Analysis of Experimental Dynamic Soil-Pile Interaction by Approximate Numerical Solutions

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SUMMARY:
Experimental results from full-scale multi-modal vibration tests of piles in a layered soil profile are analyzed using approximate computational solutions from the literature. The approximate numerical solutions assume plane strain conditions and treat the soil as a continuum while representing the pile by a finite element formulation. The full-scale vibration tests were performed on two steel HP 10x42 piles installed to a depth of 20 ft at a soft-clay site, using random vibration methods and a new servo-hydraulic inertial shaker testing system. Experimental results are presented and a new vertical eccentric testing format is employed. Experiment and theory are compared in the form of transfer functions calculated on the basis of beam theory, rigid-body kinematics, and the impedance functions of the approximate solutions. A parametric study is conducted to determine the sensitivity of the transfer functions to soil modulus and damping.

Keywords: soil-pile interaction, dynamic soil impedances, field test, random vibration, approximate solution

1. INTRODUCTION

Deep foundations are commonly used to support important structures in problematic soils. The load bearing capacity and dynamic behavior of pile foundations is critically important to the design of structures subjected to dynamic loads such as earthquakes, wind, machine vibration, and impacts. Many analytical and numerical approaches to dynamic pile-soil interaction with different levels of complexity have been described in the literature. Due to the various assumptions made in their formulation, validation of such theories by full-scale tests with realistic soil and loading conditions is essential.

Due to the highly nonhomogeneous nature of soil deposits along with the complexities of 3D wave propagation and resulting geometrical damping, solutions and theories for dynamic soil-pile interaction are usually uncertain and quite complex. The influence of soil-pile separation and pile installation effects can also complicate the pile response. To validate and calibrate theoretical solutions which may contain many unknowns and simplifying assumptions, numerous experimental laboratory or centrifuge-scale studies and relatively fewer field-scale soil-pile interaction studies have been performed to date. Although laboratory and centrifuge-scale tests may offer good control of soil properties and efficient parametric variation, they cannot fully replicate the full-scale stress-dependent behavior of soil and resulting soil-structure interaction phenomena. On the other hand, the high cost and logistical difficulties of full-scale dynamic pile testing has resulted in a limited number of such experiments. Among the database of full-scale pile vibration tests are the studies of Blaney and O'Neill (1986), Boominathan and Ayothiraman (2006) and Manna and Baidya (2009). Ideally, such testing programs and accompanying theories would produce comprehensive yet practical simplified solutions for use in engineering practice. To address this need, approximate solutions such as those given by Gazetas (1984) and Novak and El Sharnouby (1983) have provided useful predictions with negligible computing costs and relative simplicity.

This paper presents some key results from full-scale pile vibration tests using random vibration techniques and a new servo-hydraulic inertial shaker testing system. Separate tests of vertical and coupled horizontal-rocking modes as well as hybrid multi-mode tests were performed. Three excitation techniques were examined using a range of forcing intensities to determine the optimal testing configuration. A multi-modal vertical-eccentric test format was investigated as an efficient alternative to the traditional approach of separate tests with vertical and horizontal forcing. The design of the experimental testing system is briefly described below, followed by a comparison of results from the various test configurations. The approximate solution proposed by Novak and Aboul Ella (1978) is employed based on site characterization data and compared to observed pile behaviour. A parametric study is then performed to examine the sensitivity of the solution to variations in soil parameters.

2. FULL-SCALE FIELD TESTS

Full-scale elastodynamic vibration tests were performed on two steel HP 10x42 piles installed to an approximate depth of 20 ft in a soil profile featuring soft clay in Miami, Oklahoma in September 2010. One pile was tested in the natural unimproved soil profile while the other was surrounded by an improved cement deep-soil mixed (CDSM) zone over the upper 13 ft. A newly developed servo-hydraulic inertial shaker system and National Instruments (NI) dynamic signal analyzer were used to deliver various forms of broadband excitation to a pile cap mounted on the test piles. Separate tests of vertical and coupled horizontal-rocking modes (referred to as VC and HC tests, respectively) were performed, as well as multi-modal tests of general planar motion via vertical eccentric excitation (VE tests). These three test configurations are depicted in Figure 2. To determine the optimal testing configuration, three different excitation waveforms were examined for a range of forcing intensities (Ashlock & Fotouhi 2011). Data and detailed documentation for the more than 100 tests are available for public download on the NEEShub at http://nees.org/warehouse/project/940. Test data may also be plotted online using the inDEED data exploration tool. To plot the frequency domain data using inDEED, the template builder command should be used to create a new template with frequency as the x-axis.

2.1. Soil and Pile Properties

The test site consisted of 3.6 ft of lean clay with gravel and occasional construction debris, followed by 9.2 ft of a silt-clay mixture underlain by 8.2 ft of sandy gravel, with limestone bedrock starting at 21 ft. In-situ testing and sampling performed at the site included SPT tests, CPT and DMT soundings, and piston samples. Laboratory tests were carried out for grain-size analysis, Atterberg limits, and unconfined compressive strength. Each of the HP 10x42 piles tested in this study has a total length of approximately 25 feet and was installed using a vibratory hammer to embedment depths of 245 inches for pile I in improved soil and 242 inches for pile U in the unimproved natural soil. The above-ground unembedded lengths are 55.75 inches for both pile I and pile U. The cylindrical CDSM improved zone around pile I has a diameter of 48 inches and a depth of 13 ft. The water table depth was measured at 6.6 ft in a standpipe near the time of testing. Figure 1 shows the soil profile and test set up for the horizontal-centric (HC) tests.

2.2. Design of Excitation and Instrumentation System

A two-piece modular pile cap was constructed to test the piles under dynamic loading. The pile cap is 3x3x3 ft in size and has a recess on each half forming a cavity to permit the two pieces to fit around the H-pile and be bolted together with six ¾ inch threaded rods. As mentioned above, this study is focused on random vibration techniques applied to dynamic pile-soil interaction problems. Random vibration techniques typically involve excitation of a physical or electrical system over a wide range of frequencies, using complementary measurement techniques to efficiently characterize the system response in terms of Fast-Fourier Transforms (FFTs), correlation functions, spectral densities, transfer functions, coherence functions, and impulse response functions. Compatible excitation types can deliver energy over the measurement bandwidth of interest; random (white noise or pink noise), impulse or swept-sine signals are typically used. In this study, a servo-hydraulic inertial shaker system was developed which is portable and can be used at a variety of sites in the future without the need for reaction frames or a large generator to run an electric hydraulic pump. The dynamic force delivered by the shaker is proportional to the product of the moving mass and its acceleration. The system is capable of delivering user-specified broadband dynamic forcing of up to 2,000 lbf over a bandwidth of 1-200 Hz (Ashlock & Fotouhi 2011). A complete description of the experiments and shaker system can be accessed from the NEEShub project page at http://nees.org/warehouse/project/940.

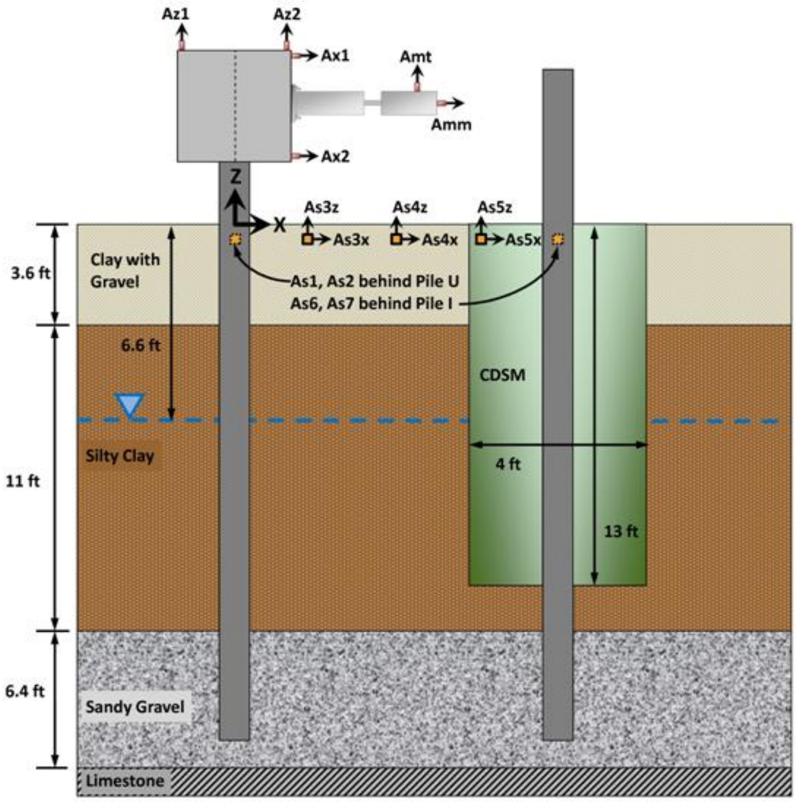

Figure 1. Soil profile and orientation and notation for accelerometers on shaker and pile cap (shaker shown in HC test position on pile U).

2.3. Test Naming Convention and Response Functions

To refer to the various tests, a naming convention of (Pile Type)-(Test Type)-(Excitation Type and Level) will be used through the paper. For example, U-VE-S3 refers to a test performed on pile U in unimproved soil with the shaker in the VE configuration with swept-sine (S) excitation at the highest intensity (level 3). In the naming convention, test types VC and HC can replace VE, and excitation types R (random) and C (chaotically timed impulse) can be used in place of S. Tests were performed at low, medium and high excitation levels, referred to as levels 1, 2 and 3, respectively.

During testing, time-domain accelerations of the shaker's moving mass, pile cap, and soil surface (see Figure 1) were recorded by the NI analyzer and processed in the form of power spectral densities (PSD), coherence functions (COH), and transfer functions (XFER). For transfer and coherence functions, the acceleration of the shaker's moving mass (Amm) was taken as the stimulus, with all other pile-cap and soil directional accelerations treated as responses. For example, the VC/VE transfer function is defined as the vertical centric (VC) acceleration of the pile cap normalized by the vertical eccentric (VE) acceleration Amm of the shaker moving mass in the VE test configuration. The acceleration transfer functions can be divided by the shaker's moving mass of 499 lb/32.2 ft/s^2 = 15.5 lb-s^2/ft, to give acceleration-to-force transfer functions commonly referred to as accelerance. The coherence function is an indicator of measurement quality which takes a value of unity for a perfectly linear, time-invariant system without added noise (Bendat and Piersol, 1986).

2.4. Test Results and Interpretation

A total of 109 full-scale vibration tests were performed on the piles using the three excitation types and different loading levels. Figures 3 through 6 show typical experimental results. The averaged horizontal power spectral densities from accelerometers Ax1 and Ax2 are shown in Figure 3 for all three excitation types for pile U in unimproved soil. As can be seen in the figure, the random and swept-sine excitations generally provide the most uniform power spectral densities for this lateral-rocking mode of vibration, while chaotic impulse delivers the least uniform PSD. As shown in the figures, the VE tests successfully generated significant horizontal rocking motion.

Acceleration transfer functions and related coherence functions corresponding to the vertical mode of vibration are shown in Figure 4 for three forcing levels. These VC/VE transfer functions are ratios of VC acceleration of the center of the pile cap to VE acceleration of the shaker moving mass. As demonstrated in these figures, good coherence was obtained over a wide frequency range, and a small extra bump can be seen prior to the major vertical resonance peak. It can be shown that this phenomenon is theoretically related to the lack of perfect symmetry of the pile cap. Similar behavior was observed in the centrifuge pile vibration tests reported in (Ashlock 2006 and Ashlock and Pak 2009). As can be seen in the transfer functions of Figure 4, the three different levels of forcing resulted in small nonlinear effects, as evidenced by slight changes in the shapes and frequencies of the peaks.

Figure 5 contains the vertical and horizontal responses obtained from separate VC and HC tests compared to those from a single VE test. As shown in Figure 5a, the multi-modal VE test is successful in capturing the vertical mode of vibration traditionally obtained from a single-mode VC test. Again, there is a small extra bump that may be explained by the slight lack of inertial symmetry of the pile cap owing to the offset shaker position in the VE tests. As shown in Figure 5b, the HC test produces a strong fundamental peak around 8 Hz, while the VE test is also effective in determining this resonant frequency by activating the coupled lateral-rocking mode of vibration. It should be noted that the magnitudes of the HC/VE and HC/HC responses differ, which can be shown theoretically to result from the different locations and directions of the excitation force in these two tests. The resonant frequencies, however, should closely agree. Both HC and VE tests exhibit additional peaks having frequencies and amplitudes dependent upon the components of the lateral-rocking impedance matrix. The parameters of the computational soil-pile model can be varied to minimize the discrepancy between such measured and theoretical transfer functions, to determine the optimum soil-pile interaction model via a system identification approach.

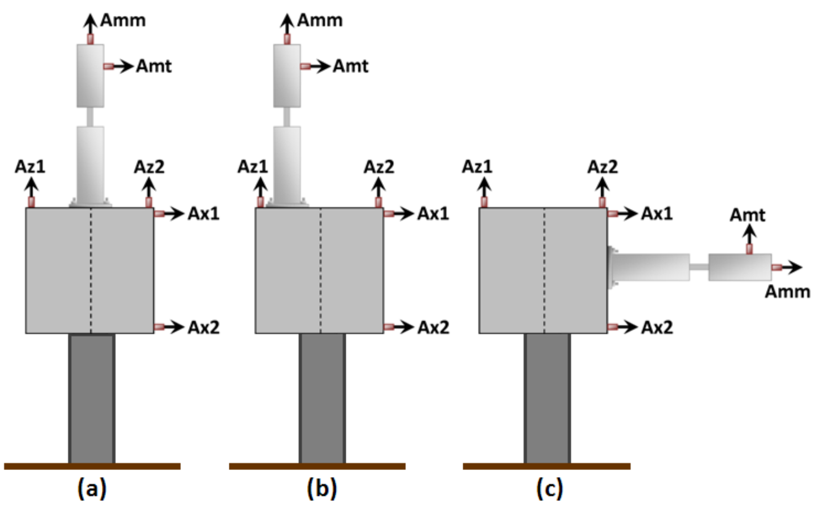

Figure 2. Inertial shaker configurations for the three test types; (a) VC test, (b) VE test, (c) HC test.

3. THEORETICAL APPROACH

In this phase of the study, the dynamic response is analyzed by taking into account the dynamic pile-soil interaction and wave propagation in the soil, as well as the dynamics of the shaker, pile-cap and elastic unembedded pile segment. This will help to better understand the complex behaviour of the soil-pile system and generate a framework for predicting the response of similar systems. The method of sub-structuring is used to formulate the theoretical transfer functions for the pile-cap-shaker system. The structure above the soil surface is decomposed into a rigid body pile-cap and a deformable beam-column pile stem (Figure 6), with compatibility conditions enforced at their interface.

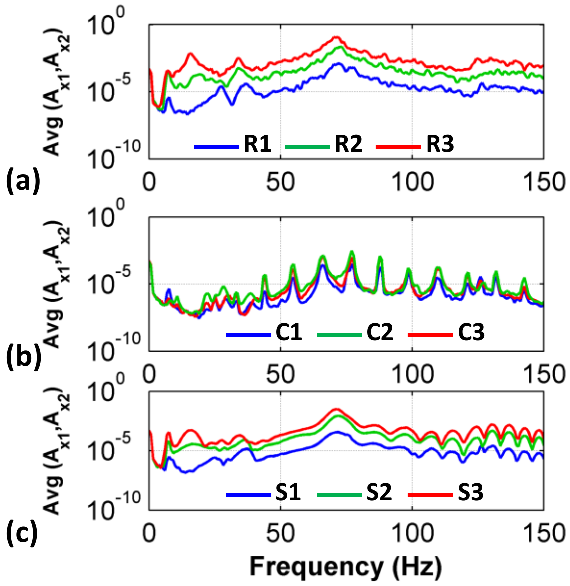

Figure 3. Averaged horizontal PSD ($(m/s^2rms)^2/Hz$) for VE excitation of pile U in unimproved soil. (a) Random, (b) Chaotic impulse, (c) Swept-sine loading at intensity levels 1, 2, and 3.

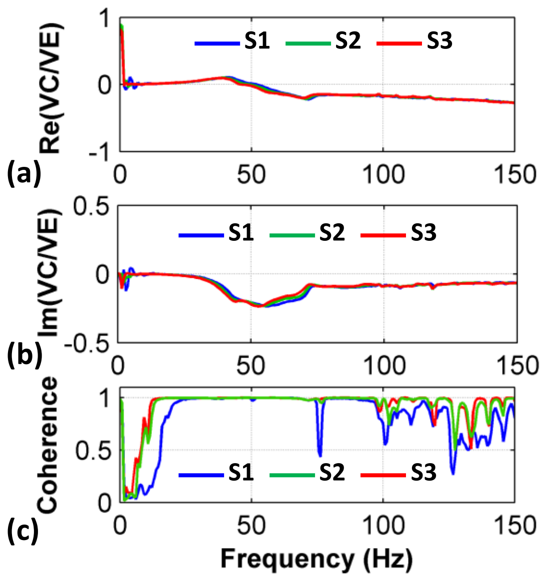

Figure 4. Vertical transfer functions and coherence for VE swept-sine excitation of pile U in unimproved soil at excitation levels 1, 2, and 3. (a) Real, (b) imaginary components of transfer function, (c) coherence.

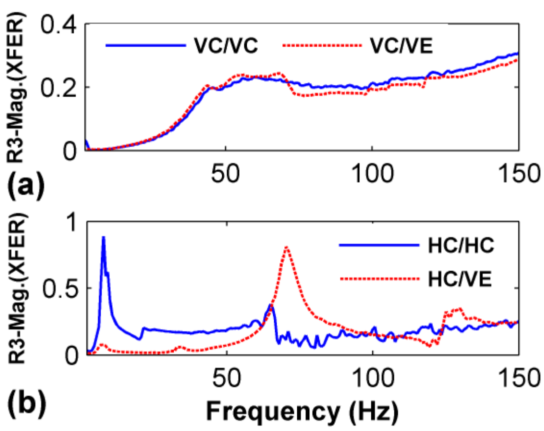

Figure 5. Comparison of VE test to combination of VC and HC tests for random excitation in unimproved soil. (a) VC response from VC and VE tests, (b) HC response from HC and VE tests.

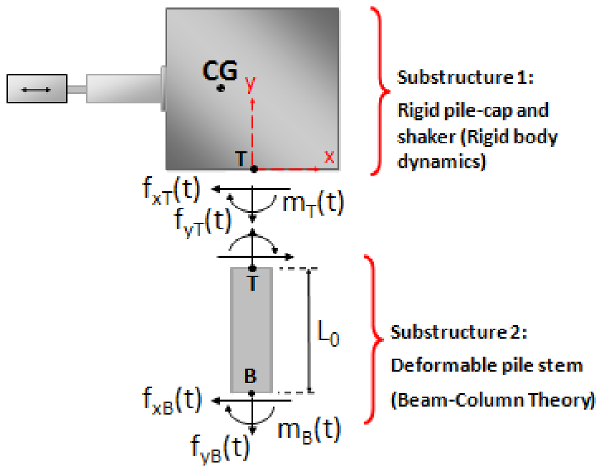

Figure 6. Substructures and interfacial forces used in transfer function formulation.

To ascertain the performance of some currently available methods of analysis, the dynamic interaction between the pile and soil layers is implemented in a numerical model based on the matrix stiffness method for piles in layered media (Novak and Aboul Ella, 1978). The formulation incorporates geometric and material damping and allows for variation of soil properties with depth. The output of the analysis is the soil-level impedance matrix at the base of the unembedded pile segment. The model was programmed in MATLAB and requires negligible computing cost, making it a useful tool for parametric studies.

3.1. Theoretical Transfer Function Formulation:

The frequency-domain rigid-body equations of motion may be written for the moving mass of the shaker and the pile cap. The the forcing components at the top of the unembedded pile segment may then be related to the pile-cap and shaker properties to give matrix equation 3.1.1. In this equation, $\mathbf{T_j}$ are rigid-body motion matrices for transforming displacement vectors to different points on the pile cap. They contain the pile-cap centroid coordinates, the dimensions of the shaker, and its offset distance from the centroid. $\mathbf{M_{mm}}$ and $\mathbf{M_R}$ represent the mass matrix of the moving (inertial) mass of the shaker, and the mass of the rigid pile cap plus stationary portion of the shaker, respectively. $\mathbf{U}$ and $\mathbf{F}$ represent the displacement and force vectors as detailed below, and ω denotes angular frequency.

$$\mathbf{F}_T = \omega^2 \left(\mathbf{T}_1 \mathbf{M}_R \mathbf{U}_R + \mathbf{T}_2 \mathbf{M}_{mm} \mathbf{U}_{mm} \right) \tag{3.1.1}$$

For the pile segment subjected only to end loadings, one may use an Euler-Bernoulli beam-column formulation and solve the differential equations of motion in the frequency domain. The force and displacement vectors at the top and bottom of the deformable pile-stem (points T and B in Figure 6) may then be related through

$$\begin{Bmatrix} \mathbf{U}_T \\ \mathbf{F}_T \end{Bmatrix} = \mathbf{S} \begin{Bmatrix} \mathbf{U}_B \\ \mathbf{F}_B \end{Bmatrix}, \tag{3.1.2}$$

where $\mathbf{S}$ is a 6x6 matrix describing the effects of elastic pile properties and dimensions (see Ashlock, 2006). Introducing the soil-pile impedance matrix $\mathbf{K}$, one may relate $\mathbf{F_B}$ to $\mathbf{U_B}$ at the soil surface through

$$\underbrace{\begin{Bmatrix} F_{yB} \\ F_{xB} \\ M_B \end{Bmatrix}}_{\mathbf{F_B}} = \underbrace{\begin{bmatrix} k_{vv} & 0 & 0 \\ 0 & k_{hh} & k_{hm} \\ 0 & k_{mh} & k_{mm} \end{bmatrix}}_{\mathbf{K}} \underbrace{\begin{Bmatrix} U_{yB} \\ U_{xB} \\ \Theta_B \end{Bmatrix}}_{\mathbf{U_B}} \tag{3.1.3}$$

The $\mathbf{K}$ matrix components are calculated using the soil-pile interaction model described below.
As defined previously, the transfer function of a point on the pile cap is defined as the ratio of the acceleration in that point to the acceleration of the moving mass, i.e.

$$H_i = \frac{\ddot{U}_i}{\ddot{U}_{mm}} = \frac{-\omega^2 U_i}{-\omega^2 U_{mm}} = \frac{U_i}{U_{mm}} \tag{3.1.4}$$

Combining equations 3.1.1 to 3.1.4 and assuming that the rotation of the shaker's moving mass and the pile-cap are equal, one may form a system of 6 equations and 6 unknowns that can be solved for each frequency. The unknowns of this system are the transfer functions at the top and bottom of the pile segment, H_T and H_B, from which the transfer functions at the point of each accelerometer can be found from the rigid body motion of the pile-cap. These transfer functions are compared below to those obtained experimentally.

3.2. Dynamic Soil Impedances

The pile-soil interaction model introduced by Novak and Aboul Ella (1978) was used to calculate the soil impedances in equation 3.1.3. This model derives the soil reactions from elasticity theory and represents the pile as a set of finite elements. Hysteretic damping is used for the soil, and the formulation is for a circular pile. Soil and pile properties are assumed to be constant for each element. The soil model can take into consideration the various soil modulus profiles and layering at a negligible computing cost, making it a useful tool for parametric studies. The solution cannot, however, model the pile installation effects or soil-pile separation. The variation of the soil profile below the pile tip is also not included in the formulation. A MATLAB program was developed for this study to perform the following steps:

1- Find complex soil reactions for the plane strain case (Novak, Aboul-Ella & Nogami, 1978)
2- Find the reaction of the soil acting on the tip of pile (Veletsos & Verbic, 1973)
3- Construct the complex frequency parameters and functions (Novak & Abou-Ella, 1978)
4- Construct the element stiffness matrices (Novak & Abou-Ella, 1978)
5- Assemble the global stiffness matrices of pile (Novak & Abou-Ella, 1978)
6- Find the pile head impedances by solving the global matrix equations for unit displacements and rotations at the soil-surface (Novak & Abou-Ella, 1978)

To verify the accuracy of the program, the simple problem of a circular pile in a soil with a nonlinear square-root shear modulus profile was solved using the program as well as the rigorous 3D boundary-element program BEASSI (Pak & Guzina. 1999). The results match well, provided one uses the slight modification suggested by Novak and Aboul Ella for low frequencies, as shown in Figure 7.

3.2. Parametric Study

To examine the sensitivity of the calculated pile transfer functions to the soil properties, a parametric study was conducted in which the soil shear modulus profile, soil shear modulus magnitude, and soil damping ratios were varied. For the shear modulus profile, a model having a constant modulus equal to the average values for each of the four major soil layers described above is compared to the finer layered profile shown in Figure 8. The finer layered profile of Figure 8 was calculated from the CPT data using correlations to shear wave velocity presented in NCHRP Synthesis 368 (TRB, 2007). Twenty-eight segments for the pile and soil were input to the program and the resulting transfer functions are presented in Figure 9. Damping was set to 5% for all layers in this analysis. As shown in Figure 9, using a uniform modulus profile in each layer changes the transfer functions minimally, resulting in a slightly stiffer response in the horizontal mode of vibration and a softer response in the vertical mode. For comparison, the experimental transfer functions from tests U-HC-S2 and U-VC-S2 are presented in the figure. It can be seen that the theory can predict the experimental first HC/HC peak frequency reasonably well. However further analysis may be needed to explain the discrepancies at higher frequencies through more advanced modelling considering 3D wave propagation, pile installation effects and soil-pile separation. The experimental curves also exhibit a few spurious peaks which may be related to the dynamics of the shaker.

Since the correlations used between CPT resistance and shear-wave velocity are not exact, the input values for the soil modulus may include some error. To examine how this error could affect the system response, the shear modulus profile was varied by ±50%, and the results are compared in Figure 10. As expected, increasing the modulus makes the system stiffer and moves the peak response toward higher frequencies. This has a significant effect for the VC test and the second peak of the HC test, while the first peak of the HC test shifts only slightly. Finally, different material damping ratios for the soil layers were examined and their effect on the entire system response is presented in Figure 11. The trend is as expected; increasing the damping decreases the peak frequencies of the system as well as the peak amplitudes. However, this parameter is not very crucial, especially for the HC test, since the main source of damping in the model arises from geometric damping through wave propagation in the soil. Further modelling is currently being performed using the 3D boundary element program BEASSI to account for soil inhomogeneity and stress-state dependence, as well as pile installation effects.

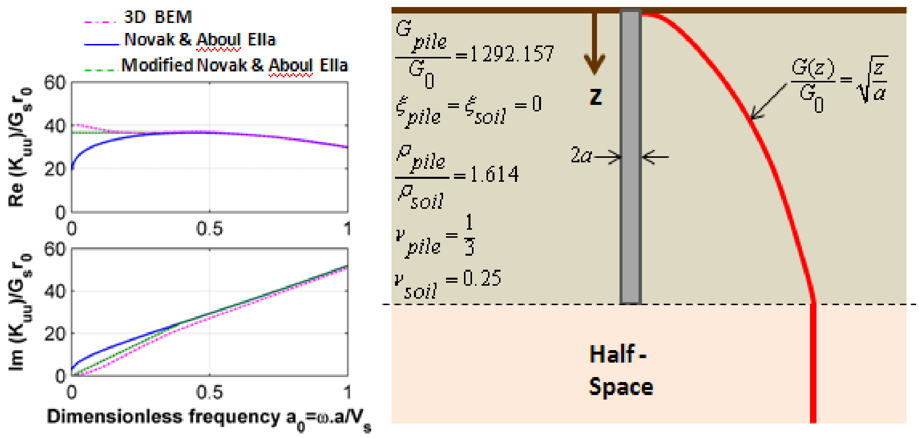

Figure 7. Verification of the MATLAB program written for the approximate solution by comparison against the rigorous 3D boundary element program BEASSI. Horizontal impedance Kuu is shown.

4. CONCLUSIONS

A full-scale dynamic soil-pile interaction problem has been investigated both experimentally and theoretically using random vibration techniques. The experimental setup is described and selected results are presented in the frequency domain . It is verified that a VE testing technique can be used to simultaneously measure both vertical and horizontal vibration modes. The recorded data from the experiment is used to verify and calibrate the analytical and computational solutions. A theoretical model for the soil-pile-pile cap system with inertial shaker is introduced using rigid body dynamics, beam theory for the unembedded region of the pile, and an approximate continuum-finite element solution for the soil-pile interaction. This model is then used for a parametric study of soil modulus and damping. It is shown that using a uniform modulus within each soil layer can be a good approximation for the actual modulus profile. Finally, it is shown that increasing the soil damping tends to soften the system response, with a more pronounced effect in the vertical mode of vibration.

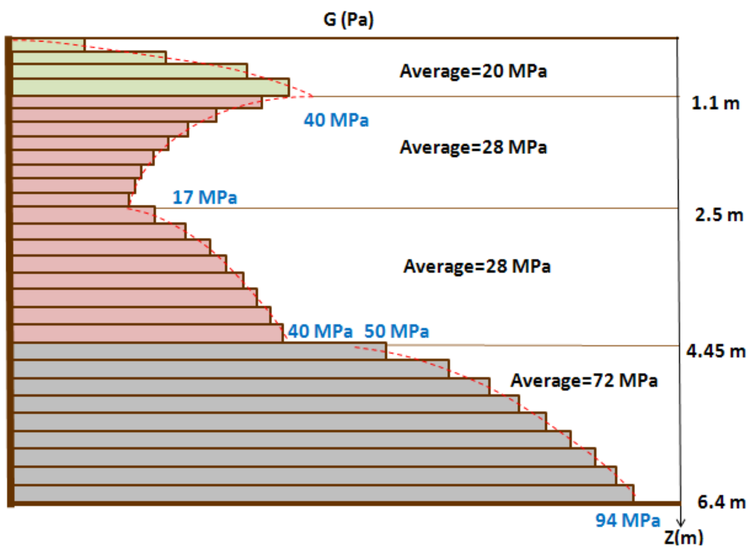

Figure 8. Shear modulus profile from CPT test data

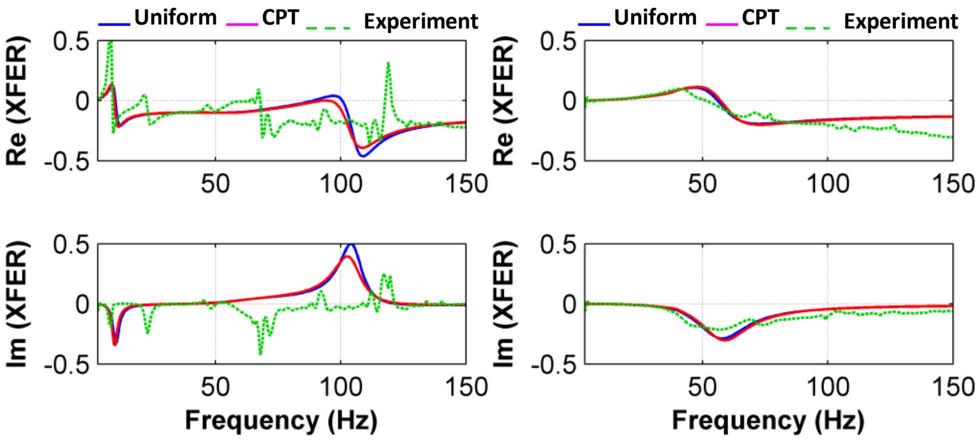

Figure 9. Effect of soil modulus variation on the transfer function of the system and comparison with U-HC-S2 and U-VC-S2. (a) Horizontal transfer function at A_{x2} in HC test, (b) Vertical transfer function at A_{z2} in VC test.

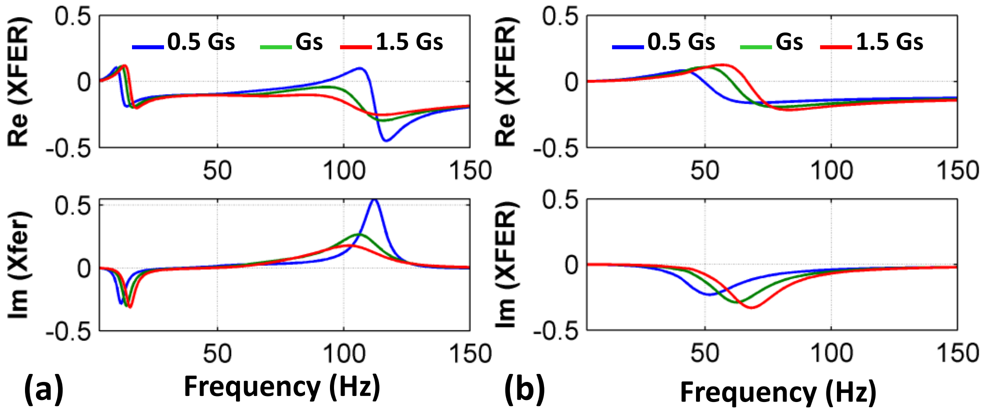

Figure 10. Effect of soil modulus value on the transfer function of the system. (a) Horizontal transfer function at A_{x2} in HC test, (b) Vertical transfer function at A_{z2} in VC test.

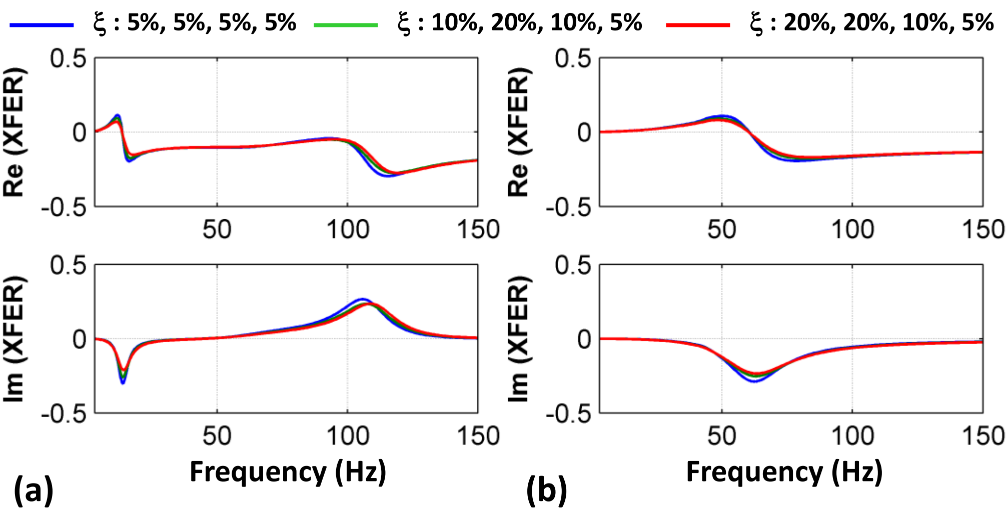

Figure 11. Effect of soil damping on the transfer function of the system. Damping ratios shown in the legend correspond to the four soil layers, from top to bottom. (a) Horizontal transfer function at A_{x2} in HC test, (b) Vertical transfer function at A_{z2} in VC test.

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REFERENCES

Bendat, J. S. and Piersol, A. G. (1986), *Random Data*, Wiley-Interscience Publications, New York.

Blaney, G.W. and O'Neill, M.W. (1986). Analysis of Dynamic Laterally Loaded Pile in Clay. *Journal of Geotechnical & Geoenvironmental Engineering*. **112:9**, 827-840.

Boominathan, A. and Ayothiraman, R. (2007). Dynamic response of laterally loaded piles in clay. *Proc. Institution of Civil Engineers, Geotechnical Engineering*. 159**:GE3**, 233-241.

Mannal, B. and Baidya, D. K. (2009). Vertical Vibration of Full-Scale Pile—Analytical and Experimental Study. *Journal of Geotechnical & Geoenvironmental Engineering*. **135:10**, 1452-1461.

Gazetas, G. (1984). Seismic response of end-bearing single piles. *International Journal of Soil Dynamics and Earthquake Engineering*. **3:2**, 82–93.

Novak, M. and El Sharnouby, B. (1983). Stiffness Constants of Single Piles. *Journal of Geotechnical & Geoenvironmental Engineering*. **109:7**, 961-974.

Ashlock, J.C. and Fotouhi, M.K. (2011). Characterization of Dynamic Soil-Pile Interaction by Random Vibration Methods: Experimental Design and Preliminary Results. *2011 NSF Engineering Research and Innovation Conference, Atlanta, Georgia (CMMI Grantee Conference)*. 11 pages.

Ashlock, J.C. (2006). Computational and Experimental Modeling of Dynamic Foundation Interactions With Sand. *Ph.D. Thesis, University of Colorado at Boulder*.

Ashlock, J.C. and Pak, R.Y.S. (2009), Experimental Response of Piles in Sand under Compound Motion, *Journal of Geotechnical and Geoenvironmental Engineering*, 135 (6), 799-808, ASCE.

Novak, M. and Aboul-Ella, F. (1978). Impedance Functions of Piles in Layered Media. *Journal of the Engineering Mechanics Division*. **104:6**, 643-661.

Novak, M., Aboul-Ella, F. and Nogami, T. (1978). Dynamic Soil Reactions for Plane Strain Case. Journal of the Engineering Mechanics Division. 104: 4, 953-959.

Veletsos, A.S. and Verbič ,B. (1973). Vibration of viscoelastic foundations. *Earthquake Engineering and Structural Dynamics*. **2:1**, 87-102.

Pak, R. Y. S. and Guzina, B. B. (1999), Seismic soil-structure interaction analysis by direct boundary element methods. *International Journal of Solids and Structures*. **36:31-32**, 4743–4766.

Transportation Research Board (TRB) (2007). National Cooperative Highway Research Program (NCHRP) Synthesis 368, Cone Penetrating Testing.