Finite element analysis of ductile fuses for W-shape steel bracing members

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SUMMARY:
Capacity design provisions have now been implemented in modern codes for the design of earthquake-resistant building structures, causing increasing costs compared to past practice. The impact has been particularly severe for steel concentrically braced frames as brace connections, beams, columns and other connections have to be designed to resist lateral load effects corresponding to the probable tensile and compressive resistances of the braces. The tensile resistance of a bracing member being generally markedly higher than its design compressive strength used for selecting the member, capacity design requirements may result in significant increase in design loads, implying increases in steel tonnage as well as in costs related to shop fabrication and assembly on the site. Ductile brace fuses can be created by locally reducing the brace cross-section area to bring both tensile and compressive strengths closer. However, this modification to the braces may further local buckling response. Brace fuses must therefore be confined to prevent local instability. The paper presents a nonlinear finite element analysis study carried out to characterize the inelastic cyclic response of fuses for W-shaped bracing members.

Keywords: Steel structures, earthquake, brace, fuse, ductility

1. INTRODUCTION

The concept of adding ductile fuses in bracing members of steel concentrically braced frames (CBFs) resisting seismic loads arose some fifteen years ago, following the advent of seismic capacity design based methods for steel structures. Current code provisions require that steel CBFs be designed and detailed to exhibit ductile energy dissipation through brace yielding in tension and inelastic buckling of braces in compression. In capacity design of CBFs, the bracing members are selected based on their design compressive strength. Limits on brace overall slenderness and cross-section width-to-thickness ratios must also be satisfied to achieve ductile brace cyclic inelastic response. Once braces are selected, connections are designed for brace axial forces corresponding to the probable brace tensile and compressive resistances to prevent connection failure when the braces reach their capacities in the inelastic range. Similarly, beams and columns must resist gravity loads acting together with the lateral loads that are expected to develop when the braces reach their probable strengths. Implementation of this design approach may result in significant increases in design loads for brace connections, beams and columns in CBFs. In particular, the need to design brace connections for the expected brace tensile resistance has a major impact on costs in view of the difference that typically exists between the tensile yield resistance and design compressive strength of braces.

Introducing ductile fuses in bracing members has been proposed to control their probable axial resistances with the objective of reducing seismic design loads and, thereby, costs related to steel tonnage, shop fabrication and assembly on the site. This can be achieved by locally reducing the brace cross-section area (e.g., Rezai et al. 2000; Kassis and Tremblay 2008; Vincent 2008; Desjardins and Légeron 2010; Giugliano et al. 2011) or by introducing ductile components that yield in both compression and tension (e.g., Vayas and Thanopoulos 2005; Bonetti and Matamaros 2008; Gray et al. 2012). Brace overall buckling is eliminated when adopting the second approach, which typically leads to symmetrical hysteretic brace response, with no or limited strength degradation. A similar response can also be attained with a local reduction of the brace cross-section but the reduced brace segment must be properly confined to prevent local buckling and have sufficient length to prevent premature low-cycle fatigue due to cumulated cyclic plastic deformations. Alternatively, the reduced brace cross-section segment can be sized to yield in tension while remaining essentially elastic in compression. In that case, the fuse is only subjected to successive monotonically applied yield tension excursions, eliminating low-cycle fatigue limit states. Local buckling of the fuse segment must still be prevented, however, upon global buckling of the brace. This type of fuse can be implemented in HSS (tubular) or W-shape bracing members. W-shaped profiles, or I-shapes, typically exhibit higher ductility than their HSS counterpart and are available in sizes that can cover heavy applications. Vincent (2008) proposed a fuse for controlling the tension resistance of W-shape bracing members (Fig. 1). As shown, the material is removed at the flange to web intersection to minimize the impact on the brace flexural stiffness and buckling resistance. This paper presents a finite element study that was conducted on a typical W-shape brace with the proposed fuse detail. The design of the fuse is first illustrated for the sample brace. The cyclic buckling response of the brace is examined, including local buckling of the fuse region. Finite element analysis is then used to propose a local buckling restraining system (LBRS) for the fuse and develop a design procedure for the system.

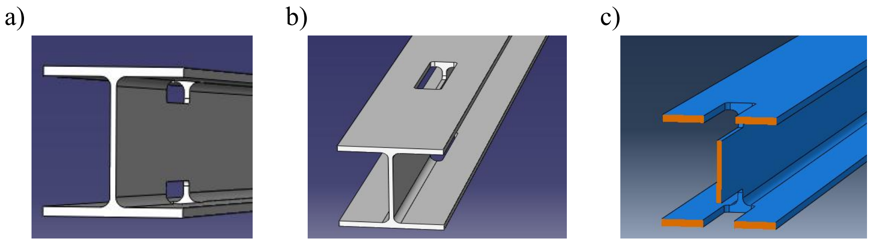

Figure 1. Ductile fuse for W-shape bracing members (Vincent 2008):
a) View from the side; b) View from above; and c) cross-section.

2. FUSE DESIGN AND PRELIMINARY ANALYSIS

2.1 Fuse Design

Bracing members are designed to resist axial tension and compression forces due to factored loads, C_f and T_f, respectively. In CSA-S16 standard (CSA 2009), the factored tensile and compressive resistances are respectively given by:

$$T_r = \phi A F_y \tag{1.1}$$

$$C_r = \phi A F_y (1+\lambda^{2n})^{-1/n} \qquad with: \lambda = \sqrt{F_y / F_e} \;\; and \;\; F_e = \frac{\pi^2 EA}{\left(KL/r\right)^2} \tag{1.2}$$

In these expressions, ϕ is the resistance factor for steel ($\phi = 0.9$), A is the cross-section area of the bracing member, F_y is the minimum yield stress, λ is the non-dimensional brace slenderness, F_e is the elastic buckling stress, n = 1.34 for hot-rolled shapes, E is the steel Young's modulus, KL is the brace effective length, and r is the radius of gyration of the brace cross-section. In most applications, C_f is similar to or exceeds T_f. Strength design is then governed by resistance to compression. The selected brace section must also meet limits on brace overall slenderness and the width-to-thickness ratios of the cross-section elements. In the context of capacity design, the brace probable resistances are:

$$T_u = A R_y F_y \tag{1.3}$$

$$C_u = 1.2 A R_y F_y (1+\lambda^{2n})^{-1/n} \le A R_y F_y \qquad with: \lambda = \sqrt{R_y F_y / F_e} \tag{1.4}$$

where R_yF_y is the probable yield stress. For ASTM A992 steel used for W-shaped members, F_y = 345 MPa and R_yF_y = 385 MPa. The forces C_u and T_u can exceed by a large margin the design loads C_f and T_f. In particular, when compression resistance governs the design of the brace, the brace overstrength in tension corresponding to the difference between T_u and C_f may compel designers to oversize brace connections, beams and columns to ensure elastic response of these components. To minimize this difference, the brace cross-section area is locally reduced to A_F in the fuse region, A_F being determined to achieve a yield tensile resistance for the fuse, $T_{uF} = A_FR_yF_y$, equal to or slightly greater than the brace probable compressive resistance, C_u:

$$A_F R_y F_y \ge C_u \;\Rightarrow\; \frac{A_F}{A} \ge 1.2\,(1+\lambda^{2n})^{-1/n} \;\le 1.0 \;,\; with:\; \lambda = \sqrt{R_y F_y / F_e} \qquad (1.5)$$

The fuse length is determined such that the anticipated brace axial deformation, including inelastic effects, can be accommodated without exceeding an axial strain of 10% in the fuse.

2.2 Brace Studied

The bracing member studied is a W310x129 shape. The complete brace assembly with the fuses and end connections is illustrated in Fig. 2a. The brace connections are designed to trigger brace buckling about weak axis. The brace effective length, KL, taken as the c/c distance between the plastic hinges expected to develop in the gusset plates upon buckling, is equal to 6000 mm. The brace probable resistances are given in Table 1. Using R_yF_y = 385 MPa and equations 1.3 and 1.4, the value of T_u and C_u of the original brace without fuses are equal to 6353 and 4216 kN, respectively. Using equation 1.5, the minimum fuse cross-section area ratio A_f/A = 0.66. A ratio A_f/A = 0.70 was selected, which resulted in a probable fuse tensile resistance, T_{uF} = 4435 kN = 1.05 C_u, a reduction of 30% compared to T_u. Removal of the brace material in the fuse reduced the brace moment of inertia and the moment about weak axis by only 1% and 8%, respectively. As shown, fuses were used at both ends of the brace. By doing so, shorter fuse lengths are needed to accommodate the expected plastic strain, which makes the fuses less prone to local instability. Each fuse has a length of 340 mm, based on an expected brace extension of 2% of the 6000 mm brace length.

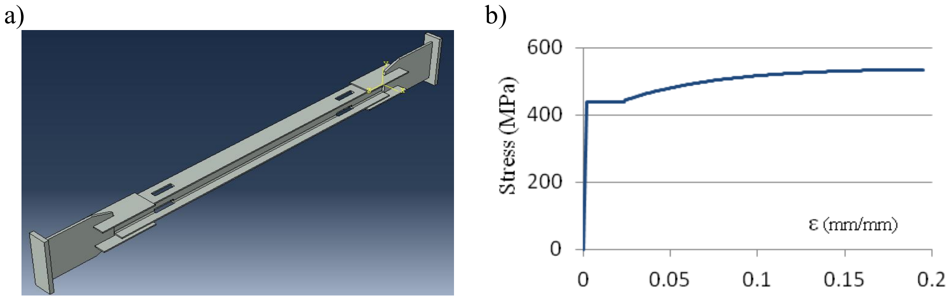

Figure 2. a) 3D Abaqus model of the brace; b) Steel properties from coupon test.

Table 1: Computed resistance of the brace

Predictive method	Condition		Without fuse A = 16500 mm^2	With the fuse A_f = 11520 mm^2
Design equations 1.3 to 1.5	R_yF_y = 385 MPa (CSA-S16-09)	T_u (kN)	6353	4435
		C_u (kN)	4216	4216[1]
	F_y = 439 MPa (mill-test)	T_u (kN)	7244	5057
		C_u (kN)	4461	4461[1]
Finite element analysis with unconfined fuse	F_y = 439 MPa (mill-test)	T_u (kN)	7406	5375
		C_u (kN)	4364	4110
Finite element analysis with confined fuse	F_y = 439 MPa (mill-test)	T_u (kN)	7406	5399
		C_u (kN)	4364	4227

Note: [1]Assumed unaltered by the presence of the fuse.

2.3 Numerical Model

The inelastic cyclic response of the bracing member was examined using the finite element analysis software ABAQUS (Dassault, 2010). The brace was modeled using 8-node brick elements. The brace shown in Fig. 2a corresponds to specimens to be tested in a subsequent phase of the project. The actual steel properties of the profile that will be used for the fabrication of the specimens were available and adopted for the analysis: E = 205000 MPa, F_y = 439 MPa, and F_u = 535 MPa.

In the numerical model, the stress-strain relationship in Fig. 2b was established with these values, assuming the following relation for the strain hardening range (St-Onge 2012):

$$\begin{aligned} &\sigma(\varepsilon) = F_y + (F_u - F_y) * (1 - C * (1 - r) * \exp(A * (\varepsilon - \varepsilon_{sh}))), \qquad \varepsilon_{sh} < \varepsilon < \varepsilon_u \\ &r = (\varepsilon - \varepsilon_{sh}) / (\varepsilon_u - \varepsilon_{sh}), \text{ with} : C = 0.96 \text{ and } A = -13 \text{ for I} - \text{shapes.} \end{aligned} \tag{2.1}$$

In this expression, $\varepsilon_y = F_y/E = 0.214\%$, $\varepsilon_{sh} = 10\times\varepsilon_y = 2.14\%$, $\varepsilon_u = 90\times\varepsilon_y = 19.3\%$, and $\varepsilon_{rupt} = 23.7\%$, where ε_{sh} is the strain at onset of strain hardening, ε_u is the strain when the stress reaches F_u, and ε_{rupt} is the strain at rupture. The brace resistances determined with the measured yield strength (F_y = 439 MPa) are also given in Table 1. Interestingly, the fuse axial resistance still exceeds the brace probable compressive strength, as both resistances are based on the same yield strength, indicating that the intended behaviour of the fuse-brace assembly will be achieved, with brace overall buckling occurring prior to fuse yielding in compression, regardless of the actual brace yield strength.

In the numerical model, an initial brace out-of-straightness of 1/1000 of the brace length was specified; however, for simplicity, residual stresses were not modelled. In the analysis, the brace was subjected to a cyclic displacement sequence with amplitudes corresponding to a fraction of the 6000 mm brace length: one cycle with an amplitude of 1.35% of the brace length and one cycle with an amplitude of 2.7% of the brace length. The displacement protocol started in compression.

2.3 Analysis Results

The behavior of the bracing member with the unconfined fuse is illustrated in Fig. 3. The axial load-axial deformation responses without and with the fuse is shown in Fig. 4. The maximum compression and tension resistances reached in the analyses are given in Table 1.

In Fig. 3a, local buckling of the remaining portions of the brace flanges in the fuses occurred early in the first compression phase, without overall buckling of the brace, forming a three-hinge mechanism with two hinges in the gusset plates and one hinge in one of the fuses. In Fig. 4, the presence of the fuse has no significant effect on the ultimate compressive strength, likely because the brace flexural stiffness was only marginally affected by the presence of the fuses. In Table 1, the peak compressive strengths with and without the fuses are close to each other, 4110 kN *vs* 4363 kN, i.e., 6% reduction. They are also close to the 4461 kN prediction from equation 1.4. However, in Fig. 4, the fuse-provided brace shows a steeper degradation of the brace compressive strength after local buckling in the fuses. This more pronounced strength degradation is attributed to fuse local buckling and the loss of resistance was not recovered in the subsequent loading cycle.

During the following tension phase, a reduction of 30% of the tensile strength is noticed compared to the original brace without fuse, as intended in design. In Table 1, the peak tension force in the analysis slightly exceeds the design prediction (5375 kN *vs* 5057 kN) due to strain hardening of steel in the fuses. This behavior can be distinguished in Fig. 4, at the end of the first excursion in tension. Necking in the fuse initiated in that same excursion, at a brace elongation of 56 mm (0.93% of the brace length), which is less than half the brace deformation at which F_u was expected to develop in the fuses, according to the measured steel properties (0.193 x 340 mm x 2 = 131mm). This difference is attributed to the fuse local buckling response which induced large plastic strains before the tension phase, thus reducing the ductility of the fuse and, thereby, the brace ductility in tension.

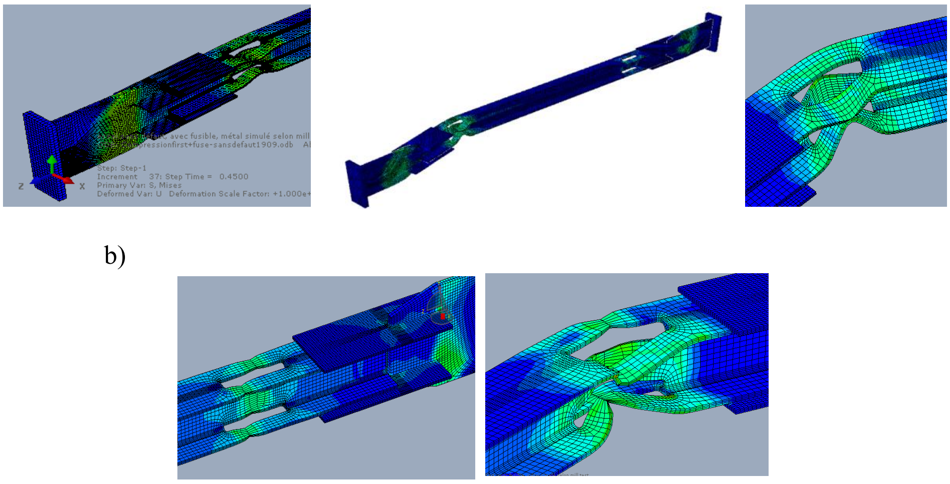

Figure 3. Inelastic response of the unconfined brace fuses: a) Local buckling response in the first compression excursion; b) Necking in tension (left) and buckling in compression (right) in the second cycle.

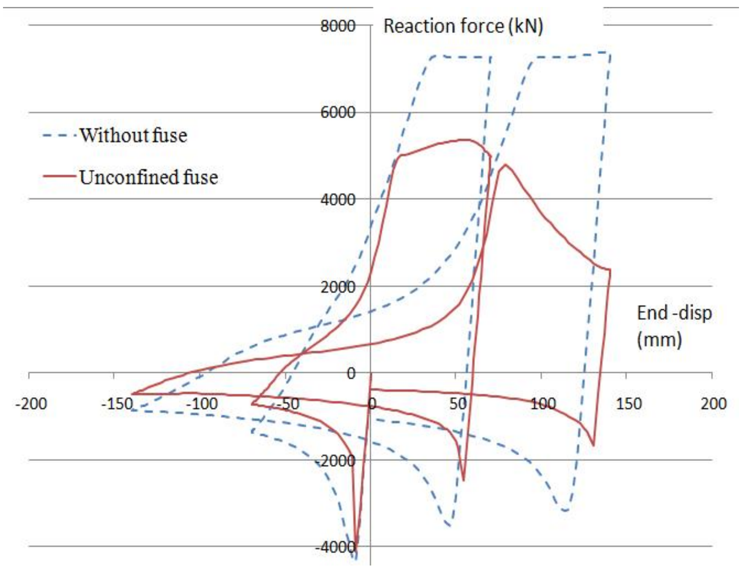

Figure 4. Axial load-deformation response of the brace without fuse and with unconfined fuse.

3. BRACE RESPONSE WITH CONFINED FUSES

3.1 Proposed Fuse Local Buckling Restraining Mechanism

The preliminary finite analysis of the brace with unconfined fuses clearly showed that brace fuses have to be protected against local buckling, mainly because the brace flange portions in the fuse are unsupported. A cost-effective local buckling restraining system (LBRS) was developed that could be easily fabricated and installed in the shop, prior to shipping the braces to the construction site. The system includes two cold-formed channels that support the brace web from either side and the brace flanges from the inner side. With cold-formed C sections, the flange-to-web transition of the brace cross-section can be easily accommodated by selecting a suitable bending radius. To prevent outward local buckling of the brace flanges, cover (outer) plates are bolted to the channels. As shown in Fig. 5, the width of these outer plates and the width of the flanges of the channels are selected so that both exceed the brace flange width, allowing bolting of the outer plates to the channels along the tips of the brace flanges. Splice plates are used in this connection, which also provide lateral support to the brace flange segments in the fuse. The fuse LBRS can slip longitudinally with respect to the brace so that it does not attract any axial force. The LBRS exceeds the fuse length by one half the brace flange width on either side of the fuse, plus the expected fuse extension length. Half the flange width is the length required for additional axial stresses due to the bending moments developing in the brace flanges upon brace buckling to reduce to zero. This aspect is discussed further later.

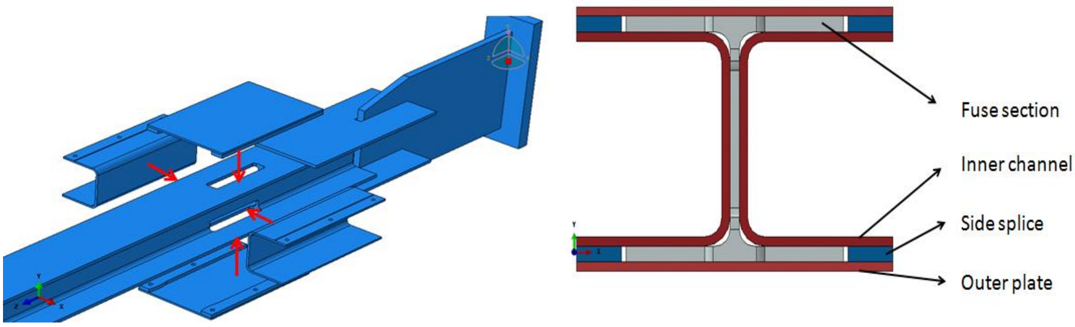

Figure 5. Proposed fuse local buckling restraining mechanism.

3.2 Analysis Results

Extensive finite element analysis was performed to verify the performance of the proposed brace fuse local buckling restraining system. The same material properties and imposed displacement protocol were considered in these analyses. In the study, the influence of several parameters on the behaviour of the fuse restraining system response was investigated, including the thickness of the channels, the thickness of the outer plates, and the spacing of the bolts connecting these two components. The influence of gaps between the LBRS components due to the permissible variations in cross-section dimensions for the brace W-shape was also examined.

In Fig. 6a, the axial load-deformation responses obtained using a numerical model with perfect fit between the brace cross-section and the LBRS components are presented for various outer plate thicknesses. As shown, the behaviour is nearly the same for all plate thicknesses considered. Only the brace with the 6 mm thick outer plates lost its compressive strength during the second cycle. All braces with the confined fuses exhibited superior post-buckling resistance compared to the same brace without the fuse LBRS. The confined fuses also permitted to reach higher brace ductility compared to the brace with unconfined fuses, with necking developing at an extension of 90 mm, instead of 56 mm, that is to say 60% gain.

Figure 7a shows the brace buckled shape without gaps between the LBRS and the brace and 12 mm thick outer plates. In this analysis, the outer plates were connected (welded) to the brace flanges along their edge closest to the gusset plates. As shown, overall buckling developed with a plastic hinge forming at the brace mid-length, without damage to the fuses. In Fig. 7b, the analysis was rerun without this welded connection. The LBRS slipped during the cyclic response, which detrimentally affected its effectiveness: local buckling of the fuse was observed when removing the outer plates from the model at the end of the analysis. This indicates that the restraining system must be longitudinally attached to the brace at one of its ends, preferably at the outer ends.

Figure 8 shows three gap conditions resulting from permissible variations in the brace sectional dimensions. Responses in Fig. 6b indicate that a small (1 mm) gap between the fuse and the LBRS is acceptable with regards to the ductility. When the system is not tightly maintained on both sides of the web, it is observed, however, that the fuse tends to lose both its tensile and post-buckling capacity.

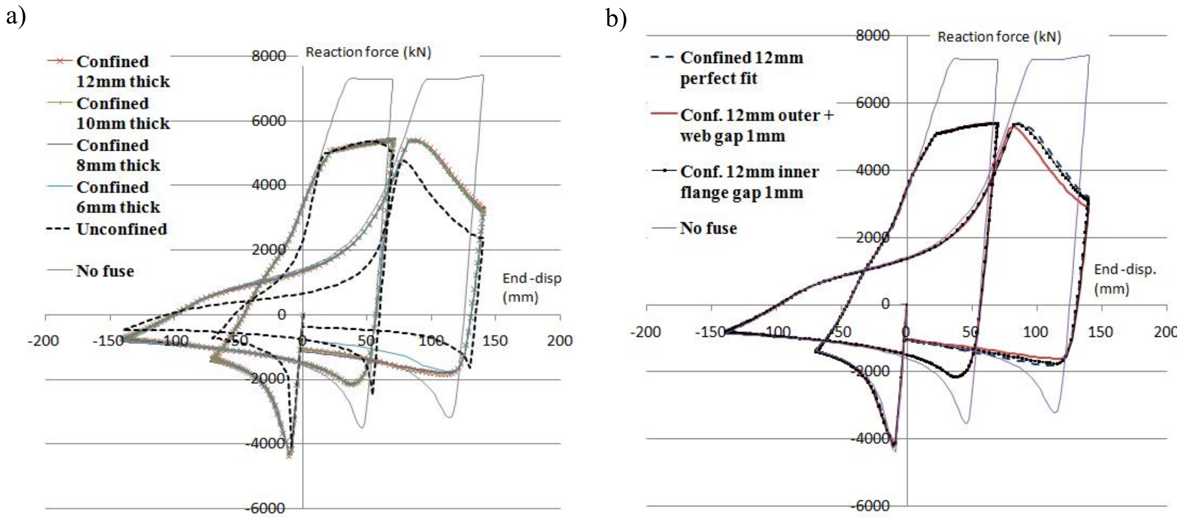

Figure 6. Axial load-deformation response of the brace with confined fuses: a) Influence of the thickness of the outer plates (fuse with perfect fit); b) Influence of gaps between components.

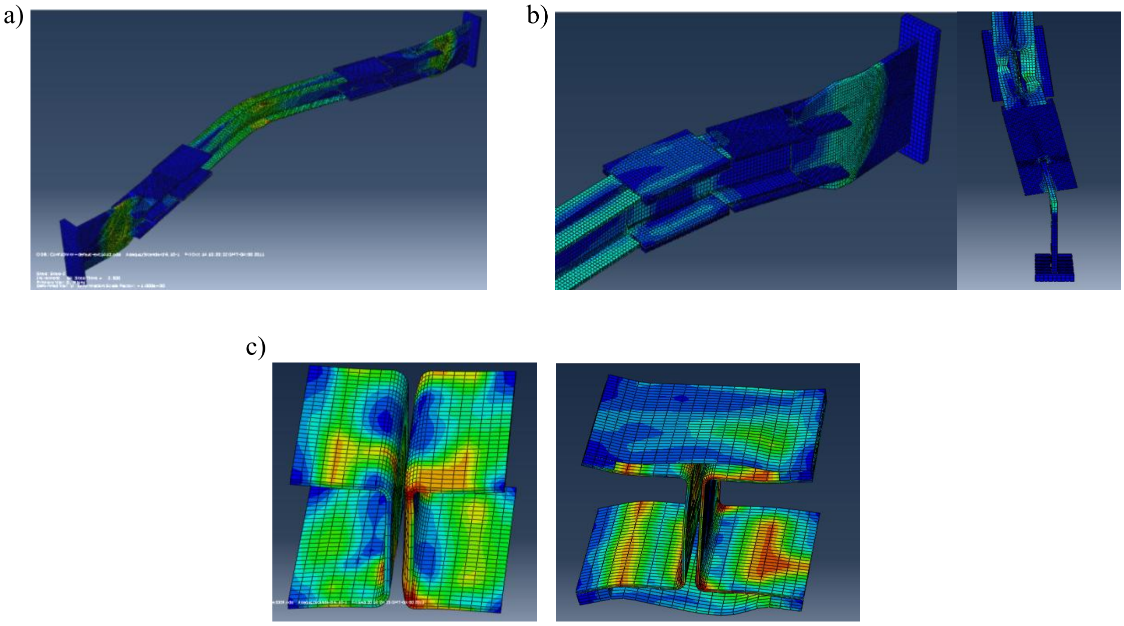

Figure 7. a) Brace overall buckling in the first cycle with 12 mm welded outer plates and no gaps; b) Local buckling of the fuse due to slippage of the LBRS ; and c) Von Mises stresses due to flexure in the channels and deformation of the weak 6mm-thick LBRS upon second occurrence of buckling.

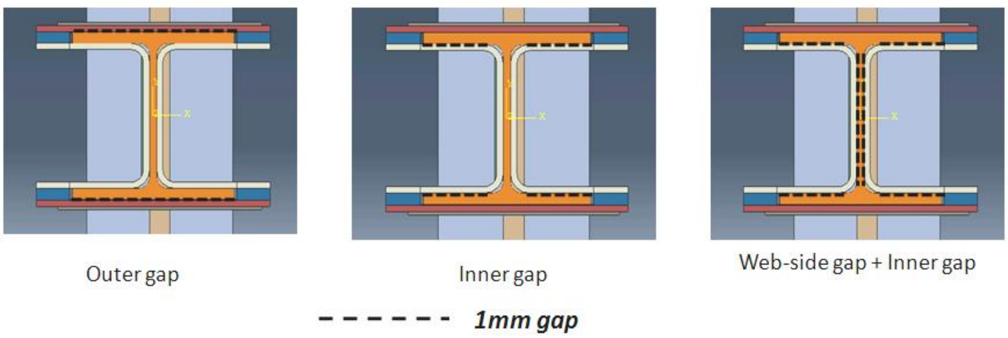

Figure 8. Gap conditions considered due to variations in cross-section dimensions

In Fig. 7c, local buckling of the fuse was observed when a gap was left between the brace and channel flanges. In that case, the 6 mm thick outer plates did not have sufficient out-of-plane flexural stiffness to constraint outward local buckling of the brace flanges. Upon brace overall weak axis buckling, the axial compression demand on the brace flange segments located on the intrados is amplified by the flexural demand on the brace. Initial buckling of that brace flange segment due to the gap was accentuated when overall brace buckling occurred, inducing large outward forces against the outer plates. The fuse in this example was so damaged that it could not withstand any additional cycle.

3.3 Further Improvements to the Fuse Local Buckling Restraining System

The analyses showed that the performance of the fuse-brace assembly could be affected when a gap was present between the webs of the brace and the cold-formed channels. To mitigate this effect, it was decided to bolt the two channels together through the brace web at each end of the fuse, as illustrated in Fig. 9a. Slotted holes are used in the brace web to allow for the fuse to axially deforming without imposing axial loads in the channels.

Upon inelastic buckling, the brace is subjected to bending moments about weak axis resulting from the action of the axial force on the laterally deformed brace. The moment is maximum at the brace mid-length, where it is equal to the plastic moment capacity of the brace, as reduced by the axial force. It decreases towards the two brace ends to reach nearly zero at the plastic hinges forming in the gusset plates. The analyses revealed that these bending moments impose additional axial stresses in the fuses, thereby promoting local buckling. The LBRS can also be used to reduce the bending moment in the fuses by taking advantage of the flexural stiffness and strength of the channels and outer plates. In Fig. 9b, the channels being bolted together through their webs can resist part of the bending moment through clamping action against the web of the brace. This induces tension forces in the web bolts. Bending moment in the fuse is also relieved by the outer plates and the side splice plates, as illustrated in Fig. 10. The outer plates are welded onto the brace flanges along their outermost edges with regards to the end connections. They can then attract portion of the bending moment through transverse contact bearing of the brace flanges against the splice plates at the opposite fuse ends.

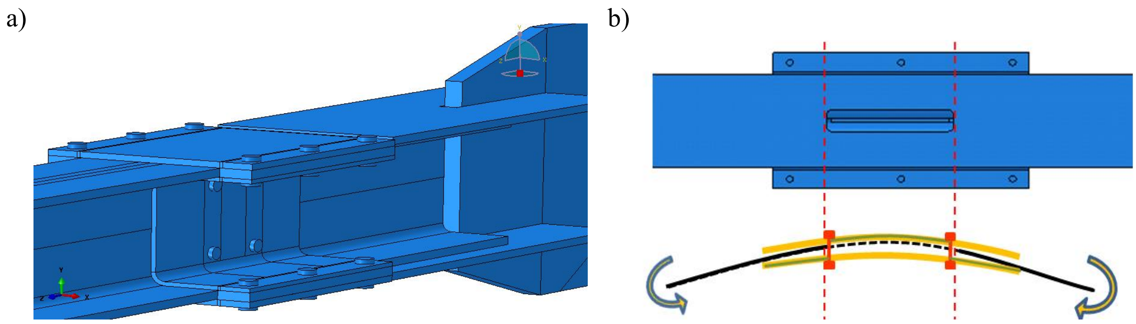

Figure 9. a) Channel webs bolted together through the brace web; b) Channels resisting bending moments.

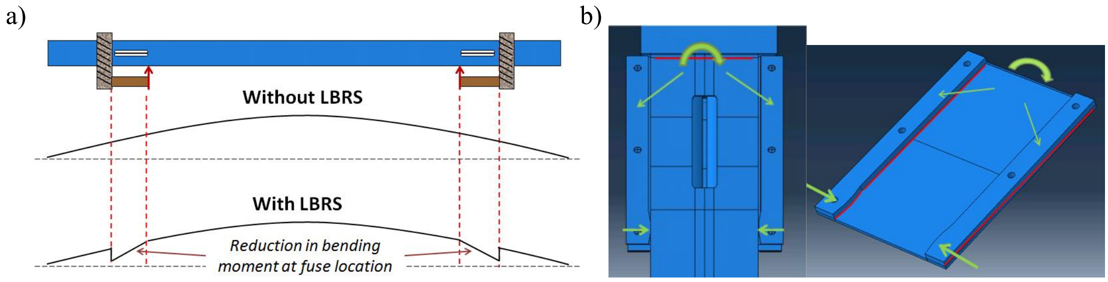

Figure 10. a) Brace plan view with bending moment demand on the brace without and with LBRS; b) Moment resisted by the outer plates by the welded connection and transverse contact bearing.

4. DESIGN PROCEDURE FOR THE FUSE AND THE FUSE LBRS

The numerical study showed that the main role of the LBRS is to avoid local buckling of the fuse by relieving it from a large portion of the bending moment acting in the brace upon brace buckling in compression. However, the tendency for local buckling of the fuse elements will still exist and the LBRS components must also be provided with sufficient strength and stiffness to mitigate this stability phenomenon and resist the resulting out-of-plane bearing and punching forces developing between the components. A diagram summarizing the design procedure for the fuse and the fuse LBRS is presented in Fig. 11. Once the brace member is selected, the required fuse cross-section area and length are first determined. The bending moment diagram along the brace can also be determined based on the brace properties and the expected brace axial deformation in compression.

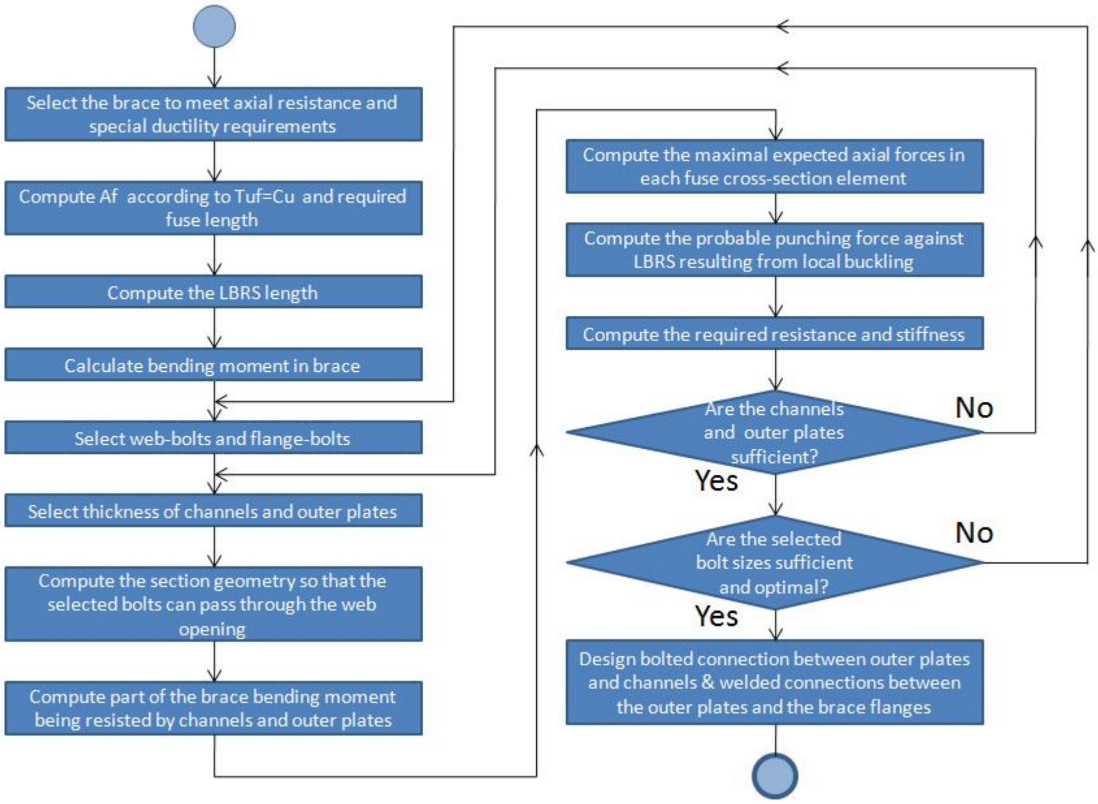

Figure 11. Overview of the design procedure for the fuse and the fuse local buckling restraining system

An iterative process is then initiated to select the outer plates, the channels and the bolts connecting these components. In particular, the web connecting bolts pass through the part of the web that is cut to obtain the fuse, meaning that the depth of the web cut depends on the diameter of these bolts. This bolt diameter depends on the tension forces in the bolts which, in turn, depend on the amplitude of the brace bending moments resisted by the channels. The portion of the bending moment resisted by the channels and the outer plates depends on the relative flexural stiffness of these components, with respect to each other and with respect to the brace flexural stiffness. The number, size and spacing of the bolts connecting the outer plates to the channels along the edges of the brace flanges vary with the brace bending moment resisted by the outer plates as well as with the outward normal forces that develop between the brace flange elements in the fuse and the outer plates. The welds connecting the outer plates to the brace flanges are designed for the bending moments attracted by the outer plates.

In view of the interdependency between all these design parameters, several iterations may be required before an optimal solution is achieved. Therefore, the design process in Fig. 11 has been automated using an Excel based Visual Basic application.

CONCLUSION

The cyclic inelastic response of a bracing member detailed with a ductile fuse proposed to reduce tension forces transmitted by heavy W-shape braces to seismic force resisting systems has been examined through detailed finite element analysis. This fuse detail should allow for significant savings when applying capacity design principles in the seismic design of steel braced frame structures.

The numerical study showed that local buckling is likely to form in the proposed brace fuse when the brace is subjected to compression axial forces. Fuse local buckling is accentuated by the bending moment that develops in the brace upon overall buckling. A local buckling restraining system has been proposed to mitigate this response and its performance has been verified through additional finite element analysis. The system comprises back-to-back cold-formed channels and outer plates. These components and their connections must be designed to resist local buckling effects. They must also resist a portion of the brace bending moment acting at the fuse location. A design procedure has been proposed to enable a systematic sizing of the fuse and its local buckling restraining system.

The subsequent phases of this project will comprise full-scale quasi-static cyclic testing of the sample brace examined in this study to finalize the validation of the concept and the design procedure. Parametric studies will then be conducted to assess the effectiveness of the fuse detail for various brace sizes and brace slenderness. The increase in costs resulting from the fabrication of the fuse detail has to be set off by the savings on the brace connections and the rest of the seismic force resisting system. A comparative study will also be performed for a prototype high-rise steel building designed with bracing members with and without the proposed fuse detail.

ACKNOWLEDGMENTS

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