

Seismic Protection Design of Nonlinear Structures Using Hybrid Simulation



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SUMMARY:

Seismic protective devices in the form of isolation and supplemental damping devices can be effective means in mitigating the seismic responses of structures. The required stiffness and damping values to achieve optimal performance depend on the characteristics of structures and ground motions. Instead of using actively controlled devices, one can select the passive devices to roughly mimic the behaviour exhibited by active devices. In order to achieve this, the structural control of structures with controllable devices needs to be obtained first. However, the structural control of nonlinear structures can not be easily conducted due to the difficulties in realistic modelling of complex structures and in implementing the control algorithms within the typical finite element programs. Utilizing the hybrid simulation, the response of a complex nonlinear structure can be obtained by integrating various numerical and physical components or numerical components in different computational platforms. Based on this methodology, this study will investigate a seismically protected nonlinear building model where the structure is realistically modelled in OpenSees while the seismic protective devices and the control algorithm are implemented in Matlab. Through hybrid simulation, the responses and structural control of the building are conducted. An equivalent passive stiffness and damping parameter set is obtained for this nonlinear building and it shows much improved structural performance. An experimental program is also in planning to verify the numerical results and structural performance obtained through hybrid simulation.

Keywords: structural control, hybrid simulation, nonlinear structures

1. INTRODUCTION

In light of damages of structures observed in past earthquakes, seismic protection strategies are needed to retrofit existing structures or to improve the design of new structures. In addition to the traditional strengthening/stiffening method, structural control technology can be implemented to improve the performance of structures, e.g. adding damping to structure to reduce drift and deformations during the seismic response. Structural control technologies are typically classified into the passive, active/hybrid and semi-active systems (Housner et al. 1997).

To date, a number of passive systems have been implemented in buildings and other civil engineering structures. Two of the most popular approaches are to use supplemental energy dissipation or base isolation for vibration reduction and energy dissipation. Most passive seismic protective systems are based on the general idea of increasing the damping of structures (Constantinou et al. 1998). Because

ground motions are stochastic in nature, passive systems might have a limited range of effectiveness. Active control systems are more efficient in this regard. However, except for protecting small or light weight objects, such as aerospace equipments, the solution on how to deliver large active counter forces is needed before wide use of this technology in civil structures. Semi-active systems, such as magnetorheological (MR) dampers, include smart mechanical and material components whose physical parameters can be modified in real-time through switching or on-off operations (Spencer and Nagarajaiah 2003). Due to the variability and use of passive forces, semi-active control is becoming a promising technology of seismic hazard mitigation for civil engineering structures.

In order to optimally select the stiffness and damping values for control devices in design, the structural control of structures with controllable devices needs to be performed first. However, for structures exhibiting nonlinearity, the structural control can not be easily conducted within the typical finite element analysis program. Although current FEM programs typically have various elements of modelling complex nonlinear structural components and control devices, there is no well established approach to apply control algorithms in most existing commercial codes. Instead, the structural controls were often conducted on simplified structural models that can be generated in the same simulation platform for control algorithms. For example, efforts have been made to develop the benchmark problems for several structures to allow for a platform to compare various control strategies (Agrawal et al. 2009; Ohtori et al. 2004). Nevertheless, the ability to use advanced and realistic structural models in conjunction with structural control is currently lacking, which also limit the adoption of structural control.

Hybrid simulation is a method for examining the seismic response of structures using a hybrid model comprised of either both physical and numerical sub-structures, or numerical sub-structures only (Saouma and Sivaselvan 2008). This alternative way of physical testing or numerical modelling of an entire system allows for numerical simulations of complex coupled systems performed separately on different computational platforms. In this paper, a novel approach utilizing the hybrid simulation is proposed to take advantage of modelling ability of existing finite element software and realize the structural control algorithm at the same time. As shown in Figure 1.1, a complex nonlinear structure can be modelled in any existing finite element software, such as OpenSees, Abaqus, etc, while the structural control devices of viscous fluid dampers, base isolators or MR dampers are simulated in other software, such as Matlab, where the control algorithms can be easily formulated and implemented using the built-in toolboxes. The main nonlinear structure and the control devices, as two substructure parts, can communicate with each other by transferring force and displacement information through a platform designed for hybrid simulation: UI-SIMCOR. A nonlinear structure equipped with linear fluid dampers, nonlinear fluid dampers or base isolators is studied using the hybrid simulation hereafter. The structural control is implemented and the equivalent passive parameters are derived. The study verifies the validity of the hybrid numerical simulation scheme in efficiently developing seismic protection strategies for nonlinear structures.

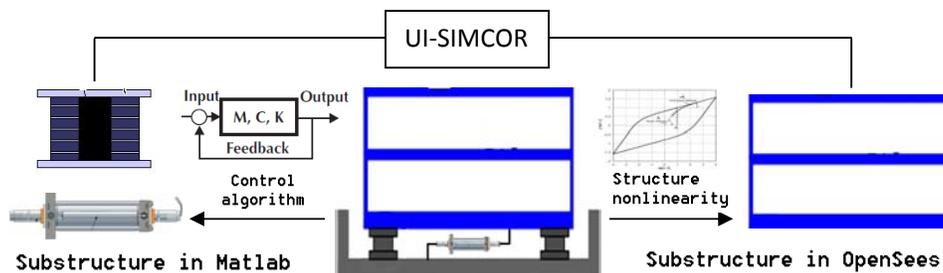


Figure 1.1. Hybrid numerical simulation scheme

2. HYBRID SIMULATION PLATFORM: UI-SIMCOR

The paper builds on an existing hybrid simulation platform and has implemented changes to enable the consideration of nonlinear seismic protective devices. The UI-SIMCOR was originally developed to facilitate geographically distributed pseudo-dynamic (PSD) hybrid simulation. It has been widely used for PSD hybrid simulation and multi-platform simulation with OpenSees, Matlab, Abaqus, etc (Kwon et al. 2008). UI-SIMCOR can control distributed PSD test in several sites. The simulation can be either all experiments, combination of experiments and analyses, or all analyses.

The UI-SIMCOR program solves the equation of motion of a dynamic system generated by static condensation (Kwon et al. 2007). A portal frame with 16 nodes (plus 2 constrained nodes) and 17 beam elements shown in Figure 2.1 (a) is used for the purpose of illustration. Lumped masses are located at beam column joints and ground acceleration is applied in horizontal direction. There will be mass and stiffness matrices with 48 by 48 elements in the equation of motion of the frame. However, the structure's mass and stiffness matrix can be reduced to 6 by 6 using static condensation if the stiffness and mass matrices are known, as shown in Figure 2.1 (b).

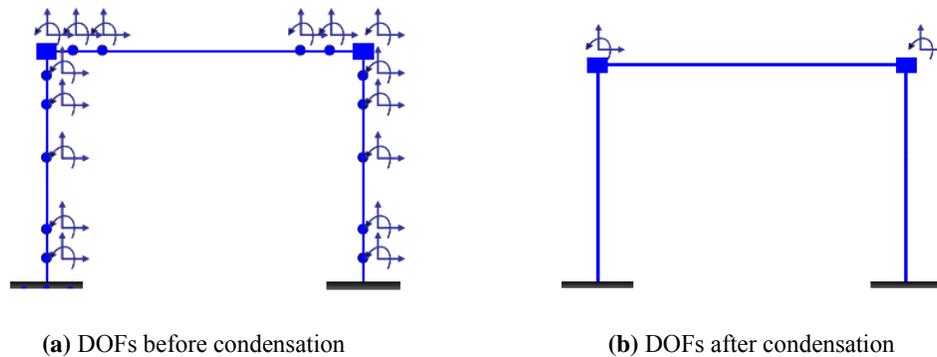


Figure 2.1. Static condensation of multiple DOF structure

By defining the effective DOFs, which are the DOFs where lumped masses are defined as shown in Figure 2.1 (b), the mass matrix is easily formulated. At the same time, the condensed stiffness matrix can be determined by applying a pre-specified displacement to each effective DOFs and measuring reaction forces as shown in Figure 2.2 (a). For the hybrid simulation of the frame which is divided into two segments on two sites or two analysis modules, the initial stiffness of a certain DOF can be calculated by applying certain displacement to the segmented structure and take summation of reaction forces from each segment of structure as shown in Figure 2.2 (b). The dynamic analysis can then be performed for the structure using reduced DOFs. This is very important concept for the application of hybrid simulation and testing using UI-SIMCOR.

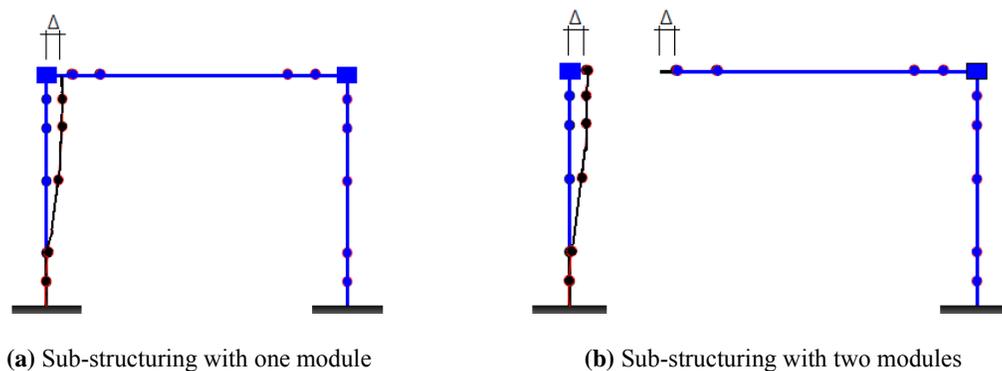


Figure 2.2. Formulation of stiffness matrix in UI-SIMCOR

3. MODIFIED INTEGRATION SCHEME FOR NONLIENAR PROTECIVE DEVICES

When nonlinear seismic protective devices (e.g. nonlinear damper) are used for inelastic structures, the existing integration algorithms in UI-SIMCOR need to be modified. Since it is an open source software, UI-SIMCOR can accommodate different integration schemes that are fit for the hybrid simulation of given structure. The α operator-splitting method is written in UI-SIMCOR for solving equation of motion and ready to be modified. For the hybrid simulation scheme proposed in Figure 1.1, it's convenient to formulate the control devices in Matlab since UI-SIMCOR is also a Matlab-based computational platform. The computational elements that model the nonlinear viscous dampers and base isolators could be incorporated in the time integration section of UI-SIMCOR according to their roles in the equation of motion. The modification to the built-in α operator-splitting method is shown in the following to incorporate nonlinear viscous dampers. The base isolators can also be incorporated similarly through modifying the integration scheme.

The following derivation is based on a nonlinear N DOF structure equipped with nonlinear dampers on each DOF. By setting damper force to be zero, the locations where dampers are installed can be adjusted accordingly. The equation of motion of this nonlinear system can be expressed as:

$$\mathbf{M}\mathbf{a}(t) + \mathbf{f}_d(t) + \mathbf{r}(t) = \mathbf{f}(t) \quad (3.1)$$

where \mathbf{M} is the mass matrix, $\mathbf{a}(t)$ is the system acceleration vector, \mathbf{f}_d is the nonlinear damper force vector, $\mathbf{r}(t)$ is the inelastic structural restoring force vector and $\mathbf{f}(t)$ is the external force vector. In α operator-splitting method, the numerical solution of Equation (3.1) is obtained by a two-step scheme: the predictor step and the corrector step. Knowing the displacement vector \mathbf{d}_n , velocity vector \mathbf{v}_n and acceleration vector \mathbf{a}_n of previous time step t_n , the predictor displacement and velocity vectors of t_{n+1} are expressed as:

$$\tilde{\mathbf{d}}_{n+1} = \mathbf{d}_n + \Delta t \mathbf{v}_n + \frac{\Delta t^2}{2} (1 - 2\beta) \mathbf{a}_n \quad (3.2)$$

$$\tilde{\mathbf{v}}_{n+1} = \mathbf{v}_n + \Delta t (1 - \gamma) \mathbf{a}_n \quad (3.3)$$

where $\beta = (1 - \alpha)^2 / 4$ and $\gamma = (1 - 2\alpha) / 2$ are integration constants. The parameter α controls the numerical damping of the method and is equal to 0.05 in this paper. The *corrector* step yields the true solution of displacement and velocity vectors of t_{n+1} :

$$\mathbf{d}_{n+1} = \tilde{\mathbf{d}}_{n+1} + \Delta t^2 \beta \mathbf{a}_{n+1} \quad (3.4)$$

$$\mathbf{v}_{n+1} = \tilde{\mathbf{v}}_{n+1} + \Delta t \gamma \mathbf{a}_{n+1} \quad (3.5)$$

where \mathbf{a}_{n+1} is solved from the time discretized form of equation (3.1) as following:

$$\mathbf{F}(\mathbf{a}_{n+1}) = \mathbf{M}\mathbf{a}_{n+1} + \mathbf{f}_{d,n+1}(\mathbf{a}_{n+1}) + \mathbf{r}_{n+1}(\mathbf{a}_{n+1}) - \mathbf{f}_{n+1} = 0 \quad (3.6)$$

In general a nonlinear viscous damper can be modeled by:

$$f_d = c_d |v_d|^{\alpha_d} \text{sign}(v_d) \quad (3.7)$$

where c_d is the damping coefficient, v_d is the velocity of the nonlinear damper and α_d is a constant that controls the force-displacement loop of the damper. For a N DOF system including nonlinear viscous dampers, the damper forces are in the vector form:

$$\mathbf{f}_d = [f_{d1} \quad f_{d2} \quad \cdots \quad f_{di} \quad \cdots \quad f_{dN}]^T \quad (3.8)$$

Nakashima et al. (1990) first proposed and implemented the non-iterative α operator-splitting method for PSD testing with the approximation of the restoring force \mathbf{r}_{n+1} is:

$$\mathbf{r}_{n+1}(\mathbf{d}_{n+1}) \approx \mathbf{K}^1 \mathbf{d}_{n+1} + [\tilde{\mathbf{r}}_{n+1}(\tilde{\mathbf{d}}_{n+1}) - \mathbf{K}^1 \tilde{\mathbf{d}}_{n+1}] \quad (3.9)$$

where \mathbf{K}^1 is the initial stiffness. The derivative of Equation (3.6) about \mathbf{a}_{n+1} is given by:

$$\mathbf{F}'(\mathbf{a}_{n+1}) = \mathbf{M} + \frac{\partial \mathbf{f}_{d,n+1}}{\partial \mathbf{v}_{n+1}} \frac{\partial \mathbf{v}_{n+1}}{\partial \mathbf{a}_{n+1}} + \frac{\partial \mathbf{r}_{n+1}}{\partial \mathbf{a}_{n+1}} \quad (3.10)$$

where $\frac{\partial \mathbf{f}_{d,n+1}}{\partial \mathbf{v}_{n+1}}$ is a $N \times N$ vector, $\frac{\partial \mathbf{v}_{n+1}}{\partial \mathbf{a}_{n+1}} = \Delta t \gamma \mathbf{I}$, and $\frac{\partial \mathbf{r}_{n+1}}{\partial \mathbf{a}_{n+1}} = \Delta t^2 \beta \mathbf{K}^1$. For the i^{th} nonlinear damper, its derivative is given by:

$$\frac{\partial f_{di}}{\partial v_{di}} = \alpha_i c_{di} |v_{di}|^{\alpha_i - 1} \quad (3.11)$$

where the velocity across of the i^{th} nonlinear damper v_{di} can be related to global velocity vector \mathbf{v} according to which two DOFs it is installed to. Once $\mathbf{F}'(\mathbf{a}_{n+1})$ is obtained, Newton's iteration is applied to obtain the converged solution for \mathbf{a}_{n+1}

$$\mathbf{a}_{n+1}^{k+1} = \mathbf{a}_{n+1}^k - [\mathbf{F}'(\mathbf{a}_{n+1}^k)]^{-1} \mathbf{F}(\mathbf{a}_{n+1}^k) \quad (3.12)$$

The displacement and velocity vectors can then be obtained by Equation (3.4) and (3.5) once the acceleration vector \mathbf{a}_{n+1} is solved.

4. VALIDATION OF PROPOSED HYBRID SIMULATION SCHEME

To validate the hybrid simulation scheme proposed above, i.e. modelling the main nonlinear structure in OpenSees and modelling the control devices in Matlab, a three story nonlinear frame structure is considered as in Figure 4.1. This model is the test structure located in Harbin Institute of Technology, which will be used to validate experimentally the hybrid simulation. Detailed description on the model can be found in another paper (Ozdagli et al. 2012). The beams and columns in the frame are modelled by beam-column elements with bilinear force-displacement material property. The nonlinear viscous damper follows the definition of Equation (3.7), and the linear dampers and base isolators are modelled by Equation (4.1) and (4.2), respectively.

$$f_d = c v_d \quad (4.1)$$

$$f_b = \varepsilon K_b d(t) + (1 - \varepsilon) K_b D_y Z(t) \quad (4.2)$$

where Z is the dimensionless hysteretic parameter of Bouc-Wen model which is governed by:

$$\dot{Z}(t) = -\frac{1}{D_y}(\gamma|\dot{d}(t)|Z(t)|Z(t)|^{n-1} + \beta\dot{d}(t)|Z(t)|^n - \dot{d}(t)) \quad (4.3)$$

The following structural control strategies are adopted to investigate the structural response under earthquake excitation: (a) linear viscous dampers are installed between floors; (b) nonlinear viscous damper is installed between the first floor of the structure and outside fixture; and (c) base isolators are installed on the structure at the base level. Figure 4.1 illustrates the above control strategies for the structure to be analyzed. In implementing the hybrid simulation scheme for this structure, two effective DOFs are selected corresponding to each floor (6 DOFs in total). The equation of motion is solved with the modified integration algorithm in UI-SIMCOR described in section 3.

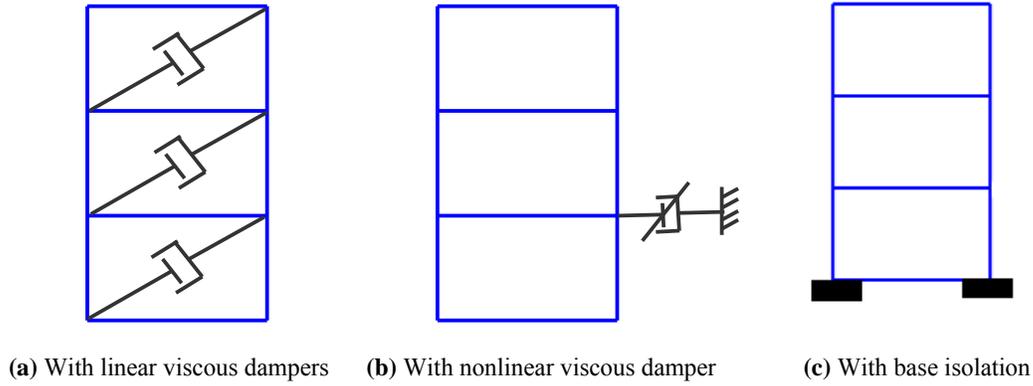


Figure 4.1. Structural control strategies for numerical simulation

The 1940 El-Centro earthquake record is used as the input ground motion for all the analysis reported here. The dynamic structural response from the hybrid simulation is compared with that of the whole OpenSee model, which both main structure and control devices are modelled in OpenSees. The comparisons of the first story displacements from both methods are shown in Figure 4.2. It is seen that the proposed hybrid simulation scheme results in almost the same solution as the whole model in OpenSees, therefore, it's correct and reliable for the further application of advanced structural control strategies. Figure 4.3a also shows the column responses when linear dampers are used where nonlinearity in structure can be observed. Figure 4.3b shows the force-displacement loop of the nonlinear damper. It again shows that the hybrid simulation yields same component responses as computed in the complete OpenSees model. It is noted that the advantages of hybrid simulation scheme implemented here are to take advantage of advanced existing finite element analysis programs to realistically simulate the responses of nonlinear structures.

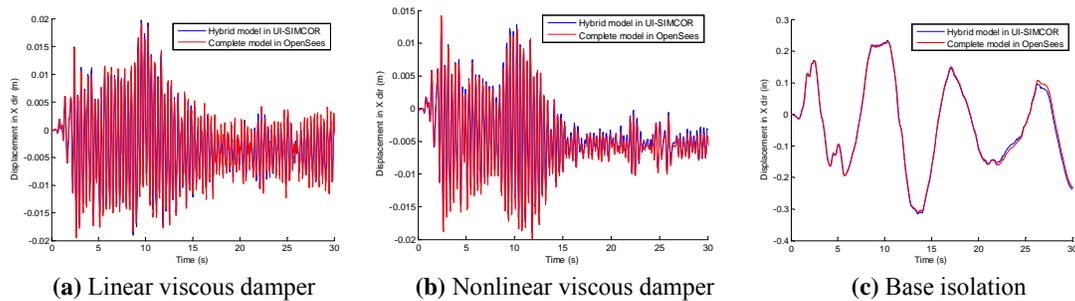
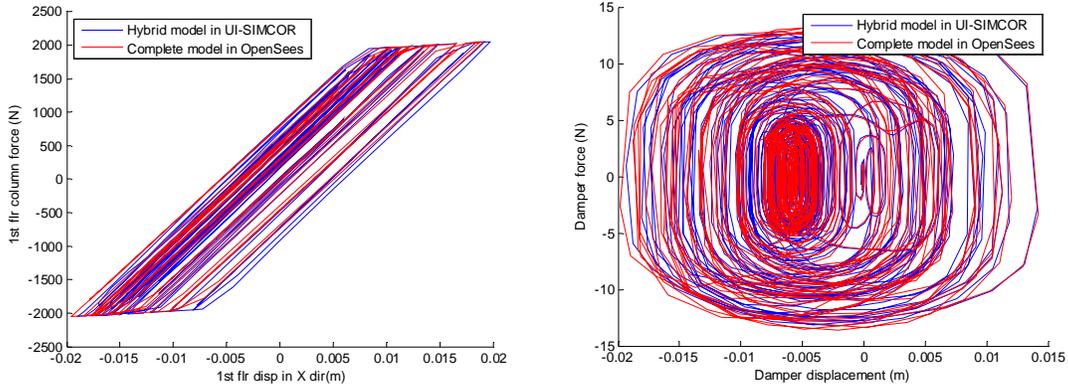


Figure 4.2. Displacement history of 1st floor of different control strategies



(a) 1st floor column reaction of control strategy (a) (b) Single nonlinear damper of control strategy (b)

Figure 4.3. Force displacement loop of hybrid simulation model

5. IMPLEMENTATION OF STRUCTURAL CONTROL WITH HYBRID SIMULATION

In the hybrid simulation scheme stated above, the control devices are modeled separately in Matlab and control algorithms can be applied for the design of their parameters. A three-storey (3DOF) shear building model excited by earthquake motion is numerically analyzed (shown in Figure 5.1) and the active structural control is implemented using LQR theory. The goal is to obtain the stiffness and damping coefficients of the passive devices added to the structure that can provide the response reduction effect most close to that of active control method.

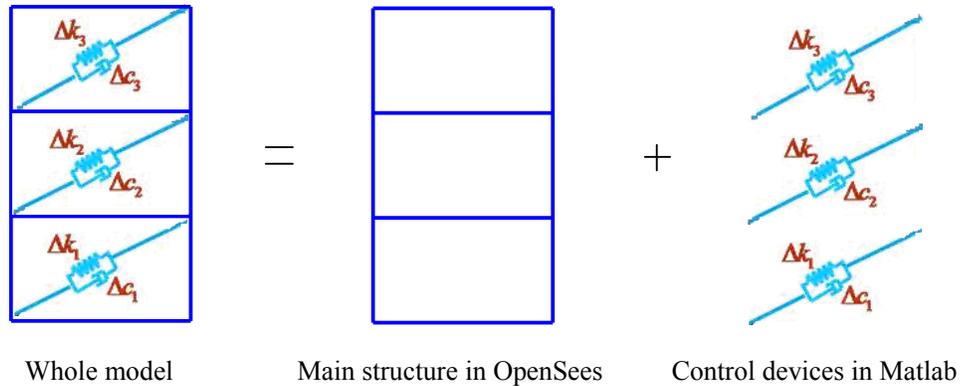


Figure 5.1. Numerical example of structural with hybrid simulation

5.1 Classical linear optimal control theory (LQR)

In classical linear optimal control, the control force $\mathbf{u}(t)$ is chosen in such a way that a performance index J is minimized:

$$J = \int_0^{t_f} [\mathbf{z}^T(t)\mathbf{Q}\mathbf{z}(t) + \mathbf{u}^T(t)\mathbf{R}\mathbf{u}(t)] dt \quad (5.1)$$

where \mathbf{z} vector is the structural response in state space, \mathbf{Q} and \mathbf{R} are referred to as weighting matrices, whose magnitudes are assigned according to the relative importance attached to the state variables and control forces. The equation of motion with control forces applied in state space is:

$$\dot{\mathbf{z}}(t) = \mathbf{A}\mathbf{z}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{H}\mathbf{f}(t) \quad (5.2)$$

where \mathbf{A} is system matrix. \mathbf{B} and \mathbf{H} are location matrices specifying, respectively, the locations of the control forces and external excitations in the state space. $\mathbf{f}(t)$ is a vector representing external excitation. The optimal control force is given by:

$$\mathbf{u}(t) = \mathbf{G}\mathbf{z}(t) \quad (5.3)$$

The gain matrix \mathbf{G} is given by:

$$\mathbf{G} = -\frac{1}{2}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P} \quad (5.4)$$

and \mathbf{P} is the solution of Ricatti equation:

$$\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} - \frac{1}{2}\mathbf{P}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P} + 2\mathbf{Q} = \mathbf{0} \quad (5.5)$$

5.2 Equivalent optimal passive control approximation

Equivalent optimal passive control theory uses LQR method in active control theory to design linear passive stiffness and damping devices (Gluck et al. 1996). The design is aimed at minimizing the difference between the control force from active control theory and those for passive control devices. By considering displacement and velocity feedback, both stiffness and damping devices are designed.

As stated in section 5.1, the active control forces are obtained as

$$\mathbf{u}(t) = \mathbf{G}\mathbf{z}(t) = [\mathbf{G}_x \ : \ \mathbf{G}_{\dot{x}}]\mathbf{z}(t) = \mathbf{G}_x\mathbf{x}(t) + \mathbf{G}_{\dot{x}}\dot{\mathbf{x}}(t) \quad (5.6)$$

where the gain matrix \mathbf{G} are decomposed to two sub-matrices \mathbf{G}_x and $\mathbf{G}_{\dot{x}}$ which correspond to the stiffness and damping information for the control devices.

If the same control forces are supplied by passive devices and they are denoted by

$$\mathbf{u}^*(t) = \mathbf{K}_x\mathbf{x}(t) + \mathbf{C}_{\dot{x}}\dot{\mathbf{x}}(t) \quad (5.7)$$

where \mathbf{K}_x and $\mathbf{C}_{\dot{x}}$ are the matrices containing the stiffness and damping coefficients of the passive devices. Intuitively, the elements in \mathbf{K}_x and $\mathbf{C}_{\dot{x}}$ could be derived by elements in \mathbf{G}_x and $\mathbf{G}_{\dot{x}}$.

Applying the least square approximation to the difference between Equation (5.6) and (5.7), the stiffness and damping parameters of the diagonal control devices, Δk_i and Δc_i , can be determined by the following two approximation approaches.

(1) The response spectrum approach. It includes the influence of all or several modes of vibration relevant. In applications involving structures in earthquakes, sometimes only one mode of vibration is relevant. Then the control device design are governed by

$$\Delta k_i = \frac{\sum_j g_{ij,d} \varphi_{jm}}{\varphi_{im}}, \quad \Delta c_i = \frac{\sum_j g_{ij,d} \varphi_{jm}}{\varphi_{im}} \quad (5.8)$$

where φ is the mass normalized modal shapes. $g_{ij,d}$ and $g_{ij,\dot{d}}$ are elements in the transformation form of gain matrix in terms of interstory drift by multiplying the gain matrix with a transformation matrix \mathbf{T} :

$$\mathbf{G}_d = \mathbf{T}^T \mathbf{G}_x \mathbf{T}, \quad \mathbf{G}_{\dot{d}} = \mathbf{T}^T \mathbf{G}_{\dot{x}} \mathbf{T} \quad (5.9)$$

(2) Truncation approach. This is a much more simplified formulation obtained if only a single gain factor in gain matrix of active control is considered. Design parameters can be obtained directly from truncating all off-diagonal terms in the transformation form of gain matrix in terms of interstory drift. In such case

$$\Delta k_i = g_{ii,d}, \quad \Delta c_i = g_{ii,\dot{d}} \quad (5.10)$$

5.3 Numerical results

A sample earthquake excitation (the 1940 El Centro record) is selected to conduct the time history analysis of structure equipped with the designed supplemental stiffness and damping. The relative displacement and total acceleration time histories are shown for different floors in Figure 5.2 and 5.3 respectively. It is seen that the optimal passive control designs result in much better structural responses in terms of relative displacement and total acceleration than the uncontrolled case. Single mode design achieves close control effect as that of truncation design.

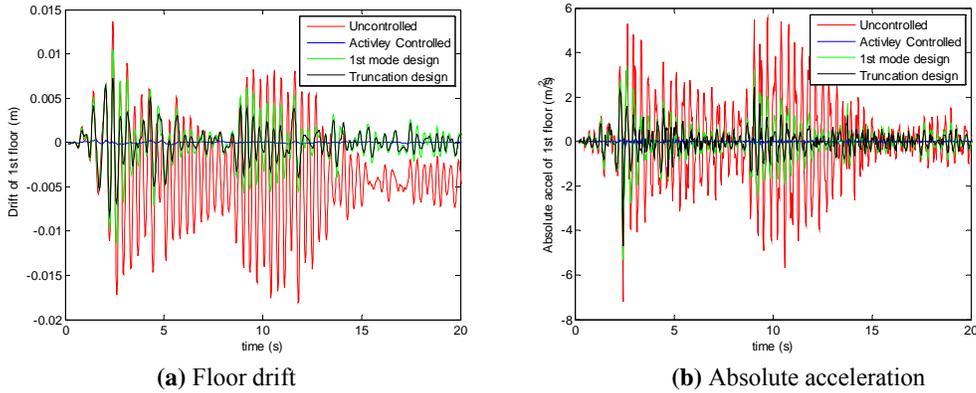


Figure 5.2. Response history of 1st floor with different control designs

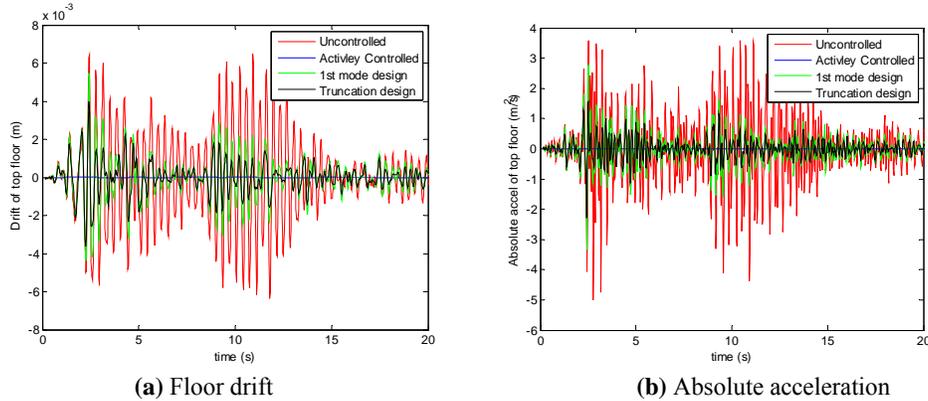


Figure 5.3. Response history of top floor with different control designs

6. CONCLUSIONS

This paper explores using the hybrid simulation to conduct structural control of nonlinear structures so as to derive the optimal passive stiffness and damping values to mimic the actively controlled devices. In order to achieve this objective, existing hybrid simulation software (UI-SIMCOR) is adopted and modified to enable the integration algorithm to include nonlinear seismic protective devices such as nonlinear dampers and base isolation devices. While the realistic behavior of nonlinear structures can be modeled separately in current finite element analysis software package (e.g. OpenSees, Abaqus etc.), the nonlinear seismic protective devices can be modeled in Matlab and pieced together through hybrid simulation to produce the overall structural responses. The control algorithms can also be implemented under this framework. Using a real test structure equipped with various protection devices, the paper demonstrated the accuracy and versatility of hybrid simulation. Furthermore, this leads to the easy application of different control algorithms that can yield the optimal selection of stiffness and damping values for control devices in design.

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