Seismic Damage Detection via Finite Element Model Updating

Saurabh Shiradhonkar
Indian Institute of Techonlogy Bombay, Mumbai, India

Manish Shrikhande
Indian Institute of Techonlogy Roorkee, Roorkee, India

15 WCEE
LISBOA 2012

SUMMARY:
The study is aimed at detecting and locating the damage in the beams with the aid of vibration based system identification and finite element model updating method, using limited number of responses recorded during strong motion earthquakes. The response of the structure at unmonitored degrees of freedom is estimated by interpolating recorded responses in time domain. Modal parameters are identified using frequency domain decomposition and empirical transfer function estimates. It is found that a combination of a suite of system identification techniques with sensitivity based finite element model updating can potentially locate and quantify the damage in a moment resistant frame.

Keywords: Structural health monitoring, modal identification, model updating

1. INTRODUCTION

Structural health monitoring using vibration based system identification methods has gain immense importance in last couple of decades to examine integrity of aging infra-structure and to assure safety of structure during future seismic events. Structural health monitoring is based on tracking changes in the modal parameters identified from the acquired vibration signatures of the structural system. System identification procedures identify modal response parameters such as natural frequencies, modal damping and mode shapes of the structure from recorded vibration signatures. However the inverse problem of system identification is inadequately constraint as the structural response is measured at a few carefully chosen locations (Datta et al, 2002). Further the recorded signatures do not represent the true structural response but with contamination from noise of transducers and recording system.

Humar (2006) and Brownjohn (2007) summarized several approaches explored to solve the inverse problem of system identification. Adams and co-workers (1978, 1979) used ratios of the reduction in measured natural frequencies of two modes to locate the damage by correlating it with the results of a finite element model of plates with holes. However for large-scale civil engineering structures, it is very difficult to locate damage only from changes in modal frequencies as the damage at different locations may produce comparable changes in the natural frequencies. To locate the damage in simple beam type structures methods such as curvature mode (Pandey et al, 1991), flexibility matrix methods (Pandey and Biswas, 1994) etc. were proposed. Modal parameters identified from system identification methods can also be used in finite element (FE) model updating methods, which provide probable location of damage by adjusting critical parameters of analytical model, so that predictions obtained from updated analytical model are consistent with those obtained from measurements. Mottershead and Friswell (1994) summarised vibration response based FE model updating methods. One class of FE updating methods identify stiffness matrix of structural system by minimising the difference between analytical and experimental responses. Though identified stiffness matrix provides good agreement with measured quantities, the correlation with the physical location of the damage was not satisfactory. Other class FE updating methods is known as sensitivity based parameter estimate methods. These methods estimate changes in model parameters depending on sensitivity of the model parameters against the modal characteristics of the structures.

Though conceptually sound, the application of these procedures to large-scale civil engineering structure is challenging due to difficulties in prototype testing and experimental data analysis. The problem of prototype testing is partly alleviated for some structures like bridges, for which ambient vibrations caused by wind, vehicular traffic are useful for deriving modal characteristics. For building structure ambient vibrations do not yield essential modal characteristics. The only source which guarantees modal characteristics of building frame is strong motion earthquake. The primary goal of earthquake resistant design of building frame is to avoid formation of collapse mechanism in the structures. Accordingly the frames are designed on the basis of strong column-weak beam concept, wherein the vibration energy is dissipated due to inelastic deformation by formation of plastic hinges in beams while the columns are not expected to incur any damage. As hinge formation in a beam is a localized damage, it is better captured by the higher modes. But a reliable estimate of higher modes is very difficult as the structural response is recorded only at limited number of floors thereby limiting the spatial resolution of the data available for modal estimation. Secondly, the structural response is usually dominated by the lower modes of vibration with very little contribution coming from the higher modes.

Hence the study aimed at improving the reliability of a sensitivity-based model updating method for the detecting and locating damage in a moment resistant framed building designed according to the strong column-weak beam philosophy, from few recorded responses during a strong motion earthquake.

2. PROBLEM DESCRIPTION AND DATA RECONSTRUCTION

To simulate damage scenario, a 7 story, 2-D RC fames has been modelled in SAP2000 (1995). The span of the bay considered as 5 m and the height of the each floor is taken as 3m. The beam and columns dimensions are respectively 0.3 m x 0.4 m and 0.3 x 0.55 m and these dimensions assures a strong column-weak beam system. Bi-linear moment-rotation relationship from FEMA-356 (2000) (shown in Fig. 2.1) is taken as non-linear constitutive models for the plastic hinges in beams. For monitoring seismic response, it is assumed that the accelerometers are installed at the mid-span location of the 1st, 3rd, 5th and 7th floors to record both the horizontal and vertical vibrations. Non-linear time history analysis of the frame has been carried out using Hilber-Hughes-Tatlor (HHT-α) method with parameters $\gamma = 0.7$, $\beta = 0.36$ and $\alpha = -0.2$; for San Fernando (February 09, 1971, 14:00 UTC, Pacoima Dam-CDMG station 279) (horizontal PGA 0.375 g, and vertical PGA 0.214 g) and for Loma Prieta (October 18, 1989, 04:15 UTC, Santa Cruz Mountains Station No. 58135) (horizontal PGA 0.338 g and vertical PGA 0.253 g) base motion. For the San Fernando plastic hinges are formed in the beams of 1st, 2nd, 3rd and 4th floors of the frames and for the Loma Prieta plastic hinges formed in the beams of 1st , 2nd, 3rd, 4th and 5th floors of the frames. The damage states of the building frames for the two earthquakes are as shown in Fig. 2.2. To simulate the effect of signal contamination due to the noise of transducers and recording system, a Gaussian white noise is added to each analytically computed absolute acceleration response time history so as to have a signal to noise ratio of 13 db.

The data at few locations causes low spatial resolution of the structural response and adversely affects the reliability of identified normal modes (Allemang, 2006). Hence, missing data has been reconstructed by time domain interpolation of each discrete time sample of the recorded responses to estimate the corresponding discrete time sample of responses at intermediate floors, using trend preserving piece-wise cubic Hermite interpolating polynomial (PCHIP). The relative error between the interpolated and analytically computed (true) acceleration response time history is computed as,

$$e = \frac{\sqrt{\sum_{i=1}^{N}(\ddot{u}_i - \tilde{\ddot{u}}_i)^2}}{\sqrt{\sum_{i=1}^{N}\ddot{u}_i^{\,2}}} \tag{2.1}$$

where, $\ddot{u}_i$ and $\tilde{\ddot{u}}_i$, $i = 1,2,\dots,N$ are discrete time samples of analytically computed and interpolated absolute acceleration time history, respectively. Interpolated and the true time histories of horizontal component for the strong motion part at 2nd floor, for San Fernando excitation, are compared in Fig. 2.3.

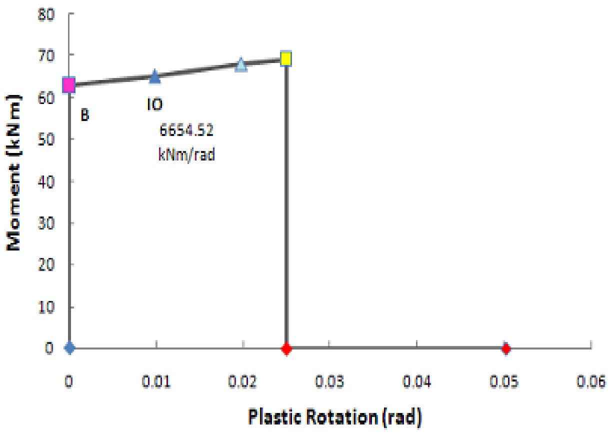

Figure 2.1. Moment-rotation curve defined for hinges in SAP-2000

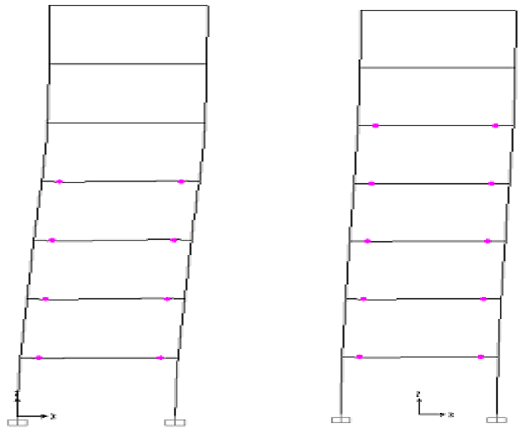

Figure 2.2. Damage pattern of example frame for San Fernando and Loma Prieta excitations

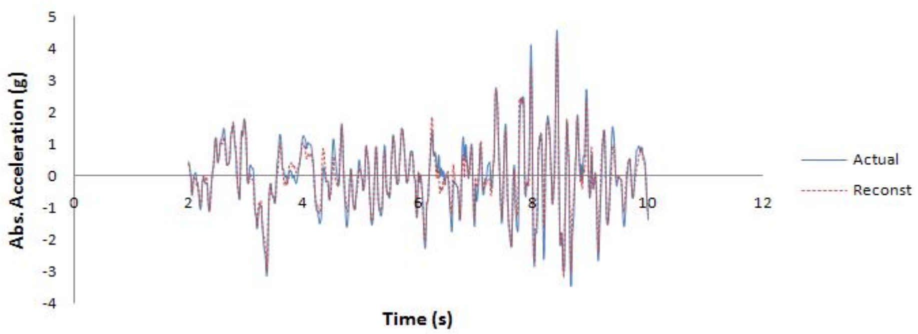

Figure 2.3. Comparison of the strong motion part of the true and time domain interpolated absolute horizontal acceleration time histories at 2nd floor, for San Fernando excitation

The relative error at various floor levels are summarised in Table 2.1. The relatively large differences in the case of San Fernando motion are due to large amplitude inelastic excursion on account of the presence of strong directivity pulse. However, these interpolated time histories even in presence of strong nonlinearities improve spatial resolution of the building response data, which is useful in modal parameter identification.

Table 2.1. Relative error between true and time domain interpolated time histories

Floor	% Error	
Level	San Fernando	Loma Prieta
2	26.46	15.52
4	27.46	10.89
6	11.16	10.53

3. MODAL IDENTIFICATION

Modal characteristics play vital role in structural health assessment. Time domain and frequency domain methodologies are available for modal identification. In this study frequency domain methods such as frequency domain decomposition (FDD) (Brincker, 2001) and empirical transfer function estimate (Ljung, 1987) are used for modal identification. To use modal characteristics for health monitoring a set of benchmark values of these modal parameters corresponding to the undamaged state are essential to draw any inference. In present study aleatory uncertainties associate with material and geometric uncertainties in analytical model are ignored and therefore calibration of analytical model from undamaged state modal characteristics is not required. Modal properties of analytical model corresponding to undamaged state are chosen as benchmark values. In field problem, initial building phase of the earthquake ground motion does not incur damage in structure and hence response data prior to time of initiation of damage (usually taken as the S-wave arrival time) can be used to determine undamaged state modal characteristics.

Rodgers and Çelebi (2006) have reported changes in the modal characteristics of a framed building identified from recorded vibration response to different earthquake events, which could not be attributed to any structural damage. Therefore, it is important to explore the possibility of tracking changes in the dynamic characteristics of the structural system from one event. This is done by using moving time window analyses of response data. Also the formation of plastic hinges in different locations does not take place simultaneously rather it is generally spread across the strong motion duration of the motion and accordingly dynamic properties of the structure experience changes over the earthquake duration. These changes in dynamic properties are identified from short time windows. A size of time windows are determined by examining the temporal evolution of frequency content in the structural response time history as depicted by the Spectrogram – the plot of squared amplitude of the short time Fourier transform (STFT) of the response time history. From the spectrogram of the 1st floor absolute acceleration response for San Fernando excitation, a time window of size 4 s is selected to capture the time varying characteristics. Hence, four time windows 0-4 s, 4-8 s, 8-12 s and 12-16 s are determined for modal characteristics. Similarly, for the Loma Prieta excitation three time windows 0-7 s, 7-14 s and 14-21 s are considered for identifying modal characteristics.

3.1. Frequency Domain Decomposition

Frequency domain decomposition (FDD) is an output only identification method provides natural frequencies and mode shapes from response data resulting data from a broad-band excitation. The modal identification procedure reduces to estimation of the response power spectral density (PSD) matrix $\widehat{\boldsymbol{G}}_{\boldsymbol{yy}}(j\omega)$ at discrete frequencies $\omega = \omega_i$ and then decomposing it by singular value decomposition (SVD).

$$\widehat{\boldsymbol{G}}_{\boldsymbol{yy}}(j\omega) = \boldsymbol{U}_i \boldsymbol{S}_i \boldsymbol{V}_i^H \tag{3.1}$$

Where, $\boldsymbol{U}_i$ and $\boldsymbol{V}_i$ are unitary matrices of left and right singular vectors respectively, and $\boldsymbol{S}_i$ is diagonal matrix of positive, real singular values. The peaks in the plot of singular values ($\boldsymbol{S}_i$) against frequencies (ω_i) indicate the presence of a normal mode with natural frequencies (ω_n) and the first left singular vector of matrix $\boldsymbol{U}_i$ corresponding to each peak is an estimate of the corresponding mode shape. The quality of mode shapes is assessed by *modal assurance criterion (MAC)*, using analytical mode shapes as a reference. First frequency and first mode shape identified from FDD are summarised in Table 3.1. The method provides good estimation of only first two natural frequencies and first mode, but the mode shapes associated with the second frequencies are of poor quality, for both excitation cases.

3.2. SVD in Time Domain

2nd and higher mode shapes are obtained by only considering the recorded absolute acceleration time histories. The recorded acceleration response data is arranged as,

$$A = \begin{bmatrix} a_1(t_1) & a_1(t_2) & \dots & a_1(t_N) \\ a_3(t_1) & a_3(t_2) & \dots & a_3(t_N) \\ a_5(t_1) & a_5(t_2) & \cdots & a_5(t_N) \\ a_7(t_1) & a_7(t_2) & \cdots & a_7(t_N) \end{bmatrix} = \begin{bmatrix} a_1^t \\ a_2^t \\ a_3^t \\ a_4^t \end{bmatrix} \tag{3.2}$$

where, N is the total number of discrete time samples of acceleration response data and $\mathbf{a}_j$, j = 1,3,5,7 denotes the absolute acceleration response time histories recorded at the jth floor. The method based on factorisation of recorded acceleration response data matrix $\boldsymbol{A}$ using SVD in time domain (SVD in TO).

$$\boldsymbol{A} = \boldsymbol{U} \sum \boldsymbol{V}^T \tag{3.3}$$

where, vectors of left singular matrix represents the spatial distribution of the principle components describing the variation in time, namely the vectors of right singular matrix $\boldsymbol{V}$ weighted by the corresponding singular values. The $\boldsymbol{U}$ matrix obtained is of the size (4 x 4) and thus a maximum of four mode shapes can be estimated. Since the lower modes of building frames are reasonably smooth the four elements of each left singular vector are interpolated using PCHIP interpolation to estimate the pseudo-mode coordinates at 2nd, 4th and 6th floors. These 7×1 left singular pseudo-mode shapes have been expanded to the 42×1 vectors by inverting the static condensation operation to include the representation of rotational and vertical DOFs. Gram–Schmidt orthogonalization (Golub and van Loan, 1996) of these vectors with respect to the analytical mass matrix ($\boldsymbol{M}$) provides normal mode shapes. This method provided good estimation of 2nd mode shapes for both excitations. MAC values of 2nd mode shapes estimate from this method are summarised Table 3.1.

Table 3.1. Identified Modal Parameters

		San Fernando case				Loma Prieta case			Identified from
		0-4 s	4-8 s	8-12 s	12-16 s	0-7 s	7-14 s	14-21 s	
1st frequency		2.0	1.9	2.0	2.0	2.0	2.0	2.0	FDD
2nd frequency		6.2	6.2	6.1	5.9	6.2	5.9	5.9	ETFE
3rd frequency		12.5	12.6	11.4	11.1	11.7	11.5	11.1	ETFE
4th frequency		20.3	20.7	20.6	19.5	20.5	19.5	19.1	ETFE
MAC Values	1st mode	0.9917	0.9985	0.9997	0.9999	0.9951	0.9986	0.9999	FDD
	2nd mode	0.8785	0.9630	0.9668	0.9931	0.9207	0.7808	0.9636	SVD in TD

3.3. Empirical Transfer Function Estimate

As the hinge formation is a localised phenomenon, its effect will be more pronounced on higher modes in comparison with the lower once. Hence, identification of higher modes would be helpful in identifying and quantifying the localised structural damage. Since only two frequencies are identified from the FDD plot, the other higher frequencies are identified from empirical transfer function estimate (ETFE) which is defined as the ratio of smoothed estimates of the cross-power spectrum between the input and output data pairs ($P_{xy}(\omega)$) and the auto-power spectrum of the input ($P_{xx}(\omega)$),

$$H(j\omega) = \frac{P_{xy}(\omega)}{P_{xx}(\omega)} \tag{3.4}$$

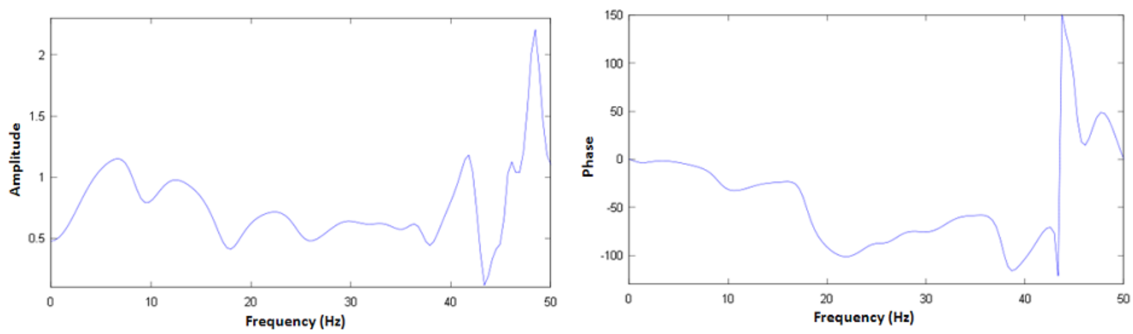

Figure 3.1. Amplitude and phase of the empirical transfer function estimate for data time window 8-12 s for San Fernando excitation.

here, ***x*** and ***y*** respectively denote the excitation and response time histories. The estimated transfer function for data time window 8–12 s corresponding to 1st floor absolute acceleration response for the San Fernando excitation is as shown in the Fig. 3.1. The peaks in the amplitude spectrum correspond to a natural frequency of the system if and only if there is a corresponding shift in the phase spectrum at the same frequency. Higher modes identified by ETFE are summarised in Table 3.1.

It is interesting to note that the first mode is consistently not identified in any of time windows for both excitation cases, this could be smearing due to spectral smoothing employed to estimate the smooth transfer function. This highlights the need for combining and cross-validating the results of system identification using different approaches.

4. DAMAGE IDENTIFICATION

Since any damage in the structural system is associated with stiffness degradation its effect should manifest as a decrease in the natural frequencies. Therefore, a classical method for damage detection is the monitoring of changes in the identified natural frequencies with respect to time.

4.1. Changes in Identified Frequencies

The percent differences between the 1st frequency obtained from the FDD and 2nd, 3rd and 4th frequencies obtained from transfer function estimate and the undamaged frequencies (in present case analytical frequencies), for San Fernando excitation are as shown in the Table 4.1. A negative relative change in identified frequencies can be construed as an indication of damage. As can be seen from the Table 4.1, the formation of hinges in beams does not reflect in any change in the fundamental frequency. The effect of damage is more substantial in the case of 3rd natural frequency. However, the estimates of the 4th frequency do not exhibit any definite trend. Similar results are obtained for Loma Prieta excitation as shown in Table 4.1 except that the 4th natural frequency also indicates a progressive degradation of the structural system. Though progressive differences in frequencies indicate presence of damage in the structure, care must be exercised in interpreting these results. Because frequency identification from empirical transfer functions are sensitive to the parameters such as data overlap, window type and size.

Table 4.1. Relative Changes in Identified Frequencies with reference to Undamaged Frequencies

	% Frequency change in window				
	San Fernando case			Loma Prieta case	
Mode	4-8 s	8-12 s	12-16 s	7-14 s	14-21 s
1	-5.00	0.00	0.00	0.00	0.00
2	0.00	-1.61	-4.84	-4.84	-4.84
3	0.80	-8.80	-11.20	-1.71	-5.13
4	1.97	1.48	-3.94	-4.88	-6.83

Other methods such as reduced order approximation of flexibility matrix (Pandey and Biswas, 1994), damage index method (Xiaodong, 2007) and changes in modal strain energy for each element (Shi et al, 1998) were tested and found unsuitable to identify the location of damage in beams of the example moment resisting frame (Shiradhonkar, 2009).

4.1. Changes in Curvature Mode Shapes

Changes in curvature modes shapes have been used to locate the damage in the example moment resistant frames. The formations of hinges in the beams are expected to cause some localized changes in curvature of the lateral vibration mode shapes of the moment resistant frame. Accordingly, the curvature modes (Pandey et al, 1991) are obtained from the displacement mode shapes by using a central difference approximation as,

$$\text{Ø}''_{if} = \frac{\text{Ø}_i^{f+1} - 2\text{Ø}_i^{f} + \text{Ø}_i^{f-1}}{h^2} \tag{4.1}$$

where, h is storey height, f is the floor level, and the subscript i denotes the mode. In this study, only the first mode has been used to determine the curvature mode. The relative differences between the curvature modes in successive data time windows are calculated. Each successive difference vector is then normalized with respect to the largest value. The normalised differences between successive time windows, for both excitation cases are shown in Table 4.2. From this table it is clear that, difference is substantially large for 1st, 2nd, 3rd and 4th compared to other floors, for San Fernando excitation. Similarly, for the Loma Prieta excitation the normalised mode difference is substantially large for 1st, 2nd, 3rd, 4th, 5th and 6th floors. The large value of normalized mode curvature for 6th floor in the case of Loma Prieta excitation is a false alarm–damage is indicated where there is none. However, there has been no missing alarms–unable to indicate the damage where there is one. This false alarm in the case of Loma Prieta excitation persists even for the hypothetical case of no contamination of vibration signatures with measurement/recording noise. This consistency indicates large sensitivity of the modal parameters to the changes in rotational stiffness at level 6 and calls for a user intervention either at the stage for identifying probable damage locations from the mode shape curvatures and/or modification of the objective function in the finite element model updating process discussed in the following.

Table 4.2. Normalized Successive Difference between 1st Curvature Modes at different Data Time Windows

	Normalized Successive Difference between 1st Curvature Mode				
	San Fernando Excitation			Loma Prieta Excitation	
	0 - 4 s & 4 - 8 s	4 - 8 s & 8 - 12 s	8 - 12 s & 12 - 16 s	0 - 7 s & 7 - 14 s	7 - 14 s & 14 - 21 s
Floor 1	0.233	0.390	0.140	1.000	0.086
Floor 2	0.491	1.000	0.525	0.400	1.000
Floor 3	1.000	0.658	0.330	0.737	0.238
Floor 4	0.037	0.845	1.000	0.531	0.226
Floor 5	0.055	0.046	0.036	0.201	0.059
Floor 6	0.042	0.047	0.076	0.183	0.062
Floor 7	0.029	0.013	0.048	0.007	0.001

5. FINITE ELEMENT MODEL UPDATING

For damage quantification of present problem sensitivity based model updating method has been used. As study assumes absence of aleatory uncertainties difference between the analytical predictions and the experimental observations is due to softening in structural elements.

For building structures, change in frequencies and mode shapes are relatively more sensitive to the stiffness of columns than beams. If the stiffness parameters of both beams and columns are retained as free variables for model updating then and identified changes in frequencies and mode shapes would be reflected as larger changes in column stiffness coefficients rather than in beam stiffness coefficients even if the damage is localized in beams thereby leading to a wrong diagnosis. As damage in beams will be in the form of development of plastic hinges in beams, rotational spring coefficients are considered as updating parameters. It is assumed that hinges will form at both ends of beam simultaneously and that both the ends will have the same rotation. The general analytical model capable of producing observed behaviour and response consists of set of seven rotational springs corresponding to seven floors of the structure. At floor level same property springs are located at both ends of the beams. The undamaged state has been simulated with assigning a very high initial stiffness

(1.43E10 Nm/rad) to the rotational springs and the results of this model are in good agreement with the rigid joint model.

5.1. Sensitivity Based Parameter Updating and Objective Function

The eigenvalue problem of structural system of mass matrix $\boldsymbol{M}$ and stiffness matrix $\boldsymbol{K}$ is formulated as,

$$\boldsymbol{K\Phi} = \boldsymbol{M\Phi\Lambda}\text{ , where }\boldsymbol{K} = \boldsymbol{f}(\boldsymbol{a}, \boldsymbol{e}, \boldsymbol{l}, \boldsymbol{x}) \tag{5.1}$$

where, $\boldsymbol{\Phi}$ is the modal matrix and $\boldsymbol{\Lambda}$ is a diagonal matrix containing eigen values. For present problem, the stiffness matrix is function of cross-section properties vector **a**, material properties vector **e**, geometric properties vector **l** and rotation spring stiffness vector **x**. As aleatory uncertainties are absent, spring coefficients x are the only free parameters available for updating. Let $\boldsymbol{g}$ be the vector containing the modal parameters used for model updating. After expanding vector $\boldsymbol{g}$ at $k+1$ iteration by using Taylor series and then by neglecting higher order terms, we have obtained linear approximation as,

$$\boldsymbol{g}(\boldsymbol{x}_{k+1}) = \boldsymbol{g}(\boldsymbol{x}_k) + \boldsymbol{S}_{nk}\Delta\boldsymbol{x}_k\text{, where }\boldsymbol{S}_{nk} = \frac{\partial \boldsymbol{g}}{\partial x_k} \tag{5.2}$$

where, $\boldsymbol{S}_{nk}$ denotes sensitivity matrix corresponding to spring coefficient in k^{th} iteration, which represents relative change in modal characteristics due to changes in the spring coefficient.

Since the model updating is based on the minimization of the model prediction error, a weighted residual vector ***r(x)*** from the difference of identified and analytical values of modal parameters is selected as objective function.

$$\boldsymbol{r}(\boldsymbol{x}_{k+1}) = \boldsymbol{W}\big(\boldsymbol{g}_{id} - \boldsymbol{g}_{ana}(\boldsymbol{x}_{k+1})\big)\text{ , where }\boldsymbol{W} = diag\,[1/\omega_{1,id}, \dots\dots, 1, \dots..] \tag{5.3}$$

where, the subscripts $()_{id}$ and $()_{ana}$ respectively denote the identified and analytical modal parameters and $\boldsymbol{W}$ is weighting matrix used is for scaling the relative contribution of various terms of the prediction error in the weighted residual vector. In the present study a diagonal weighting matrix, proposed by Weber et al (2007), has been used with the frequency terms being scaled by the observed values and unit weight for mode shape coordinates. After using linear approximation for $\boldsymbol{g}_{ana}$, the objective function will be in terms of Euclidean norm of the weighted residual vector, which will be minimised by gradient-based minimization methods to obtain spring coefficients as,

$$\Delta\boldsymbol{x}_k = \left(\overline{\boldsymbol{S}}_k^T\overline{\boldsymbol{S}}_k\right)^{-\mathbf{1}}\overline{\boldsymbol{S}}_k^T\boldsymbol{r}_k\text{ , where }\overline{\boldsymbol{S}}_k = \boldsymbol{W}\frac{\partial \boldsymbol{g}}{\partial x_k} \tag{5.4}$$

It is well known that although the product $\overline{\boldsymbol{S}}_k^T\overline{\boldsymbol{S}}_k$ is symmetric and positive definite, its condition number is significantly more than that of $\overline{\boldsymbol{S}}_k$. Therefore, to use of Eqn. 5.5 as model updating solution Tikhonov regularisation has been used.

In regularisation the quality of solution to ill-conditioned and/or ill-formed problems can be significantly improved by imposing some penalty on the complexity of the solution. The magnitude of update parameters is controlled by the regularization parameter γ. Weber et al have shown that if number of iterations exceed beyond a certain number then regularization has no effect over the solution. If regularisation is applied before linearization then the formulation does not depend on number of iterations thereby ensuring the stability of solution. The vanishing gradient criterion for the minimum leads to the modified update solution as,

$$\Delta\boldsymbol{x}_k = \left(\overline{\boldsymbol{S}}_k^T\overline{\boldsymbol{S}}_k + \boldsymbol{I}\gamma\right)^{-\mathbf{1}}\left(\overline{\boldsymbol{S}}_k^T\boldsymbol{r}_k - \gamma\boldsymbol{x}_k\right) \tag{5.5}$$

The regularization parameter is selected using Generalized Cross Validation (GCV) (Ahmadian et al, 1998), which is given by

$$GCV\,(\gamma) = \frac{(1/n)\|(\boldsymbol{A}_\gamma - \boldsymbol{I})\boldsymbol{r}_k\|_2^2}{[(1/n)trace(I - \boldsymbol{A}_\gamma)]^2}\text{, where } \boldsymbol{A}_\gamma = \boldsymbol{S}_k\left(\boldsymbol{S}_k^T\boldsymbol{S}_k + \gamma\boldsymbol{I}\right)^{-1}\boldsymbol{S}_k^T \tag{5.6}$$

The term $(\boldsymbol{A}_\gamma - \boldsymbol{I})\boldsymbol{r}_k$ in the numerator of the GCV equation is the error of regularized solution. The term in denominator of the same equation represents variation of the error. The optimal regularization parameter is chosen as that particular value of γ at which GCV function value, which balances the variation of the error against the model fit attains a minimum. This optimal regularization parameter value is obtained by plotting GCV function for series of the γ values.

5.3. Model Updating Results

From detectability index defined by Weber et al, it is observed that different amount of changes in any one spring coefficient or combination of spring coefficients will result into similar changes in frequencies and mode shapes. Therefore, spring coefficients corresponding to floor locations where substantial difference in curvature modes is observed are considered for FE model updating. In addition to this, it has been observed that the FE model updating procedure is better constrained when first two mode shapes are included in the response vector for model updating. Therefore, FE model updating is applied only to those time windows where a good quality estimate for the 2nd mode is available.

Table 5.1. Updated Spring Stiffnesses from Model Updating

Spring Location	Data time window			
	San Fernando case			Loma Prieta case
	4 - 8 s	8 - 12 s	12 - 16 s	14 - 21 s
1^{st} Floor	9.06E+07	1.57E+08	3.25E+08	9.19E+07
2^{nd} Floor	9.68E+07	1.66E+08	3.40E+08	1.17E+08
3^{rd} Floor	8.95E+07	1.53E+08	3.14E+08	1.19E+08
4^{th} Floor	8.26E+07	1.42E+08	2.92E+08	1.21E+08
5^{th} Floor	Not updated	Not updated	Not updated	1.17E+08
6^{th} Floor	Not updated	Not updated	Not updated	9.46E+07
7^{th} Floor	Not updated	Not updated	Not updated	Not updated

For San Fernando excitation, model updating using 4 spring stiffness parameters has been carried out for time windows 4-8 s, 8-12 s and 12-16 s, as 2^{nd} mode having MAC value greater than 0.95 is available for these windows. The updated values of springs' stiffness are summarised in the Table 5.1. Similarly, for Loma Prieta excitation model updating using 6 spring stiffness parameters has been carried out for 14-21 s time window. The updated values of springs' stiffness are summarised in the Table 5.1. The secant stiffness of the rotational spring at the immediate occupancy (IO) point (Fig. 2.2) is 6.6545×10^6 Nm/rad. The updated spring coefficients are larger than this value, which indicates that the sustained damage is within the IO point.

6. CONCLUSIONS

Paper addressed the problem of detection and quantification of damage in strong column-weak beam frame from recorded acceleration response at few locations during a strong motion earthquake. It is found that the piecewise cubic Hermite interpolating polynomial (PCHIP) can be used for the reconstruction of missing data at intermediate locations by interpolating recorded response time histories in time domain and to obtain 2nd mode shape by interpolating orthogonal vectors obtained from the left singular vectors of the recorded response time history data. A time window analysis is carried out to track changes in the identified modal parameters of the structural system. Numbers of system identification methods are tested to detect the presence and locations of the damages. It is observed that combined use of system identification methods confirms presence of damages and its locations. Sensitivity based model updating method worked reasonably well in quantifying the damages at identified locations. Thus, combination of a suite of system identification techniques with sensitivity based finite element model updating can potentially locate and quantify the damage in a moment resistant frame.

REFERENCES

Adams, R., Cawley, P., Pye, C. and Stone, B. (1978). A vibration technique for non-destructively assessing the integrity of structures. Journal of Mechanical Engineering Science, 20:2, 93–100.

Agarwal, P. and Shrikhande, M. (2006). Earthquake resistant design of structures, PHI Learning Pvt. Ltd, New Delhi, India.

Ahmadian, H., Mottershead, J. and Friswell, M. (1998). Regularization methods for finite element model updating. Mechanical Systems and Signal Processing, 12:1, 47-64.

Allemang, R. (2003). The modal assurance criterion: twenty years of use and abuse. Sound and Vibration, 37:8, 14–23.

Brincker, R., Zhang, L. and Andersen, P. (2001). Modal identification of output only systems using frequency domain decomposition. Smart Materials and Structures, 10:3, 441–445.

Brownjohn, J. (2007). Structural health monitoring of civil infrastructure, Philosophical Transactions of the Royal Society, Series A, 365, 589–622, http://dx.doi.org/10.1098/rsta.2006.1925.

Cawley, P. and Adams, R. (1979). The location of defects in structures from measurements of natural frequencies. Journal of Strain Analysis for Engineering Design, 20:2, 49–57.

Datta, A., Shrikhande, M. and Paul, D. (2002). On the optimal location of sensors in multi-storeyed buildings. Journal of Earthquake Engineering, 6:1, 17–30.

FEMA-356 (2000). Pre-standard and commentary for seismic rehabilitation of building, Federal Emergency Management Agency, Washington, D.C., USA.

Golub, G. and Van Loan, C. (1996). Matrix computations (3rd ed.), The John Hopkins University Press, Baltimore, USA.

Humar, J. (2006). Performance of vibration based techniques for identification of damage. Structural Health Monitor, 5:3, 215–241.

Ljung, L. (1987). System identification: theory for the user Prentice-Hall, Inc, Englewood Cliffs, New Jersey, USA.

Pandey, A., Biswas, M. and Samman M. (1991). Damage detection from changes in curvature mode shapes. Journal of Sound and Vibration, 145:2, 321–332.

Pandey, A. and Biswas, M. (1994). Damage detection in structure using changes in flexibility. Journal of Sound and Vibration, 169:1, 3–17.

Rodgers, J. and Çelebi, M. (2006). Seismic response and damage detection analyses of an instrumented steel moment-framed building. Journal of Structural Engineering, ASCE, 132:10, 1543–1552.

SAP-2000 (1995), CSI Analysis Reference Manual, Computers and Structures, Inc., University Avenue Berkeley, California, USA.

Shi, Z., Law, S. and Zhang, L. (1998). Structural damage localization from modal strain energy change. Journal of Sound Vibration, 218:5, 825–844.

Shiradhonkar, S. (2009). Structural health assessment via finite element model updating, M.Tech. Dissertation, Department of Earthquake Engineering, Indian Institute of Technology Roorkee, India.

Weber, B., Paultre, P. and Proulx, J. (2007). Structural damage detection using nonlinear parameter identification with Tikhonov regularization. Structural Control and Health Monitoring, 14:3, 406–427.

Xiaodong, J., Jiaru, Q. and Longhe, X. (2007). Damage diagnosis of a two storey spatial steel braced frame model. Structural Control and Health Monitoring, 14:8, 1083–1100.