ij}[2q^2(2\xi_i^2 - 1) - 2(2\xi_j^{\,2} - 1)]}{1-q^4} \qquad 1.13$$

$$A'_{ij} = q^2(2\xi_j q - 2\xi_i - q^3 D_{ij}) \qquad 1.14$$

These factors also depend only on the modal frequencies and critical damping ratios, and satisfy $A'_{ij} = -D'_{ji}$, $B'_{ij} = -E'_{ji}$. Introducing the response coefficients:

$$\alpha_{ikl} = \sum_{j=1}^{n}\left(c_{k,i}c_{l,j} + c_{l,i}c_{k,j}\right)A_{ij}\ ,\quad \beta_{ikl} = \sum_{\substack{j=1\\ j\neq i}}^{n}\left(c_{k,i}c_{l,j} + c_{l,i}c_{k,j}\right)B_{ij} \qquad 1.15$$

$$\alpha'_{ikl} = \sum_{j=1}^{n}\left(c_{k,i}c_{l,j} - c_{l,i}c_{k,j}\right)A'_{ij}\ ,\quad \beta'_{ikl} = \sum_{j=1}^{n}\left(c_{k,i}c_{l,j} - c_{l,i}c_{k,j}\right)B'_{ij} \qquad 1.16$$

and using (8) and (6) in (5) we can show that

$$S_{Z_dZ_d}(\omega) = \sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n}\left[\alpha_{ikl} + \beta_{ikl}\left(\frac{\omega}{\omega_i}\right)^2\right]\left|H_i(\omega)\right|^2 C_{k,l}$$
$$-\sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n}\left[\alpha'_{ikl}\left(\frac{\omega}{\omega_i}\right) + \beta'_{ikl}\left(\frac{\omega}{\omega_i}\right)^3\right]\left|H_i(\omega)\right|^2 Q_{k,l}(\omega) \qquad 1.17$$

Power spectral density of the response of a simple beam (Figure 1.1) subjected to horizontal ground motion is estimated using both Eqs 1.4 and 1.17, in order to corroborate partial fractions replacement. The beam has uniform mass and stiffness properties along its longitude. It is assumed that mass is concentrated in two point of the beam and no mass moments of inertia are associated with rotational degrees of freedom. The ground motions of the three supports are modeled by the modified Kanai Tajimi power spectrum. It is supposed that the ground motion model is the same for the three supports of the beam (see properties in table 1.1). Cross covariance is modeled with a loss of coherency and wave passage effect as discussed below.

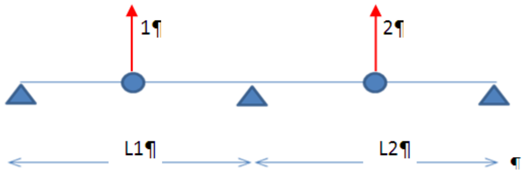
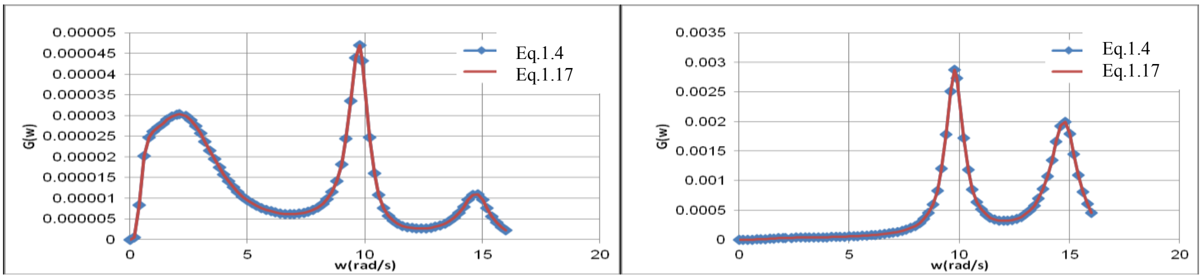


Figure 1.1: Degrees of fredom of beam subjected to seismic ground motion.

Table 1.1 Properties of ground motion in supports

Type of Soil	ω_f (rad/s)	ξ_f	ω_g (rad/s)	ξ_g
Soft	π	0.2	0.5	0.6
Stiff	15	0.6	1.5	0.6

Figure 1.2 shows the analytical spectral density of the horizontal displacement of the DOF #1. It can be seen that for both soil conditions, the spectral density obtained using the partial fraction expansion coincides with the theoretical one. Peaks at frequencies equal to 9.79 and 14.81 rad/s correspond to the modal natural frequencies of the structure. Furthermore, for soft soil conditions, the spectral density exhibits a peak around the dominant frequency of the soil.



(a) (b)

Figure 1.2: Power spectral density of the displacement response in (a) soft soil (b) and stiff soil

2. GROUND MOTION CROSS CORRELATION

The normalized ground acceleration cross-spectral density is called the coherency spectrum:

$$\gamma_{\ddot{u}_k\ddot{u}_l} = \frac{S_{\ddot{u}_k\ddot{u}_l}}{\sqrt{S_{\ddot{u}_k\ddot{u}_k} S_{\ddot{u}_l\ddot{u}_l}}} \tag{2.1}$$

Several functional forms for the coherency spectrum have been proposed based on theoretical and empirical approaches. Der Kiureghian (1996) proposed a theoretical model based on the superposition of incoherence, wave passage and site effects:

$$\gamma_{\ddot{u}_k\ddot{u}_l} = \gamma_{\ddot{u}_k\ddot{u}_l}{}^{incoherence} \cdot \gamma_{\ddot{u}_k\ddot{u}_l}{}^{wave.passage} \cdot \gamma_{\ddot{u}_k\ddot{u}_l}{}^{site.effects} \tag{2.2}$$

The first factor in the right side of Eq. 2.2 is a real valued function and models the incoherence effect due to the scattering of waves in the heterogeneous medium of the ground and the differential wave arriving at each point from different segments of an extended source. The wave passage and site effect components are complex functions. The following expression was deduced by Der Kiureghian (1996):

$$\gamma_{\ddot{u}_k\ddot{u}_l}{}^{incoherence} = \left|\gamma_{\ddot{u}_k\ddot{u}_l}\right| = \cos[\beta]\exp\left\{-\frac{1}{2}\alpha^2\right\} \tag{2.3}$$

where α and β are increasing functions of frequency, ω ,and distance between points k and l, d_{kl} and

$$\lim_{d_{kl}\to 0} \alpha = \lim_{\omega\to 0} \alpha = 0 \text{ and } , \lim_{d_{kl}\to\infty} \alpha = \lim_{\omega\to\infty} \alpha = \infty \qquad 2.4$$

$$\lim_{d_{kl}\to 0} \beta = \lim_{\omega\to 0} \beta = 0 \quad , \lim_{d_{kl}\to\infty} \beta = \lim_{\omega\to\infty} \beta = \frac{\pi}{2} \qquad 2.5$$

The first term in the right side of Eq. 2.3 represents the random variation of wave amplitudes, the second term represents the random variation in phases angles. Based on Uscinski's (1977) theoretical model of shear waves propagating in a random medium, Luco and Wong (1986) derived a function similar to the second term in Eq 2.3 with:

$$\alpha = \sqrt{2}K_2 d_{kl}\omega \ , \qquad 2.6$$

Where $K_2 = \dfrac{\eta}{Vs}$ is a model parameter, η is an incoherence parameter and V_s is the typical shear wave velocity of the medium. Notice that this expression satisfies the conditions in 2.4. The random variation of amplitudes depends on function β. It is proposed that:

$$\beta = \arctan\left(K_1 \omega d_{kl}\right) \qquad 2.7$$

which must satisfy the conditions in 2.5. The model defined in 2.3, 2.6 and 2.7 satisfy that: (1) has a zero slope at the origin, (2) decays from unity at d_{kl}=0 or ω=0 to a value of zero at $d_{kl} \to \infty$ or $\omega \to \infty$. This model was fitted to strong ground motion data from records of two events in Mexico City. Table 3 shows the characteristic of the events and the estimated values of K_1 and K_2 for the proposed model. Figures 5 shows the fitted curves with Der Kiureghian models for one event. It is observed that the model fits quite well for the Mexico City data over the frequency and separation intervals considered. For comparison purposes, figures 3 to 6 also show the fitted curves presented in Santa Cruz et al (2000) using coherency models proposed by Hindy and Novak (1980), Luco and Wong (1986), Harichandran and Vanmarcke (1986) and Abrahamson (1992).

Results show that for the estimated K_2 values, the random variation in phase angles has little influence in the computation of $\gamma_{\ddot{u}_k \ddot{u}_l}^{incoherence}$, since $\exp\left\{-(K_2 d_{kl}\omega)^2\right\} \approx 1$ for the distance and frequency domain analyzed. It suggests that the main contribution to the incoherence effect of ground motion comes from the random variation of amplitudes. This result is compatible with previous works that show that the Luco and Wong model is not flexible enough over the range of separations considered (Santa Cruz et al , 2000). Ignoring the random variation in phase angles we obtain:

$$\gamma_{\ddot{u}_k \ddot{u}_l}^{incoherence}(\omega) = \frac{1}{\sqrt{1+(K_1 \omega d_{k,l})^2}} \qquad 2.8$$

where K_1 5.5 ~7.5 x 10^{-4}

The wave passage effect is modeled as a complex function:

$$\gamma_{\ddot{u}_k \ddot{u}_l}^{wave:passage} = \exp\left\{-\iota\left(\frac{d_{k,l}{}^{L}\omega}{V_{app}}\right)\right\}, \qquad 2.9$$

where V is the propagation velocity of the waves. Pérez Rocha and Chávez-Garcia have reported apparent velocity of 2000-2600 ms for very soft soil in Mexico City. The contribution of the site effects is modeled as a relation between the real and imaginary part of the product of transfer functions of the soil at points k and l as proposed by Der Kiureghian:

$$\gamma_{\ddot{u}_k \ddot{u}_k}{}^{site\ effects} = \exp\left\{ \iota (\tan^{-1}\left(\frac{\mathrm{Im}\left(H_k(\omega) H_l(-\omega)\right)}{\mathrm{Re}\left(H_k(\omega) H_l(-\omega)\right)} \right)) \right\} \tag{2.10}$$

where H_m is the frequency response function of the soil column at point m, $m=k$,l.

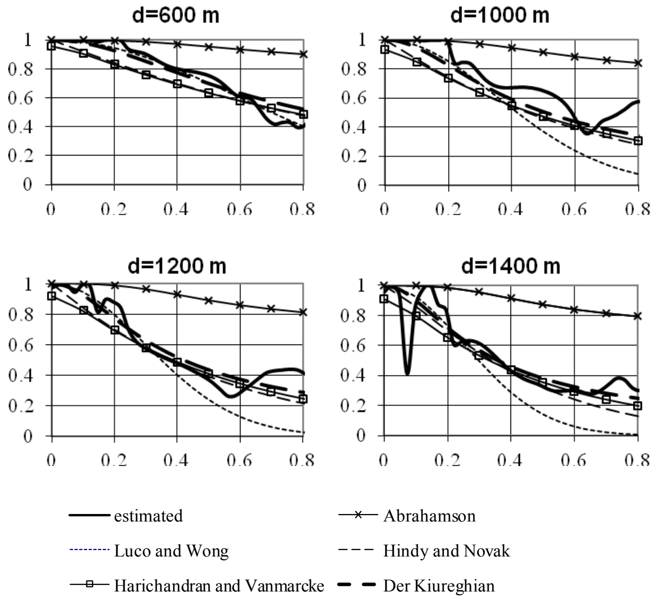


Figure 2.1: Event M-1

3. RANDOM VIBRATION METHODOLOGY

The variance of the dynamic response is the zero-th spectral moment of $S_{Z_d Z_d}$ expressed in eq. 1.17.

$$\sigma^2{}_{Z_d} = \sum_{k=1}^{m} \sum_{l=1}^{m} \sum_{i=1}^{n} \alpha_{ikl} \lambda_{0,ikl} + \frac{\beta_{ikl}}{\omega_i^2} \lambda_{2,ikl} - \frac{\alpha'_{ikl}}{\omega_i} \lambda_{1,ikl} - \frac{\beta'_{ikl}}{\omega_i^3} \lambda_{3,ikl} \tag{3.1}$$

where $\lambda_{0,ikl}, \lambda_{2,ikl}, \lambda_{1,ikl}, \lambda_{3,ikl}$ are spectral moments of modal responses

$$\lambda_{0,ikl} = \int_{-\infty}^{\infty} \left| H_i(\omega) \right|^2 C_{k,l}(\omega) d\omega \qquad \lambda_{1,ikl} = \int_{-\infty}^{\infty} \omega \left| H_i(\omega) \right|^2 Q_{k,l}(\omega) d\omega$$

$$\lambda_{2,ikl} = \int_{-\infty}^{\infty} \omega^2 \left| H_i(\omega) \right|^2 C_{k,l}(\omega) d\omega \qquad \lambda_{3,ikl} = \int_{-\infty}^{\infty} \omega^3 \left| H_i(\omega) \right|^2 Q_{k,l}(\omega) d\omega \tag{3.2}$$

Eq. 3.1 shows that given the cross power spectrum of support ground motions, computing the dynamic response variance can be reduced to the uncoupled analysis of modal responses yet accounting fully for cross-modal contributions through coefficients $\alpha_{ikl}, \beta_{ikl}, \alpha'_{ikl}, \beta'_{ikl}$ which only depend on structural properties. Notice that for a single support, k=l, $\alpha'_{ikk} = \beta'_{ikk} = \lambda_{1,ikk} = \lambda_{3,ikk} = Q_{kk} = 0$, $C_{k,k}$

$= S_{\ddot{u}_k,\ddot{u}_k}$ is the power spectral density of the ground acceleration, and ,. $\lambda_{0,ikk} = \sigma^2_{y_{ki}}$ and $\lambda_{2,ikk} = \sigma^2_{\dot{y}_{ki}}$; then the expression in 3.1 reduces to

$$\sigma^2 z_d = \sum_{i=1}^{n} \alpha_{i11}\sigma^2_{y_{ki}} + \frac{\beta_{i11}}{\omega_i^2}\sigma^2_{\dot{y}_{ki}} \qquad 3.3$$

which is the expression for the response variance derived by Heredia-Zavoni (2011).

The spectral moments in 3.2 can be interpreted in terms of the response displacements Y_{ki} and Y_{li} of a SDOF modal oscillator with natural frequency, ω_i, damping ratio ξ_i, to support ground accelerations $\ddot{u}_k(t)$ and $\ddot{u}_l(t)$. Consider first the cross power spectrum between response displacements Y_{ki} and Y_{li}

$$S[Y_{ki}Y_{li}](\omega) = |H_i(\omega)|^2 S_{\ddot{u}_k\ddot{u}_l}(\omega) = |H_i(\omega)|^2[C_{kl}(\omega) + \imath Q_{kl}(\omega)] \qquad 3.4$$

By definition, the covariance between the response displacements is

$$COV(Y_{ki}Y_{li}) = \int_{-\infty}^{\infty} S[Y_{ki}Y_{li}](\omega)d\omega = \int_{-\infty}^{\infty} |H_i(\omega)|^2[C_{kl}(\omega) + \imath Q_{kl}(\omega)]d\omega \qquad 3.5$$

Since $|H_i(\omega)|^2$ is an even function and considering 3.2, we have that

$$COV(Y_{ki}, \mathrm{Y}_{li}) = \lambda_{0,i,k,l} \qquad 3.6$$

Similarly, the covariance of the velocity responses of a SDOF modal oscillator to ground accelerations $\ddot{u}_k(t)$ and $\ddot{u}_l(t)$ is:

$$COV(\dot{y}_{k,i}\dot{y}_{l,i}) = \int_{-\infty}^{\infty} S_{\dot{y}_{k,i}\dot{y}_{l,i}}(\omega)d\omega = \int_{-\infty}^{\infty} \omega^2 |H_i(\omega)|^2 \left(C_{k,i} + \imath Q_{k,i}\right)d\omega \qquad 3.7$$

It follows from 3.2 that

$$\lambda_{2,i,k,l} = COV(\dot{y}_{k,i} \ \ \dot{y}_{l,i}) \qquad 3.8$$

Furthermore, in Heredia-Zavoni and Vanmarcke (1994) it is shown that,

$$\lambda_{1,i,k,l} = -COV(\dot{y}_{k,i}y_{l,i}) \qquad 3.9$$
$$\lambda_{3,i,k,l} = COV(\ddot{y}_{k,i}\dot{y}_{l,i}) \qquad 3.10$$

Therefore, all of the spectral moments in 3.2 and 3.3 can be interpreted in terms of covariances between various modal responses to the support ground accelerations. Let $\rho(X,Y)$ denote the cross-correlation coefficient between processes X and Y . By definition,

$$
\begin{aligned}
\lambda_{0,i,k,l} &= \rho(Y_{ki}, Y_{li})\sigma(Y_{ki})\sigma(Y_{li}) \\
\lambda_{2,i,k,l} &= \rho(\dot{Y}_{ki}, \dot{Y}_{li})\sigma(\dot{Y}_{ki})\sigma(\dot{Y}_{li}) \\
\lambda_{1,i,k,l} &= -\rho(\dot{Y}_{ki}, Y_{li})\sigma(\dot{Y}_{ki})\sigma(Y_{li}) \\
\lambda_{3,i,k,l} &= \rho(\ddot{Y}_{ki}, \dot{Y}_{li})\sigma(\ddot{Y}_{ki})\sigma(\dot{Y}_{li})
\end{aligned}
\qquad 3.11
$$

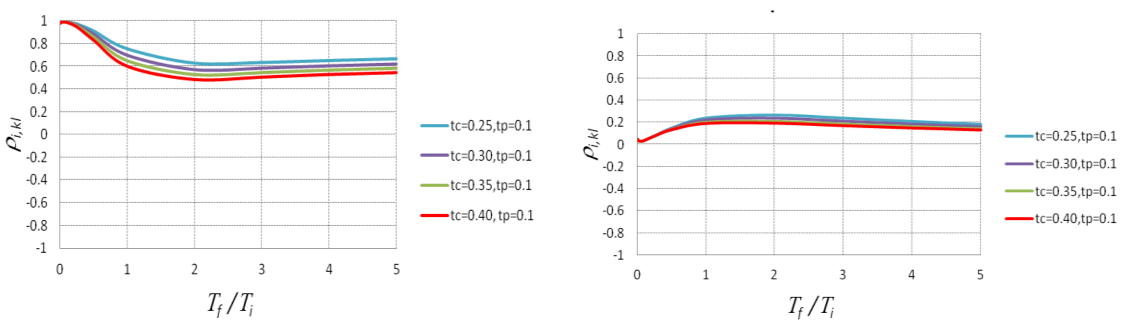
where $\sigma(Y_{ki}), \sigma(\dot{Y}_{ki}), \sigma(\ddot{Y}_{ki})$ are the standard deviations of the displacement, velocity and acceleration response of the SDOF modal oscillators to ground acceleration $\ddot{u}_k(t)$,

$$
\begin{aligned}
\sigma^2(Y_{ki}) &= \int_{-\infty}^{\infty} |H_i(\omega)|^2 S_{\ddot{u}_k\ddot{u}_k}(\omega)d\omega \\
\sigma^2(\dot{Y}_{ki}) &= \int_{-\infty}^{\infty} \omega^2 |H_i(\omega)|^2 S_{\ddot{u}_k\ddot{u}_k}(\omega)d\omega \\
\sigma^2(\ddot{Y}_{ki}) &= \int_{-\infty}^{\infty} \omega^4 |H_i(\omega)|^2 S_{\ddot{u}_k\ddot{u}_k}(\omega)d\omega
\end{aligned}
\qquad 3.12
$$

Using (25) in (15) we can write the response variance as,

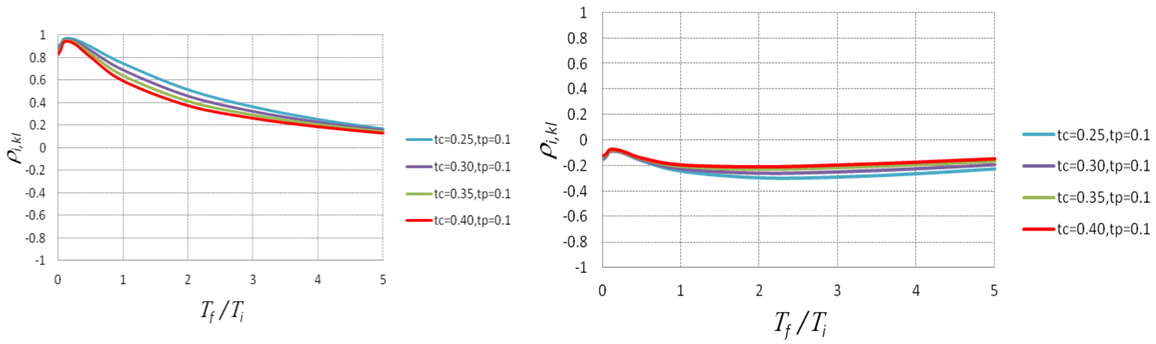
$$
\begin{aligned}
\sigma^2{}_{Z_d} = \sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n} \alpha_{ikl}\rho(Y_{ki}, Y_{li})\sigma(Y_{ki})\sigma(Y_{li}) + \frac{\beta_{ikl}}{\omega_i^2}\rho(\dot{Y}_{ki}, \dot{Y}_{li})\sigma(\dot{Y}_{ki})\sigma(\dot{Y}_{li}) + \\
\frac{\alpha'_{ikl}}{\omega_i}\rho(\dot{Y}_{ki}, Y_{li})\sigma(\dot{Y}_{ki})\sigma(Y_{li}) - \frac{\beta'_{ikl}}{\omega_i^3}\rho(\ddot{Y}_{ki}, \dot{Y}_{li})\sigma(\ddot{Y}_{ki})\sigma(\dot{Y}_{li})
\end{aligned}
\qquad 3.13
$$

The value of ρ can be estimated using ground cross correlation model in Eqs. 2.8, 2.9 and 2.10. Based on Mexican records *K1* can vary between 5.5 and 7.5 x10^{-4} and for distances around 500 m the value of the incoherence parameter $t_c = K_1 d_{k,l}$ varies from 0.25 to 0.40 . Wave passage effect parameter $t_p = \frac{d_{k,l}{}^L}{V_{app}}$ is around 0.1 and 0.2. If we suppose that the type of soil in the three supports is the same, site effects in Eq 13 have no influence in the estimation of correlation. Figures 3.1 and 3.2present the variation of the correlation ρ versus the ground period $T_f=2\pi/\omega_f$ normalized by the modal period $T_i=2\pi/\omega_i$ for soft soil.



(a) (b)

Figure 3.1 : Cross correlation between modal displacements and velocity (a) $\rho(Y_{ki}, Y_{li})$ and (b) $\rho(\dot{Y}_{ki}, Y_{li})$



(a) (b)

Figure 3.2: Cross correlation between modal velocity and acceleration (a) $\rho(\dot{Y}_{ki},\dot{Y}_{li})$ and (b) $\rho(\ddot{Y}_{ki},\dot{Y}_{li})$

For deriving a response formulation, it is convenient to express the response variance in terms of modal displacement variances,

$$\begin{aligned}\sigma^2{}_{Z_d} = \sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n}[\alpha_{ikl}\rho(Y_{ki},Y_{li}) + \frac{\beta_{ikl}}{\omega_i^2}\rho(\dot{Y}_{ki},\dot{Y}_{li})\frac{\sigma(\dot{Y}_{ki})\sigma(\dot{Y}_{li})}{\sigma(Y_{ki})\sigma(Y_{li})} + \\ \frac{\alpha'_{ikl}}{\omega_i}\rho(\dot{Y}_{ki},Y_{li})\frac{\sigma(\dot{Y}_{ki})\sigma(Y_{li})}{\sigma(Y_{ki})\sigma(Y_{li})} - \frac{\beta'_{ikl}}{\omega_i^3}\rho(\ddot{Y}_{ki},\dot{Y}_{li})\frac{\sigma(\ddot{Y}_{ki})\sigma(\dot{Y}_{li})}{\sigma(Y_{ki})\sigma(Y_{li})}]\sigma(Y_{ki})\sigma(Y_{li})\end{aligned} \quad 3.14$$

Consider now the characteristic frequencies of the modal response displacements,

$$\Omega_{2,k,i} = \frac{\sigma(\dot{Y}_{ki})}{\sigma(Y_{ki})} \quad , \quad \Omega_{4,k,i} = \sqrt{\frac{\sigma(\ddot{Y}_{ki})}{\sigma(\dot{Y}_{ki})}} \quad 3.15$$

The characteristic frequencies depend on modal frequencies and damping ratios, and on the ground motion input. If the ground motion is assumed to be Gaussian, they are associated with the mean zero-crossing rates of the modal oscillator displacement and velocity responses. In terms of the characteristic frequencies the response variance becomes

$$\begin{aligned}\sigma^2{}_{Z_d} = \sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n}[\alpha_{ikl}\rho(Y_{ki},Y_{li}) + \frac{\beta_{ikl}}{\omega_i^2}\rho(\dot{Y}_{ki},\dot{Y}_{li})\Omega_{2,k,i}\Omega_{2,l,i} + \\ \frac{\alpha'_{ikl}}{\omega_i}\rho(\dot{Y}_{ki},Y_{li})\Omega_{2,k,i} - \frac{\beta'_{ikl}}{\omega_i^3}\rho(\ddot{Y}_{ki},\dot{Y}_{li})\Omega_{4,k,i}^2\Omega_{2,l,i}]\sigma(Y_{ki})\sigma(Y_{li})\end{aligned} \quad 3.16$$

Noting that for $k = l$, $\rho(\dot{Y}_{ki},Y_{li}) = \rho(\ddot{Y}_{ki},\dot{Y}_{li}) = 0$, the response variance can be written as

$$\sigma^2_{z_d} = \sum_{k=1}^{m}\sum_{i=1}^{n}\Gamma_{k,i}\sigma^2_{y_{k,i}} + \sum_{k=1}^{m}\sum_{\substack{l=1\\k\neq l}}^{m}\sum_{i=1}^{n}\Theta_{k,l,i}\sigma_{y_{k,i}}\sigma_{y_{l,i}} \quad 3.17$$

where

$$\Gamma_{ki} = \alpha_{ikk} + \frac{\beta_{ikl}}{\omega_i^2}(\Omega_{2,k,i})^2$$

$$\Theta_{kli} = \alpha_{ikl}\rho(Y_{ki}, Y_{li}) + \frac{\beta_{ikl}}{\omega_i^2}\rho(\dot{Y}_{ki}, \dot{Y}_{li})\Omega_{2,k,i}\Omega_{2,l,i} + \frac{\alpha'_{ikl}}{\omega_i}\rho(\dot{Y}_{ki}, Y_{li})\Omega_{2,k,i} - \frac{\beta'_{ikl}}{\omega_i^3}\rho(\ddot{Y}_{ki}, \dot{Y}_{li})\Omega^2_{4,k,i}\Omega_{2,l,i} \quad 3.18$$

4. CONCLUSIONS

It has been shown that the dynamic structural response of multi-support systems can be computed based on uncoupled response analysis of modal oscillators. The dynamic response has been shown to depend on: 1) cross correlation coefficients between modal displacements, velocities and accelerations; 2) the first two characteristic frequencies of the modal response displacements; and 3) response coefficients that depend solely on modal frequencies and damping. The spatial correlation structure of the support ground motions is considered in the solution through the cross correlation coefficients. The correlation between modal responses for given support motions is rigorously accounted by the response coefficients derived from the partial fractions expansion. No simplifying assumption regarding a particular type of ground excitation has been used in deriving the solution in 3.17. It has also been shown that the response can be expressed in an exact way in terms of the contribution from all modal responses to each of the support ground motions individually, in addition to the contribution from the cross correlations between support ground motions and n between modal responses.

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