Post-seismic damage evaluation: A probability-based global damage index

S. Jerez
Escuela Colombiana de Ingeniería, ECI, Bogotá, Colombia
Université Paris-Est, Laboratoire MSME UMR 8208 CNRS, France

A. Mebarki
Université Paris-Est, Laboratoire MSME UMR 8208 CNRS, France

M. Boukri
National Centre of Earthquake Engineering, CGS, Algiers, Algeria

D. Benouar
Université USTHB, Laboratoire LBE, Algiers, Algeria

SUMMARY:
This paper proposes a strategy for global damage evaluation on the basis of observed local damage. It relies on an assumed relationship between a structural damage index and the associated residual probability of failure. This approach takes into account the significance of the damage of each component on the whole damage by means of importance factors on a two-level analysis: a storey level prior to a building level. The efficiency of the importance factors, as well as the global performance of the proposed method, have been verified by means of a comparison with a mechanical approach. A good accordance between the estimations and the reference results was obtained. Accordingly, this strategy has the potential to be implemented as a part of a wider global damage evaluation framework, as a decision-making tool which permits of doing objective and accurate post-earthquake assessments.

Keywords: Global damage, Post-seismic evaluation, probability of damage

1. INTRODUCTION

For a particular system –a building for example– the common causes of damage are the various actions supported during its lifetime, applied suddenly or steadily with different intensities and time duration. Earthquakes are among the natural events which may cause extensive damage in buildings, in structural as well as non structural elements. The extent of this kind of damage does have a significant effect in the occupation in the event aftermath. Hence the importance of the post-seismic damage assessment, as the base of the decision-making about the buildings occupancy (in regard to possible aftershocks or for permanent occupation).

Damage, as well as damage measures have been defined in several ways. The structural seismic damage is defined, according to experimental studies, as the result of the large nonlinear deformations and the energy dissipation as well as the fatigue due to the cyclic loading inherent to earthquake actions (Park and Ang, 1985). This results in functional expressions as stiffness and strength deterioration affecting the lateral performance, as well as the possibility of bearing vertical loads, leading sometimes to the failure. The usual visible indicators of damage are the presence of permanent deformations and of patterns of cracks or deformation specific to the material and the particular stress state. Some measures have been proposed as a function of the remaining life span of the structure, of microstructure measures which account for real defaults in a representative volume and of variations in physical or in mechanical properties (Besson et al., 2001). Others have been expressed in terms of economical indices as a function of repair costs (Kappos et al., 1998) or have been related to residual probabilities of failure (Mebarki and Laribi, 2008). In any case, it is clear that establishing an objective and accurate measure of such a phenomenon is a hard task.

When dealing with post-seismic damage evaluation, the procedures may be classified into rapid methods, aiming at taking in-place decisions about occupancy and identifying potentially dangerous conditions, and detailed methods, for further investigation on the serviceability state of the building. Most rapid methods use evaluation forms which must be filled by trained staff on the basis of observed damages, apparent characteristics of the structure and pre-existent available information. Regularly, occupancy decisions are taken on the basis of a global damage category or index, obtained from damage states based on visual inspection and visible damage signs (cracks, buckling, etc.).

Among the rapid methods, an integrated methodology within a probabilistic framework was recently proposed to estimate global damage (Mebarki and Laribi, 2008). An overall damage index is computed from damage in structural as well as non structural components by means of the residual probability of failure. The methodology has three main ordered stages: i) the estimation of the residual probability of failure for each component (structural and non-structural), by means of a postulated relationship $P_f = f(D_c)$, which transforms damage categories D_c into residual probability of failure, making thus possible the establishment of a quantitative parameter; ii) the estimation of the overall probability of failure P_f, which is supposed to occur when either the structural or the non structural failure occurs, under the assumption of the failure events being statistically independent. And, iii) the determination of the global damage index on the basis of the global probability of failure already obtained, by means of the inverse of the relation obtained at the first stage. The results were compared with a database of experimental field evaluations corresponding to the 2003 Boumerdès Earthquake (Mw=6.8, Algeria) finding a good accordance. This approach has the advantage of transforming, in a simple way, qualitative measures of damage into a quantitative index. It may be implemented in the post-earthquake assessment process to be a support of the tagging tasks when performing forensic evaluations.

Thereby, inspired in this integrated probability approach, this paper proposes a probabilistic-based strategy for post-seismic evaluation of structural damage. The relationship between the probability of failure and damage is postulated as a function of the mechanical properties of the building. The whole strategy has been applied to three reinforced concrete frames, obtaining results which are in good accordance with those of a mechanical approach. It is intended for being a step forward on the implementation of a decision-making tool which permits of doing accurate assessments of global damage based on damage of components.

2. GLOBAL STRUCTURAL DAMAGE ASSESSMENT OF BUILDINGS

The strategy presented hereafter is also based on a relationship between residual probability of failure and damage. In its first version it is applicable only to structural components and the interactions of the building components are treated differently. Actually, it is limited to frame buildings which are considered first as arrangements of storeys, which are in turn composed of beams and columns. Thus, hereafter two levels of study will be referenced: the storey level, depending on beams and columns, and the building level, depending on storeys (Jerez, 2011).

2.1. Global Probability of Failure

First of all, an expression for the probability of failure of a system, as a function of the probabilities of failure of its components is required. For a building floor, such an expression requires a detailed inelastic study on the possible failure mechanisms. The two classic probabilistic combinations, series and parallel, might both be unrealistic. The series combination implies that the storey fails when any of the components fails. It is then too conservative and it does not consider that vertical components may be more important on the storey behaviour, for instance. The parallel combination involves the necessity of all the components failing to cause the system’s failure. It is hence non-conservative as the storey would have failed before the whole components reach their failure. Nevertheless, for the purposes of this study the conservative approach will be used, so that each storey of the building is considered as a series system of beams and columns.

Consequently, the failure of a storey k (E_k) may be expressed in terms of the events of failure at each general component (E_b and E_c for beams and columns). Likewise, the complementary event, is derived as shown in Eq. (2.2):

$$E_k = E_b \cup \mathrm{E_c} \tag{2.1}$$

$$\overline{E}_k = \overline{E}_b \cap \overline{\mathrm{E}}_\mathrm{c} \tag{2.2}$$

In the former relationship, E_c represents the failure of the columns in the storey, so E_c and its complementary event, are given by the following expressions in terms of each element:

$$E_c = E_{c,1} \cup E_{c,2} \cup ... E_{c,i} \cup ... \cup E_{c,Nc} \tag{2.3}$$

$$\overline{E}_c = \overline{E}_{c,1} \cap \overline{E}_{c,2} \cap ... \overline{E}_{c,i} \cap ... \cap \overline{E}_{c,Nc} \tag{2.4}$$

The index N_c represents the total number of columns in a storey. Under the assumption that events are statistically independent, the relationship between the probabilities of occurrence of these events may be expressed as:

$$P(\overline{E}_c) = P(\overline{E}_{c,1}) \times P(\overline{E}_{c,2}) \times ... P(\overline{E}_{c,i}) \times ... \times P(\overline{E}_{c,Nc}) \tag{2.5}$$

Now, taking *$P(E_c)$* simply as P_c and considering that $P(E_c) + P(\overline{E}_c) = 1$, the following relationship is obtained for the probability of failure of columns (and likewise that of beams):

$$(1 - P_c) = \prod_{i=1}^{Nc} (1 - P_{c,i}) \tag{2.6}$$

Thus, under the same assumption of independence of events, Eq. (2.2) leads to:

$$(1 - P_k) = (1 - P_b) \times (1 - P_c) \tag{2.7}$$

So that the probability of failure of a storey may be computed as:

$$P_k = 1 - \left\{ \prod_{i=1}^{Nb} (1 - P_{b,i}) \right\} \times \left\{ \prod_{j=1}^{Nc} (1 - P_{c,j}) \right\} \tag{2.8}$$

Under the same assumptions of the building being a series system of the *NS* storeys and the probabilistic independence of events, the probability of failure P_G of the whole system may be expressed as:

$$P_G = 1 - \prod_{k=1}^{NS} (1 - P_k) \tag{2.9}$$

2.2. Global Damage

2.2.1. Relationship between damage and residual probability of failure

Since damage always cause a degradation which may eventually lead to a failure, it is clear that there must exist a direct relationship between damage, D_n, and the probability of failure, P_n. Then, a new relationship between P_n and D_n is assumed here. It depends on a factor α_n, intended to account for the significance of the studied component on the global system behaviour:

$$(1-P_n)=(1-D_n)^{\alpha_n} \;\rightarrow\; P_n = 1-(1-D_n)^{\alpha_n} \tag{2.10}$$

P_n is the probability of failure of the n*th* element, D_n the damage index of the same element and α_n the importance factor, whose calculation is developed below. It may be observed that this equation fulfils the correspondence between probability of failure and damage. When there is no damage, P_n = 0, and when damage is considered as complete (D_n =1), P_n is equal to 1, regardless of the α_n value.

2.2.2. Global damage index at a storey-level and at a building level

From Eqs. (2.8) and (2.10), the damage of a storey may be estimated from the damage of the components as:

$$D_k = 1-\left\{\prod_{i=1}^{Nb}(1-D_{b,i})^{\alpha_{b,i}}\right\}\times\left\{\prod_{j=1}^{Nc}(1-D_{c,j})^{\alpha_{c,j}}\right\} \tag{2.11}$$

Subsequently, according to Eq. (2.9) and defining the index β_k as the indicator of the importance of each storey *k* on the global behaviour of the building, the global damage index D_G is obtained:

$$D_G = 1-\prod_{k=1}^{NS}(1-D_k)^{\beta_k} \tag{2.12}$$

2.2.3. Damage assessment at the storey-level: α_b and α_c factors

The seismic behaviour of a storey strongly depends on the strength, stiffness and ductility of each component. The proposed factors should provide a measure of the importance of beams and columns on the overall behaviour. Thus, the effect of each of these elements needs to be separated in some way. Since damage may have a significant effect on lateral stiffness, the influence of each element class (i.e. beam, column) on the lateral stiffness of the storey is considered here as indicator of the effect of the components local damage on the storey overall damage. Therefore, a relationship between the lateral stiffness of a storey and the relative stiffness of its components is required. Getting such an explicit, separate expression is not easy since beams and columns do interact. However, a relative influence of these components may be estimated considering that the lateral stiffness of a storey reaches its maximum value when beams are stiff enough to act as a fixed end of columns (Chopra, 2007). Then the maximum lateral stiffness $K_{L,k}^{\max}$ of a storey is computed as:

$$K_{L,k}^{\max} = \frac{12E}{L_c^{\,3}}\left(\sum_{j=1}^{NC} I_{c,j}\right) \tag{2.13}$$

Where L_c is the storey height (the column's length) and $I_{c,j}$ represents the second moment of area of each column *j* cross section. According to Eq. (2.13) lateral stiffness does not depend on the beam's length, as already stated by Chopra (2007). For this case, one may say that columns have their maximum contribution, hence the maximum importance. On the opposite side, when stiffness of beams is almost negligible, so that there is no any restriction to lateral displacements, the minimum lateral stiffness value tends to:

$$K_{L,k}^{\min} = \frac{3E}{L_c^{\,3}}\left(\sum_{j=1}^{NC} I_{c,j}\right) \tag{2.14}$$

For the latter case, lateral stiffness value also depends only on columns stiffness. But, columns contribution is the smallest possible so that the associated importance must be the minimum. Now, since the current lateral stiffness K_L for a given configuration ranges between these two bounds, the ratio between the real and the maximum stiffness may provide an estimate of the influence of columns on lateral stiffness, i.e. the α_c factor (Jerez, 2011; Mebarki et al., 2011):

$$\alpha_c = \frac{K_L}{K_{L,k}^{\max}} \tag{2.15}$$

Of course this is a rough estimate since the real influence is actually more complex, and depends on all the connected elements to the columns under study. Thereby, as the summation of the importance factors for all the considered components must be equal to 1 regardless of the chosen property, the influence factor of beams is:

$$\alpha_b = 1 - \alpha_c \tag{2.16}$$

Now, the lateral stiffness K_L may be estimated with variable levels of accuracy but for the purposes of the present proposal a simplified method is selected. Several simplified relationships exist as a function of the mechanical and geometrical properties of the components. In this case a version of the Wilbur's formulae has been used (Norris et al., 1991; Bazán and Meli, 2002).

2.2.4. Damage assessment at the building-level: β factor

Concerning the influence of the storey damage on the global damage, an importance factor β is proposed. It is proportional to the gravitational load carried by each storey. As it is already stated in other research works, such an index has the advantage of reflecting the risk and the consequences of the collapse of elements on lower storeys (Bracci et al., 1989; Jeong and Elnashai, 2006). Defining W_k as the gravitational load carried by the storey being studied, that index is computed as:

$$\beta_k = \frac{W_k}{\sum_{k=1}^{NS} W_k}, \quad \sum \beta_k = 1 \tag{2.17}$$

Now, all the parameters required to compute the global damage by means of Eq. (2.12) are defined. The individual importance factors and are computed by dividing the α_b and a_c factors by *NB* and *NC* respectively. This approach for the global damage index fulfils the following requirements:

- It ranges from 0 when no damage is present on any component up to 1 for complete damage.
- It considers the relative influence of each element over the whole damage index.
- It considers the importance of the capacity to carry gravity loads through the β factor, providing more participating weight to the lower storeys given its influence on the gravity load bearing system.

3. APPLICATIONS

Three frame buildings have been tested under different patterns and distributions of local damage. They are: a 4- and a 6-storey buildings designed for seismic loads typical of high seismic risk areas; and a 8-storey frame adapted from a study about seismic collapse safety in modern RC buildings (Haselton and Deierlein, 2007). An elevation view is displayed in Fig. 3.1.

In order to verify the accuracy of the strategy, a mechanical approach will be used as the reference method. It evaluates the global damage from the variation in the stiffness of the first-mode capacity spectrum obtained from a pushover analysis, as illustrated in Fig. 3.2. Given the definition of the n*th*-mode capacity spectrum's components, the elastic stiffness of this curve coincides with the eigenvalue associated to the n*th*-mode, i.e. the square of the n*th* vibration frequency. Such a method is often used for identification and localization of structural damage in experimental studies based on finite element model updating techniques (Simoen et al., 2010; DiPasquale and Cakmak, 1990; Nielsen et al., 1992).

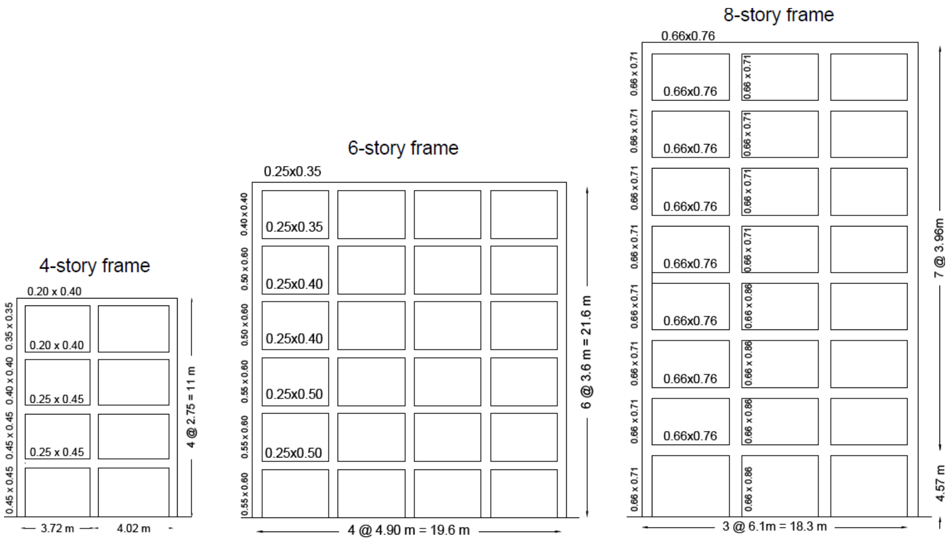


Figure 3.1. Elevation view of the three models

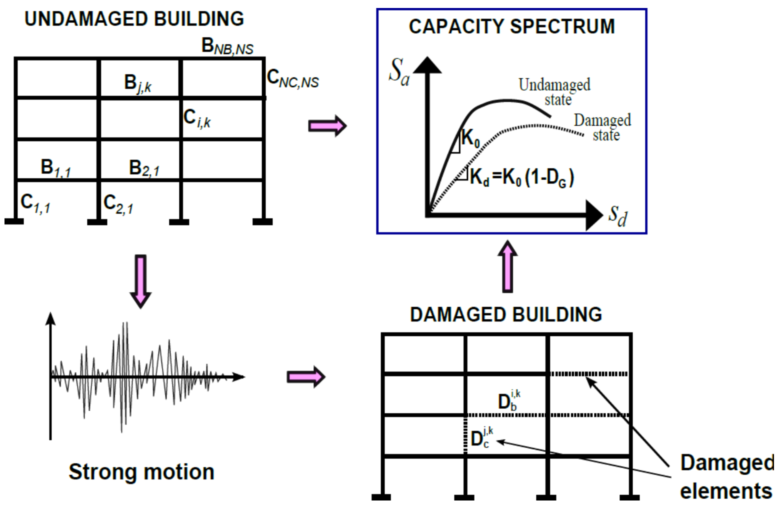


Figure 3.2. Evaluation of global damage through the variation in the capacity spectrum elastic stiffness

The overall procedure consists in computing an index D_G, as a function of the initial undamaged stiffness K_0 and the damaged stiffness K_d , according to:

$$D_G = 1 - \frac{K_d}{K_0} \tag{3.1}$$

The damage at each element, supposed to be known from the post-seismic inspection, is set directly on the mechanical model of the building. It is considered as isotropic and set as a scalar internal variable D (Besson et al., 2001), so that the relationship between moment and curvature in the elastic domain may be written as:

$$M = K_0 (1 - D) \phi \tag{3.2}$$

Where: M represents the bending moment, K_0 the elastic stiffness of the element and ϕ the curvature of the section. The moment-curvature relationship is approximated by a bilinear curve, hence described by an elastic-plastic model with strain hardening. This proposal considers that there is only coupling between elasticity and damage and no coupling between damage and strain hardening (see Fig. 3.3).

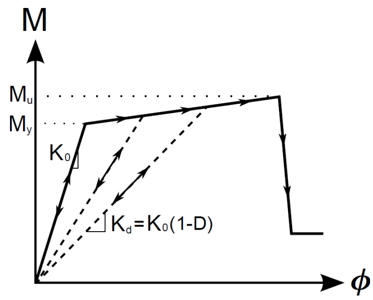


Figure 3.3. Nonlinear degrading model used to simulate damage on beams and columns

3.1. Results

Several analyses have been performed for evaluating the efficiency of α and β factors. Two damage levels have been studied: a low – medium level (LM), supposed to range between 0.10 and 0.40 and a medium-high (MH) level ranging between 0.4 and 0.8. Of course, damage levels of 0.8 are quite high so that in real situations they might be considered as complete damage. However, they have been used herein for evaluation purposes. Several patterns of damage have been investigated in order to have a significant insight on the accuracy and limits of the proposed approach. The most important results are presented hereafter.

For evaluating the β factor, which reflects the effect of damage at each storey on the global damage, uniform, concentrated damage patterns have been assigned separately to each storey. Figures (3.4) and (3.5) show the results for the 4- and the 6- storey frames. Every single point on each graph represents an analysis, i.e. the calculated (D_{CALC} –by the mechanical approach) or the estimated global damage (D_{EST} –by the proposed method) when damage is concentrated at a given storey.

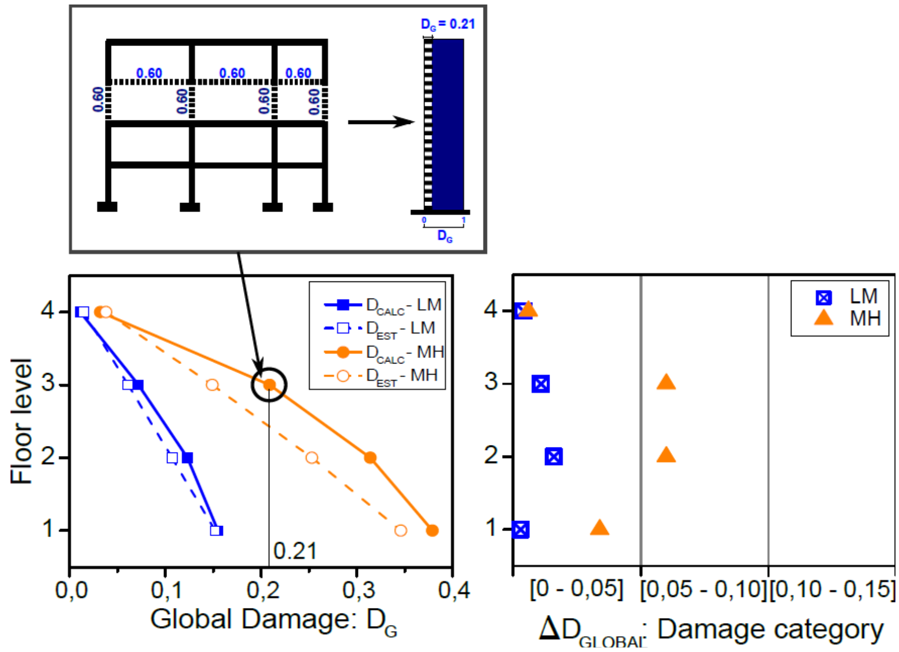


Figure 3.4. Results of global damage and damage category change for the 4-storey building

For low-medium level of damage, there is a good accordance with analytical results while for medium-high level it is observed that accuracy decreases as damage level increases. Considering the differences between calculated and estimated damage (ΔD_{GLOBAL}), one could observe that for most cases they do not exceed one category, defining here a category as a range of 0.05 of damage value.

These results are acceptable, considering that damage is concentrated only in one storey, which may not be a common configuration in a real situation. In fact, as it is observed afterwards, better results are found when damage pattern is more uniform.

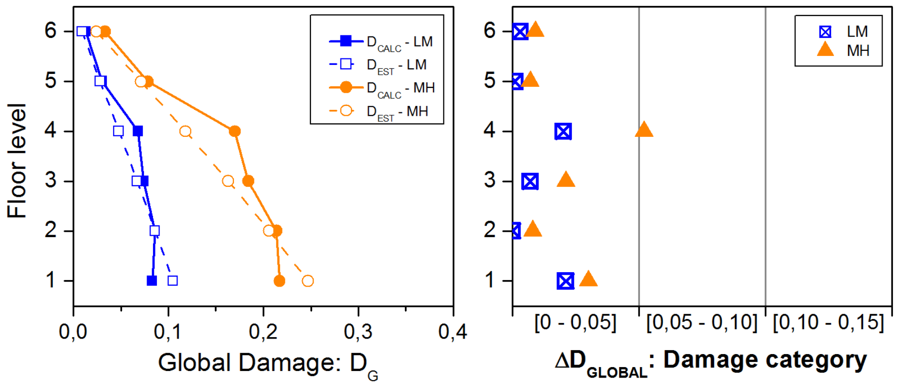


Figure 3.5. Results of global damage and damage category change for the 6-storey building

With the aim of studying the proposed approach when compared to a real distribution of damage, the studied buildings have been subjected to a set of real ground motions (see Table 3.1) in order to obtain more realistic patterns of damage. Some of them have been scaled with the aim of producing a wide range of damage magnitudes. Once the results obtained, local damage at each element was computed from peak values of the curvature, according to the model shown in Fig. 3.3. In fact, it was supposed that the peak deformation reached during the earthquake would be the "yield" deformation of the damaged model. Therefore, the following expression has been derived in order to compute the local damage at each component:

$$D_{LOCAL} = (\phi_{peak} - \phi_y)(1-b)/\phi_{peak} \qquad (3.3)$$

Where: ϕ_{peak} is the maximum curvature reached during the earthquake; ϕ_y is the yielding curvature of the undamaged component; and b represents the post-yield slope. Thus, the obtained damage patterns were taken as given patterns and were investigated within the framework of the proposed method. A wide range of damage arrangements has been observed according to the particular characteristics of the ground motions and the buildings. In the case of the 4-storey building for instance, the patterns are usually unsymmetrical and damage decreases along the building height, as shown in Table 3.2:

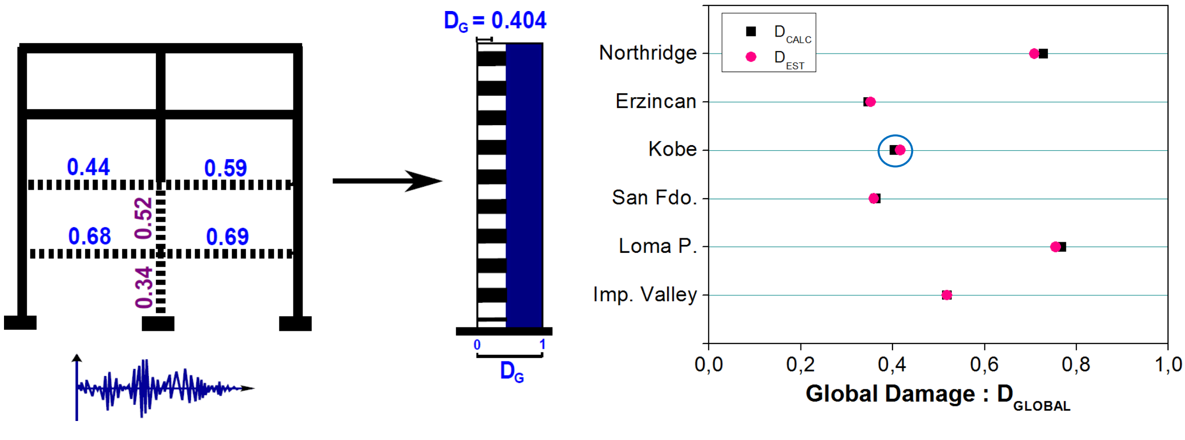
Table 3.1. Ground motion ensemble

N	Earthquake	Year	Station (Component)	Mw	PGA (g)
1	Imperial Valley	1940	El centro (180)	7.0	0.31
2	Loma Prieta	1989	Corralitos (000)	6.9	0.64
3	San Fernando	1971	Castaic (021)	6.6	0.32
4	Kobe	1995	Takaratzuka (000)	6.9	0.69
5	Erzincan	1992	Erzincan (NS)	6.9	0.52
6	Northridge	1994	Canyon country (270)	6.7	0.48

Table 3.2. Distributions of damage for the 4-storey building under Erzincan and Loma Prieta earthquakes.

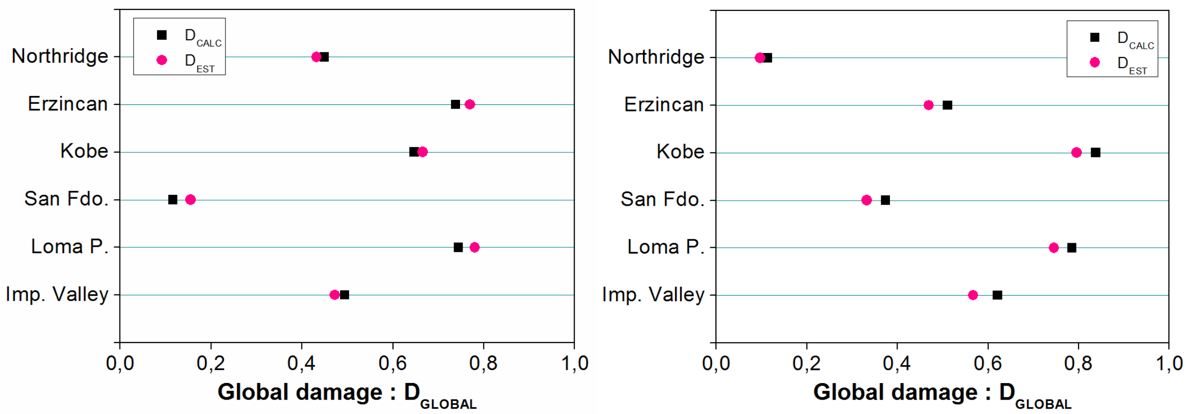
GM	Damage						GM	Damage					
	Story	Beam		Col				Story	Beam		Col		
		1	2	1	2	3			1	2	1	2	3
Erzincan	1	0,68	0,51	0	0,32	0	Loma Prieta	1	0,8	0,84	0,61	0,78	0,62
	2	0,59	0,18	0	0,48	0		2	0,85	0,82	0	0,86	0
	3	0	0	0	0,21	0		3	0,79	0,66	0,47	0,89	0,35
	4	0	0	0	0	0		4	0	0	0	0	0

When applying the proposed method for all these patterns taken as known patterns, the results for the estimated global damage index are very close to the analytical results as shown in Figs. 3.6 and 3.7, which confirms the suitability of the proposed α and β factors.



(a) (b)

Figure 3.6. (a) Damage pattern for the 4-storey building under Kobe earthquake and (b) Results of global damage for the 4-storey building under the ensemble of ground motions



(a) (b)

Figure 3.7. Results of global damage under the ensemble of ground motions for (a) the 6-storey building and (b) the 8-storey building

4. CONCLUSIONS

A probability-based strategy has been developed with the aim of computing global damage indices on the basis of given component damage indices. It relies on an assumed relationship between structural damage and residual probability of failure, considering at the same time the significance of the damage of each component on the whole damage by means of importance factors.

In the case of RC framed buildings the three proposed importance factors have proven to be suitable according to the obtained results for damage arrangements which allow identifying the individual influence of each factor. Differences between calculated and estimated global indices do not exceed, in most cases, one category of damage: very small differences between the two values of global damage are found. Concerning the general efficiency of the method, a number of damage patterns obtained from the non linear time history analyses have been tested. A good accordance between estimations and reference results was obtained, confirming the suitability of this approach and its potential for being part of a decision-making tool which helps in performing fairly accurate post-seismic assessments of global damage based on damage of components.

Further validation with experimental or field results is required to this strategy in order to be of general application, since analytical approaches are always subjected to uncertainties and usually do not account for all the phenomena inherent to this kind of task.

AKCNOWLEDGEMENT

Part of this research was supported by the research funds of the Escuela Colombiana de Ingeniería (ECI), Bogotá, Colombia. This support is gratefully acknowledged.

REFERENCES

Bazán, E. and Meli, R. (2002). Diseño sísmico de edificios, Editorial Limusa S.A. de C.V., Mexico.

Besson, J., Cailletaud, G., Chaboche, J.L. and Forest, S. (2001). Mécanique non linéaire des matériaux, Hermes Sciences Publications, Paris.

Bracci, J.M., Reinhorn, A.M., Mander, J.B. and Kunnath, S.K. (1989). Deterministic Model for Seismic Damage Evaluation of Reinforced Concrete Structures, NCEER-89-0033, N. C. f. E. E. Research, Bufalo, NY.

Chopra, A.K. (2007). Dynamics of Structures: Theory and Applications to Earthquake Engineering, Pearson Prentice Hall, Upper Saddle River, NJ.

DiPasquale, E. and Cakmak, A.S. (1990). Detection of seismic structural damage using parameter-based global damage indices. *Probabilistic Engineering Mechanics*. **5:2**, 60-65.

Haselton, C.B. and Deierlein, G.G. (2007). Assessing Seismic Collapse Safety of Modern Reinforced Concrete Moment-Frame Buildings, PEER 2007/08, P. E. E. R. Center, Berkeley, CA.

Jeong, S.-H. and Elnashai, A.S. (2006). New Three-Dimensional Damage Index for RC Buildings with Planar Irregularities. *Journal of Structural Engineering*. **132:9**, 1482-1490.

Jerez, S. (2011). Seismic vulnerability of buildings: Response and damage assessment. Phd Thesis. Université Paris-Est.

Kappos, A.J., Stylianidis, K.C. and Pitilakis, K. (1998). Development of Seismic Risk Scenarios Based on a Hybrid Method of Vulnerability Assessment. *Natural Hazards*. **17:2**, 177-192.

Mebarki, A., Jerez, S. and Laribi, A. (2011). Seismic vulnerability of buildings: Observed damages and Residual probability of failure. . *2011 World Congress on Advances in Structural Engineering and Mechanics, ASEM'11+*.

Mebarki, A. and Laribi, A. (2008). "Evaluation post-sismique des dommages structuraux: méthodologie probabiliste", Risques Naturels et technologiques, A. Mebarki, Genatios, C. and Lafuente, M., Paris, Presses de l'Ecole Nationale des Ponts et Chaussées, 155-172.

Nielsen, S.R.K., Köylüog, lu, H.U. and Çalmak, A.S. (1992). One and two-dimensional maximum softening damage indicators for reinforced concrete structures under seismic excitation. *Soil Dynamics and Earthquake Engineering*. **11:8**, 435-443.

Norris, C.H., Wilbur, J.B. and Utku, S. (1991). Elementary Structural Analysis, Mc. Graw Hill,

Park, Y.J. and Ang, A.H.S. (1985). Mechanistic seismic damage model for reinforced-concrete. *Journal of Structural Engineering*. **111:4**, 722-739.

Simoen, E., Lombaert, G., Reynders, E. and De Roeck, G. (2010). Uncertainty quantification in the vibration-based damage assessment of a reinforced concrete beam. *International Conference on Noise and Vibration Engineering, ISMA 2010*. **Vol.** 1**:** 4977-4992.