Simplified Identification Scheme for Structures on a Flexible Base

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SUMMARY:
The objective of system identification is to evaluate unknown properties of a dynamic system by developing a mathematical model for the relationship between measured responses (outputs) and measured external demands (inputs) to the system. We describe a new frequency domain procedure which utilizes a graphical bisector scheme to identify frequency and damping for the flexible base, pseudo-flexible base and fixed base behaviour of a structure, from a single set of flexible base data. The procedure is applied for analysis of both earthquake data and forced vibration test data. We can validate the bisector procedure by generating simulated data from computational models of soil-foundation-structure systems. Responses from these modelled structures can also be analyzed using traditional methods to identify flexible and fixed base modal parameters.

Keywords: soil-structure interaction, field testing, system identification, shallow foundations

1. INTRODUCTION

The objective of system identification is to evaluate the unknown properties of a dynamic system by developing a mathematical model for the relationship between measured responses of the system (outputs) and measured external demands (inputs) to the system. For applications to buildings, system identification can be used to estimate modal frequencies, damping ratios, mode shapes and participation coefficients based on recordings of system vibrations (e.g., Safak, 1991). Depending on the selected input-output and motions, modal vibration parameters can be identified that describe the behavior of the superstructure alone, or the soil-foundation-structure system. Fundamental-mode frequencies and damping ratios are distinct for the two base fixity conditions. The differences, such as the ratio of flexible/fixed base fundamental mode periods, are a useful quantification of the significance of Soil-Structure Interaction (SSI) effects.

In this paper, we first describe a new frequency domain system identification procedure to evaluate fixed- and flexible-base modal properties. The model contains unknown modal frequencies and damping ratios that are estimated using resonance considerations through a graphical scheme referred to as “bisector method”. The procedure can be applied for analysis of either earthquake data or forced vibration test data. We validate the results of the simplified bisector method for synthetic data for which both earthquake and forced vibration test data are available.

2. THEORY OF EVALUATION OF SSI EFFECTS USING BISECTOR METHOD

We first examine a linear, undamped, multi degree-of-freedom (MDOF) system with an external shaker force applied at the roof of the structure. The concept can be easily extended to incorporate foundation compliance, as shown subsequently. The relative displacements or rotations of each of the n degrees-of-freedom (DOFs) in the structure relative to the base are described by an $n\times1$ vector $\mathbf{U}$, with corresponding velocity and acceleration vectors $\dot{\mathbf{U}}$ and $\ddot{\mathbf{U}}$, respectively. In matrix form, the well-known equation of motion of the superstructure can be written as:

$$\left[\begin{array}{c|c} m_1 & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{M}_b \end{array}\right]\left[\begin{array}{c} \ddot{u}_1 \\ \hline \ddot{\mathbf{U}}_b \end{array}\right]+\left[\begin{array}{c|c} k_{11} & \mathbf{K}_{1b} \\ \hline \mathbf{K}_{b1} & \mathbf{K}_{bb} \end{array}\right]\left[\begin{array}{c} u_1 \\ \hline \mathbf{U}_b \end{array}\right]=\left[\begin{array}{c} F_{sh} \\ \hline \mathbf{0} \end{array}\right] \tag{1}$$

We have partitioned the matrices to isolate the top DOF on the structure, so k_{11} is a 1x1 matrix, and square matrix $\mathbf{K}_{bb}$ is of rank n-1 x n-1. We can apply the Fourier transform to the EOM, causing $\bar{u}$ and its derivatives to be frequency dependent, which is noted with an overbar. By converting the $\bar{\ddot{\mathbf{U}}}_b$ terms from accelerations to displacements, we can move the elements of $\mathbf{M}_b$ from the mass matrix to the stiffness matrix. This yields the equation:

$$\left[\begin{array}{c|c} m_1 & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{0} \end{array}\right]\left[\begin{array}{c} \bar{\ddot{u}}_1 \\ \hline \bar{\ddot{\mathbf{U}}}_b \end{array}\right]+\left[\begin{array}{c|c} k_{11} & \mathbf{K}_{1b} \\ \hline \mathbf{K}_{b1} & \mathbf{K}_{bb}-\omega^2\mathbf{M}_b \end{array}\right]\left[\begin{array}{c} \bar{u}_1 \\ \hline \bar{\mathbf{U}}_b \end{array}\right]=\left[\begin{array}{c} \bar{F}_{sh} \\ \hline \mathbf{0} \end{array}\right] \tag{2}$$

that leaves us with just a single nodal mass in the mass matrix and allows us to perform algebraic ("static") condensation to reduce the matrices to a single equivalent degree-of-freedom (SDOF) system. After trivial algebraic operations the condensed equation of motion is:

$$m_1\bar{\ddot{u}}_1+\left[k_{11}-\boldsymbol{K}_{1b}\left(\boldsymbol{K}_{bb}-\omega^2\boldsymbol{M}_2\right)^{-1}\boldsymbol{K}_{b1}\right]\bar{u}_1=\bar{F}_{sh} \tag{3}$$

In light of the elementary relation

$$\omega_0=\sqrt{k/m} \tag{4}$$

the fundamental frequency of the system at hand can be determined from Equation (3) as:

$$\omega_0=\sqrt{\left[k_{11}-\boldsymbol{K}_{1b}\left(\boldsymbol{K}_{bb}-\omega^2\boldsymbol{M}_b\right)^{-1}\boldsymbol{K}_{b1}\right]/m_1} \tag{5}$$

It should be noted that ω_0 is dependent on ω. By definition the resonant condition of the undamped SSI system is reached when the frequency of response is equal to the frequency of excitation If we look for solutions for which the two frequencies coincide, there are *n* distinct values that will satisfy this condition for a mass matrix of rank *n*, which correspond to the *n* fundamental frequencies. As shown in Figure 1, we can graphically solve for the resonant frequencies of the system by plotting the resonant frequency from Equation (5) as a function of the frequency of demand (red line). We plot a bisector line (black line) that indicates points at which both frequencies are identical, which are interpreted as the resonant frequencies.

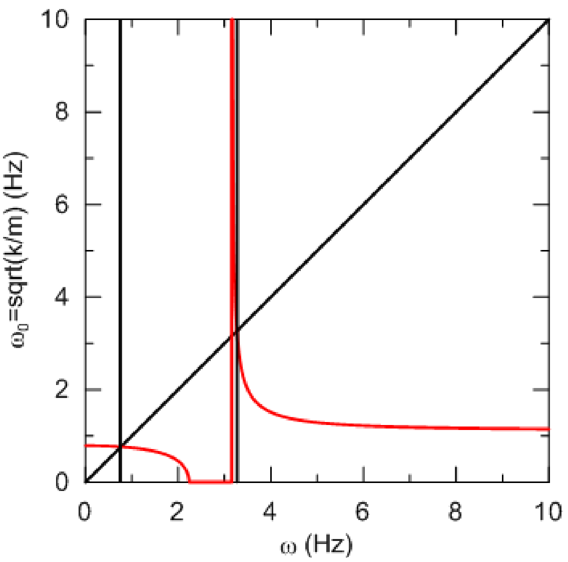

Figure 1. Schematic of bisector solution for identifying resonant frequency of a two DOF undamped system

Let us now consider a damped MDOF system with a compliant foundation as shown in Figure 2 the structure consists of *n* structural masses ($m_{s,i}$), plus two foundation degrees of freedom. The structure is subjected to an external harmonic force F_{sh} (e.g., imposed by a shaker) acting at the roof node having mass $m_{s,n}$. The compliance of the soil is modeled through frequency-dependent springs, $\mathbf{k}^*_x$, and $\mathbf{k}^*_{yy}$ that enable foundation translation (u_f) and rotation (θ_f), respectively. The soil-structure interaction is modeled by complex-valued springs of unknown modulus $\mathbf{k}^*_{i,j}$, which can be written in the form:

$$k^* = k(1 + i2\beta) \tag{6}$$

where k is the spring stiffness and β the dimensionless damping ratio. The displacement at the *n*th degree of freedom, the top of the structure, is denoted by $u_{st,n}$. The displacement is measured relative to an unmoving point on the ground.

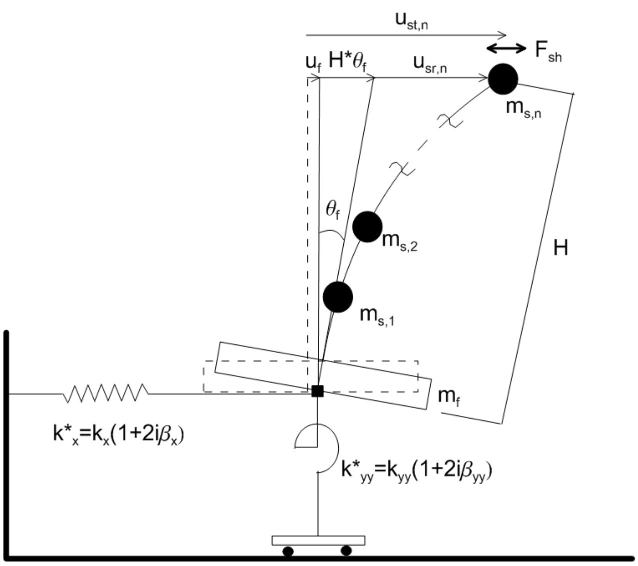

Figure 2. A simple SSI System subjected to forced vibration at roof node

The equations of motion can be written in matrix form:

$$\begin{bmatrix} m_{s,n} & \cdots & 0 & 0 & 0 \\ \vdots & \ddots & & \vdots & \\ 0 & & m_{s,1} & 0 & 0 \\ 0 & \cdots & 0 & I_f & 0 \\ 0 & & 0 & 0 & m_f \end{bmatrix} \begin{bmatrix} \ddot{u}_{st,n} \\ \vdots \\ \ddot{u}_{st,1} \\ \ddot{\theta}_f \\ \ddot{u}_f \end{bmatrix} + \begin{bmatrix} k^*_{1,1} & \cdots & k^*_{1,n-2} & k^*_{1,n-1} & k^*_{1,n} \\ \vdots & \ddots & & \vdots & \\ k^*_{n-2,1} & & k^*_{n-2,n-2} & k^*_{n-2,n-1} & k^*_{n-2,n} \\ k^*_{n-1,1} & \cdots & k^*_{n-1,n-2} & k^*_{n-1,n-1} & k^*_{n-1,n} \\ k^*_{n,1} & & k^*_{n,n-2} & k^*_{n,n-1} & k^*_{n,n} \end{bmatrix} \begin{bmatrix} u_{st,n} \\ \vdots \\ u_{st,1} \\ \theta_f \\ u_f \end{bmatrix} = \begin{bmatrix} F_{sh} \\ \vdots \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{7}$$

By converting to the frequency domain, rearranging terms, and performing static condensation as illustrated above, the system can be reduced to an equivalent SDOF one described by the equation of motion

$$m_{s,n}\bar{u}_{st,n}\left(-\omega^2\right) + k^*_{flex}\bar{u}_{st,n} = \bar{F}_{sh} \tag{8}$$

Here k^*_{flex} is the complex-valued stiffness of the flexible base SSI system.

Assuming mass $m_{s,n}$ is known, we can solve Equation (8) for the stiffness of the flexible base soil-structure system:

$$k^*_{flex} = \left(\frac{\bar{F}_{sh} + m_{s,n}\omega^2\bar{u}_{st}}{\bar{u}_{st,n}}\right) \tag{9}$$

By writing k^*_{flex} in the complex form as in Equation (6) one can solve for spring modulus

$$k_{flex} = \mathrm{Re}(k^*_{flex}) \tag{10}$$

where *Re*( ) denotes real part. Substituting k_{flex} into Equation (4) and dividing by (2π) gives the frequency-dependent un-damped natural frequency of the system:

$$f_0 = \frac{1}{2\pi}\left(\frac{k_{flex}}{m_{s,n}}\right)^{1/2} \tag{11}$$

Note that a single mass, $m_{s,n}$, is used in the above expression because the algebraic condensation has moved all other masses into the stiffness matrix.

In the same spirit, writing k^*_{flex} in the complex form as in Equation (6) and rearranging term, yields the effective frequency dependant damping ratio of the SSI system in the form:

$$\beta_s = \frac{1}{2k_{flex}} Imag(k^*_{flex}) \tag{12}$$

In this formulation the resonant frequency of the structure is dependent on the frequency of the excitation because k^*_{flex} is dependent on the excitation (shaker) frequency. As previously shown in Figure 1, we can graphically solve for the resonant frequencies of the system by plotting the resonant frequency from Equation (11) as a function of the frequency of demand (red line). We plot a bisector line (black line) that indicates points at which both frequencies are identical. By identifying the point(s) at which the frequency function and the bisector intersect, we can identify frequencies of resonance. Once the resonant frequencies are identified the appropriate damping can then be recognized by selecting beta at the identified resonant frequency. The frequencies identified through this process are un-damped frequencies of vibration, which are related to damped frequencies through the well-known relation (Chopra, 2007):

$$\omega_d = \omega_0 \sqrt{1-\beta^2} \tag{13}$$

The above derivation provides a method for evaluating the stiffness and damping for a flexible base SSI system. New inputs and outputs are needed to derive the stiffness and damping of the structure under fixed base conditions.

To illustrate how the appropriate input-outputs can be selected, we turn our attention to a simple system consisting of two springs in series, with stiffnesses of k_1 and k_2, subjected to force, F. Springs 1 and 2 experience deformations, u_1 and u_2 respectively. We can show that:

$$k_2 = k_{total} \frac{u_{total}}{u_2} \tag{14}$$

Where u_{total} is the sum of the two spring displacements, and k_{total} is the stiffness of the system. Equation (14) provides a direct relation between the stiffness of a single component spring and the stiffness of the total system. This indicates that as long as we measure the total displacement of the system and the relative deformation of one spring of interest, we can solve for the stiffness of the other component spring from the stiffness of the total system.

Returning to the MDOF SSI system, in order to determine the fixed base modal parameters, we frame the equations of motion so that the stiffness of the structure alone appears. It is convenient to formulate the equations of motion in terms of the translation of the structure relative to the foundation translations and rocking.

$$u_{sr,n} = u_{st,n} - H\theta_f - u_f \tag{15}$$

The flexible base stiffness can be thought of as the total stiffness of a system consisting of soil springs and a structure spring. If we take k^*_{flex} to be analogous to k_{total}, and the combined effects of foundation rotation and translation to be analogous to k_1, then the stiffness of the structure alone, k^*_{flex}, is analogous to k_2 . By applying Equation (14), we can then solve for k^*_{flex}:

$$k^*_{fixed} = k^*_{flex} \frac{\bar{u}_{st,n}}{\bar{u}_{sr,n}} \tag{16}$$

If we substitute Equation (16) into Equation (8) we find:

$$m_{s,n}\bar{u}_{st,n}\left(-\omega^2\right) + k^*_{fixed}\bar{u}_{sr,n} = \bar{F}_{sh} \tag{17}$$

We can solve this equation for the stiffness of the fixed base soil-structure system as:

$$k^{*}_{fixed} = \frac{\overline{F}_{sh} + m_{s,n}\omega^{2}\overline{u}_{st,n}}{\overline{u}_{sr,n}} \tag{18}$$

The stiffness of the fixed base structure can be used to solve for the natural frequency and damping in the same manner as for the flexible base case.

It is sometimes of use to investigate the structure as if it were allowed to rock, but was fixed against translation. Following prior convention (Stewart and Fenves, 1998), we call this the pseudo-flexible base case. More information about the derivation of stiffness equations for this case can be found in Star, (2011).

2.1 Fixed and Flexible Base Parameters for Earthquake Loading of a System

To extend the method developed above for earthquake loading, we consider an MDOF SSI system with a compliant foundation similar to the one illustrated in Figure 2**Error! Reference source not found.**. The equation of motion of a structure subjected to earthquake loading is given by (Chopra, 2007) as:

$$\mathbf{M}\ddot{\mathbf{U}}(t) + \mathbf{K}\mathbf{U}(t) = -\mathbf{M}\mathbf{1}\ddot{u}_{g}(t) \tag{19}$$

where **1** is a vector with ones for each of the structure translational degrees of freedom along the direction of shaking and zeros for all other degrees of freedom, and $\ddot{u}_g$ is the appropriate ground acceleration, which is discussed in more detail below. The structure consists of *n* structural masses ($m_{s,i}$), plus two foundation degrees of freedom. The compliance of the soil is modeled through springs, k^*_x, and k^*_{yy}, that enable foundation translation (u_f) and rotation (θ_f) respectively. The soil springs are complex-valued.

The displacements and accelerations on the left side of the equations are given relative to the ground. The earthquake forces can be transferred to the left side of the equations and we can rewrite the acceleration vector in terms of absolute displacements where:

$$\ddot{u}^{T}_{s,n} = \ddot{u}_{st,n} + \ddot{u}_{g} \tag{20}$$

By converting to the frequency domain, rearranging terms, and performing static condensation as illustrated in the previous sections, the system can be reduced to an equivalent SDOF system, with an equation of motion equal to:

$$m_{s,n}\overline{u}^{T}_{s,n}\left(-\omega^{2}\right) + k^{*}\overline{u}^{T}_{s,n} = 0 \tag{21}$$

where k^* is now a fictitious stiffness for degree of freedom *n* that accounts for all of the deformations within the structural system as well as ground displacements u_g.

As in the forced vibration case we are interested in solving for the fixed, flexible and pseudo-flexible base system stiffness terms. Recalling Equation (14), we can find k^*_{flex} as a function of k^*, the absolute displacement at the top of the structure, and the total displacement at the top of the structure relative to the ground:

$$k^{*}_{flex} = k^{*}\,\frac{\overline{u}^{T}_{s,n}}{\overline{u}_{st,n}} \tag{22}$$

If we substitute Equation (22) into Equation (21) we find:

$$m_{s,n}\bar{u}^{T}_{st,n}\left(-\omega^2\right)+k^{*}_{flex}\bar{u}_{st,n}=0 \tag{23}$$

We can solve this equation for the stiffness of the flexible base soil-structure system as:

$$k^{*}_{flex}=\left(\frac{m_{s,n}\omega^2\bar{u}^{T}_{s,n}}{\bar{u}_{st,n}}\right) \tag{24}$$

Here k^{*}_{flex} is the complex-valued stiffness term for the flexible base SSI system.-We can also develop equations similar to (24) for the fixed base cases:

$$k^{*}_{fixed}=\left(\frac{m_{s,n}\omega^2\bar{u}^{T}_{s}}{\bar{u}_{sr}}\right) \tag{25}$$

As was shown for the forced vibration case in Equations (11) and (12), by writing k^{*}_{flex}, and k^{*}_{fixed} in the complex form we can separate the real from the imaginary portion of the complex stiffness and we can solve for the frequency and damping of the SSI system. Unlike in the forced vibration loading case, the structure mass cancels out of the equations for frequency and damping, making the calculations more robust. One potential pitfall is that the calculations are dependent on the kinematically appropriate ground motion. Ground motions are altered by the presence of the foundation, so that the ground motions that the structure experiences are not identical to the free-field ground motions (Kim and Stewart, 2003). It is difficult to determine the true ground motion experienced by the foundation experimentally. There will be some error introduced to the results if the measured free-field motion is substituted.

3. VERIFICATION OF IDENTIFICATION METHODS USING MODELED DATA

We can validate the methods of system identification described above by generating simulated data from computational models of soil-foundation-structure systems. Responses from these modeled structures are ideal for testing the method because the structures have known properties and can be analyzed using traditional methods to identify flexible and fixed base modal parameters. The first model developed is a simple SDOF structure on top of a foundation with translational and rotational soil springs. The soil stiffness and damping are modeled using frequency dependant theoretical soil springs as developed by Pais and Kausel, (1988), with an addition hysteretic soil material damping. The structure is assigned a known stiffness, and hysteretic structural damping. Table 1 gives the soil and structure properties used in the model. The system is excited by a broadband excitation function at the top structure node. Translation and rotation accelerations at the foundation were computed as well as roof translation.

Table 1. SSI system modeling parameters (B = footing width, L= footing length, D=embedment)

Soil Parameters	Structure Parameters	Foundation Parameters
V_s = 110 m/s	m_s = 6.98 Mg	m_f = 13.36 Mg
ρ = 1.73 Mg/m^3	H = 2.9 m	B = 2.13 m
ν = 0.45	EI = 3.8 x 10^7 KN m^2	L = 4.26 m
β_{soil} = 2%	$\beta_{structure}$ = 2%	H_f = 0.6 m
G = 1.98 x 10^4 KN/m^2		D = 0
K_x, k_{yy}, β_x, β_{yy} from Pais and Kausel, 1988		

Figure 3 shows the graphical output of the bisector system identification procedure for flexible-base conditions [Equations (11) and (12)], and the analogous equations for the pseudo-flexible and fixed base conditions. The intersection of the natural frequency curve and the bisector for the flexible base case indicates the first mode frequency.

Table 2 summarizes the resulting frequencies and damping ratios. The fundamental frequency results of the system identification procedure can be checked against the results of a classical eigenvalue analysis (neglecting damping matrix) of the modeled structure with a flexible base and the modeled structure under fixed base, and pseudo-flexible conditions. The identification procedure had a discrepancy of about 1% or less compared to the eigenvalue analysis. The fixed base system identification procedure provides a damping of 2%, which is consistent with the structure damping employed in the model.

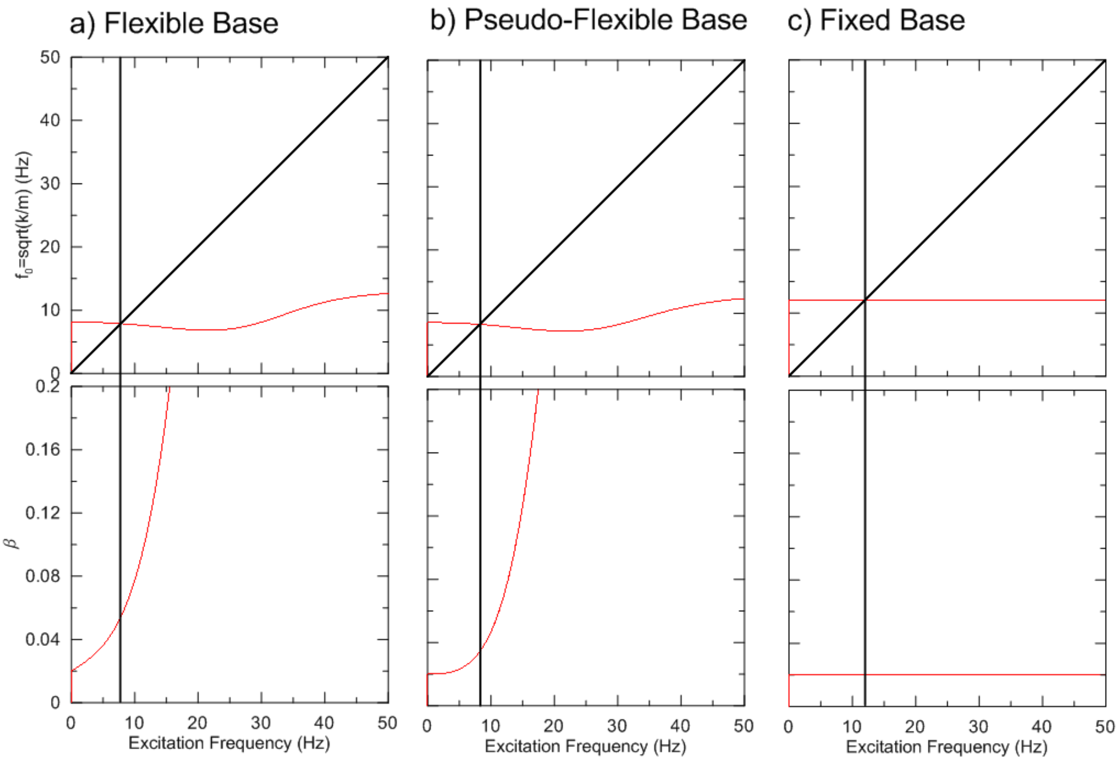

Figure 3. Bisector identification of frequency and damping for forced vibration (FV) loading of a SDOF modeled structure for (a) flexible base, (b) pseudo-flexible base and (c) fixed base conditions.

Table 2. Frequency and damping from the bisector system identification procedure and traditional eigenvalue analysis of 1 DOF modeled structure:

		Flexible	Pseudo-Flexible	Fixed	$\tilde{T}/T$	β_f
Bisector System ID	Frequency FV (Hz)	7.86	8.23	11.98	1.53	4.6
	β FV (%)	5.46	3.42	2.00		
	Frequency EQ (Hz)	7.84	8.28	11.98	1.53	3.5
	β EQ (%)	4.30	2.82	2.00		
Eigen	Frequency (Hz)	7.78	8.26	11.98	1.54	-

The same simple SSI structure can also be excited with a broadband earthquake excitation. The procedure for system identification of an earthquake excited structure can be applied. The results of the graphical output for the bisector method are shown in Figure 4. As this is the same structure as examined previously, we expect to find approximately the same frequency and damping for each base fixity condition. The results for the Eigen analysis, and the graphical identification methods are tabulated in Table 3. The system identification method for the earthquake loading produces the same damping value for the fixed base case as the forced vibration cases and marginally smaller damping levels for the other base fixity conditions.

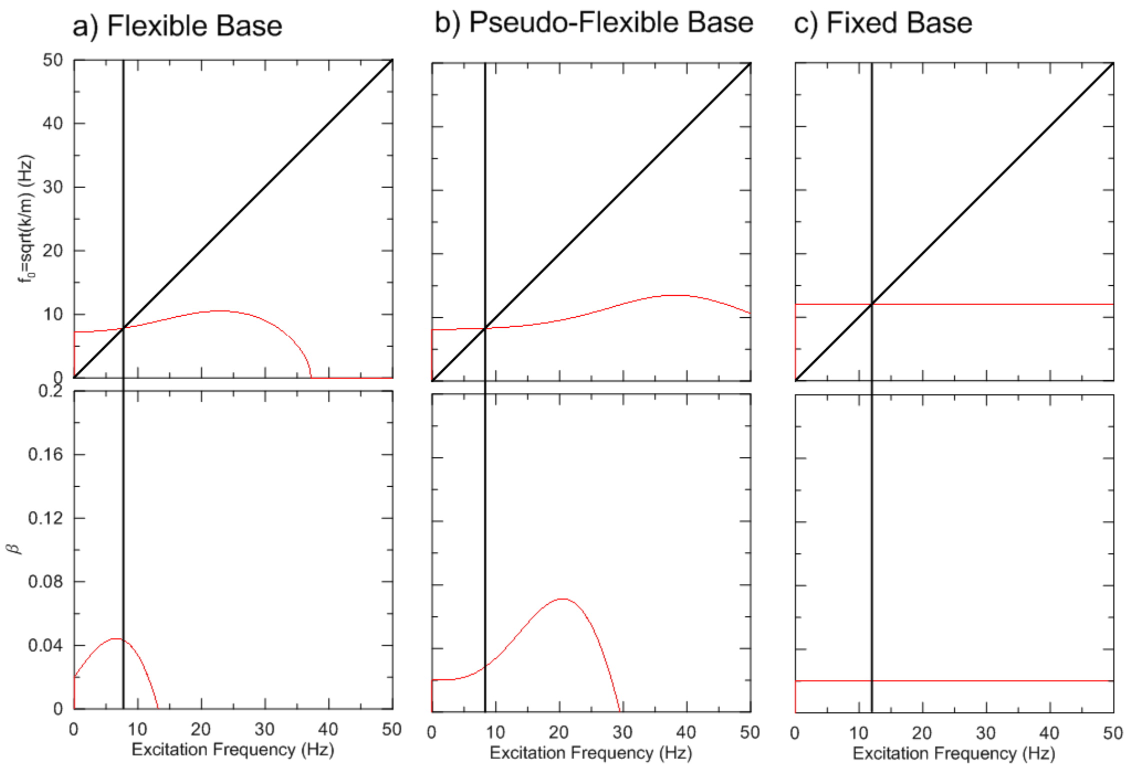

Figure 4. Bisector identification of frequency and damping for earthquake (EQ) loading of a SDOF modeled structure for: (a) flexible base, (b) pseudo-flexible base and (c) fixed base conditions.

Table 3. Frequency and damping from the bisector system identification procedure and traditional eigenvalue analysis of three DOF modeled structure (assuming β_i =2%)

		Flexible	Pseudo-Flexible	Fixed	$\tilde{T}/T$	β_f *
Bisector System ID	Frequency FV (Hz)	2.81	2.82	3.6	1.29	2.8
	β FV (%)	3.96	3.79	8.52		
	Frequency EQ (Hz)	2.81	2.84	7.94	2.84	2.7
	β EQ (%)	1.75	1.6	-		
Eigen Analysis	Frequency (Hz)	2.81	2.87	4.92	1.76	-

4. DISCUSSION AND CONCLUSIONS

A simple, physically-motivated procedure was presented to identify frequency-dependent effective stiffness and damping of structural systems for SSI applications. The method utilizes a graphical bisector scheme to identify fundamental frequency and damping for the flexible base, pseudo-flexible base and fixed base behavior of a structure, without the formidable mathematics associated with system identification theory. Implementation of the procedure requires measurements of horizontal displacements at the roof and foundation level of the structure, vertical foundation displacements (from which rotations are derived), shaker forces, and limited system masses. The procedure is applied for computer-simulated structures. The structure was loaded by forced vibration and earthquake excitation, allowing comparisons of the results from both methods.

The system identification procedure provides reasonable estimates of system frequency and damping for structures with relatively low levels of damping. The error in the first mode frequency for most structure system will be very small. The error may limit possibility of identifying higher modes in some cases. In order to identify the flexible base frequency and damping for the earthquake loading case it is necessary to know the foundation input motion, however this motion is difficult to measure experimentally. There will be some error introduced to the results because the measured free-field motion will usually be substituted.

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