Seismic Testing of Full-Scale Piles in Unimproved and Improved Soft Clay

S. Sritharan, and B. Fleming *Iowa State University, USA, Ames, IA 50011*



SUMMARY:

A desirable seismic design practice requires the foundation of structures to be limited to elastic response and that the displacements of the pile foundations are restricted to acceptable values. These criteria are difficult to achieve for piles embedded in weak soils. An effective means to mitigate this challenge is to improve the soil surrounding the piles, thereby increasing the stiffness and strength of the pile. In an on-going Network for Earthquake Engineering Simulation (NEES) project, two identical full-scale steel pipe piles were driven at a soft clay site in Oklahoma and tested for seismic resistance using dynamic and quasi-static loading. The soil surrounding one of these piles was improved using the cement deep mixing technique. Compared to the unimproved pile, the improved ground increased the system strength of the pile by 42%. Furthermore the observed responses of both piles correlated well with simulated response envelopes established using finite element methods.

Keywords: Improved, Pile, Soft Clay, Seismic

1. INTRODUCTION

Pile foundation is one of the preferred supports for many civil engineering structures including highway bridges, railroad bridges, and port wharves. These structures and their foundations are subjected to forces created by earthquakes, wind, waves, water current, vessel impact, ice, and gravity. All loads applied to the superstructure must be transmitted to the foundation, which are then transferred to the surrounding soil. With significant lateral loads, use of pile foundations may be the only option to transmit large structural loads to competent soils. Piles supported by competent soils are relatively easy to design cost effectively. However, thick layers of weak soils such as soft clays are widespread in high seismic areas (e.g., San Francisco, southern Nevada, Washington, Eastern Missouri, and Arkansas), exacerbating design challenges. Soft clays reduce the lateral resistance of the pile-soil system, making the pile foundation less cost effective. In this case, the current design practice is to use increased number larger diameter piles, which is a costly alternative (see SDC, Caltrans 2010). An innovative and more cost-efficient solution to this problem is to improve the soil within a short depth surrounding the foundation, thereby increasing its lateral stiffness. Some well-known methods for improving soil include deep soil mixing, stone columns, and simple soft soil replacement. Soil improvement techniques are not often used in design practices due to lack of fundamental understanding of the behaviour of improved and unimproved soils and the interactions between them, forcing designers to use expensive foundation solutions. To solve this problem, a Network for Earthquake Engineering Simulation (NEES) project involving researchers from six leading institutions in the United States (Iowa State University, Oklahoma University, San Jose State University, University of California, Los Angeles, University of California, Davis, and Clemson University), two engineering firms (Advanced Geosolutions Inc. and Earth Mechanics Inc.), and the Oklahoma Department of Transportation are working together to develop a design methodology for using soil improvement to reduce the seismic risk of pile foundations in soft clays. The project is composed of small-scale centrifuge testing, full-scale field testing, and comprehensive computer modelling. The

focus of this paper is to describe the full-scale field test and demonstrate the use of a simplified model to account for the increase in lateral resistance with the application of improved soil around a single pile.

1.1. Background

The concept of improving the ground where weak soils are present and its application to foundations have been studied extensively by researchers in the past four decades (Ministry of Transport, 1980). Generally, most of these studies focused on utilizing this technique to mitigate liquefaction of loosely deposited sand but without the presence of piles (Mitchel et al., 1998; Martin et al., 2001; Hatanaka et al., 1987; Adalier, 1996; Adalier et al., 1998; Iai et al., 1988; Akiyoshi et al., 1993; Liu and Dobry, 1997; Kawakami, 1996). Seismic behaviour of piles in liquefiable sands has also been extensively studied (e.g., Ashford et al., 2000a; Ashford et al., 2000b; Weaver et al., 2005; Boulanger and Tokimatsu, 2006; Ohtomo, 1996), while only a few studies have addressed the seismic behaviour of pile foundations in soft clays (Rollins et al., 2011) despite the widespread presence of the soft clay soil in high seismic regions and the frequent need to locate bridges and buildings in this soil type. Only recently, a few investigations have been carried out to determine the effectiveness of the ground improvement on increasing the lateral resistance of pile foundation embedded in soft clay.

Rollins et al. (2011) evaluated the quasi-static and dynamic behaviour of pile groups in several soil improvement types and configurations. Soil treatments, including jet grouting, soil mixing, flowable fill, compacted fill, and rammed aggregate piers, were applied to the soil surrounding embedded pile groups and pile caps. In some experiments, passive earth pressure was allowed to develop between the side of the pile cap and improved soil. These techniques were applied to configurations that accommodate both retrofit and new construction designs. In a retrofit design utilizing the jet grouting technique, where the soil improvement was applied as a wall surrounding the pile cap, the lateral resistance was 185% relative to a similar pile group in virgin soil. For new construction designs where jet grouting was applied below the pile cap, lateral resistance at the pile cap was 160% that of a pile group in virgin soil. Details related to this work are described in the National Cooperative Highway Research Program (NCHRP) Report 697. Although this research applied dynamic loads to their specimens, it did not address the dynamic behaviour of the laterally loaded pile groups at frequencies higher than 1 Hz, nor did it address the response of a single pile in improved soil, which is the focus of the study presented herein.

2. FULL-SCALE FIELD TEST

Following a series of small-scale tests to study improvement of piles in soft clay using a centrifuge test setup, a full-scale field test in the field was undertaken. A soft soil site in Miami, Oklahoma was chosen for the field experiment to observe pile response subjected to simulated earthquake loads. An illustration showing the full-scale test configuration is shown in Fig. 2.1. Two identical steel pipe piles having AISC size HSS12.75x0.375 and satisfying ASTM A106B specifications were driven into improved and unimproved soft clay and loaded laterally at the top with dynamic and quasi-static forces or displacements. A pile cap, consisting of two halves of a concrete block and acting as a seismic mass and weighing about 16.5 kN, was clamped to the head of the pile to be tested. A set of threaded rods, sent through the pile cap to avoiding the pile in the centre, were used to clamp the pile cap to the pile and to support a stiffened C15x40 (AISC notation) channel used to adapt connections between the quasi-static and dynamic actuators.

A hydraulic actuator with a high-speed valve, herein referred to as the dynamic actuator, from the Network for Earthquake Engineering Simulation (NEES) at the University of California in Los Angeles, was used to dynamically drive the soil-pile-mass system. A steel frame, which was placed between the two test piles acted as a reaction to dynamic loads, supported the dynamic actuator through the use of a cantilever (see Fig. 2.1). This allowed the reaction piles of the frame to be placed outside the influence of the test piles, as well as maintain an appropriate distance between the dynamic

actuator and pile cap. After testing was complete on one test pile, the cantilever was disassembled and reconstructed on the other side of the frame to perform testing of the other test pile. The actuator for quasi-static testing did not require the cantilever and was mounted to a cross beam supported by the reaction piles of the frame (as shown on the right half of Fig. 2.1).



Figure 2.1. Test configurations for full-scale field testing of pile foundations in improved and unimproved soft soil

2.1. Site Profile and Soil Improvement

The site profile, consisting of a relatively uniform 4270 mm layer of soft clay overlying a 2134 mm layer of sandy gravel and limestone bedrock, was examined using various geotechnical investigation techniques including piezocone soundings (CPTu), soil borings, Shelby tube sampling, and a stand pipe piezometer to measure the depth of the water table. From the investigation, lean clay with gravel and occasional construction debris was found within a depth of 1100 mm at the site. Below this existed a relatively uniform soft to very soft clay with undrained shear strengths (s_u) between 30-75 kPa according to an average of eleven CPTu soundings taken every 50 mm along the depth of the soil profile.

A technique called cement deep soil mixing (CDSM) was used to improve the soft clay around one of the test piles in the field. Prior to the piles were installed, an ABI Mobilram machine with an augur drive attachment was used to revolve a mixing tool into the soft clay. It consisted of a hollow shaft and mixing paddles (Fig. 2.2). Cement grout was pumped through the hollow shaft and ejected laterally behind the lower mixing paddle where it is mixed with the native soil. While still revolving, the mixing tool was advanced to the desired depth and retracted. This process was repeated once more to form a well-mixed uniform column of soil and cement.

CDSM installation guidelines and recommendations of Advanced Geosolutions Inc. (AGI) were followed to ensure sufficient bearing capacity to support 324 mm diameter piles. The objective was to improve the soil such that the test pile would yield and form a plastic hinge as opposed to rotating under the load.

The grout used to construct the CDSM columns consisted of a water to Portland cement ratio of 1:1 by weight. CDSM columns were arranged in a block-type configuration as shown in Fig. 2.2 with slight overlapping between columns. The diameter of the mixing tool measured 1219 mm from tip-to-tip, resulting in a plan dimension of 3962 x 3962 mm in the column arrangement. A total of 1.893 m3 of cement grout was added to each column as the mixing tool was advanced to a depth of 3962 mm and retracted. This resulted in an approximate concentration of about 25% cement grout by weight per CDSM column. A test pile was then installed while the improved soil was still wet using the same ABI Mobilram machine but with a vibrating hammer attachment. Two CDSM columns, constructed the same way, were placed in the location of two reaction piles closest to the improved test pile. This was done to increase the stiffness of the reaction frame.

Samples from the improved soil volume were extracted and tested for unconfined shear strength in the laboratory after the improved soil was allowed to cure. The average strength of the soil was approximately 2443 kPa.



Figure 2.2. Cement deep soil mixing (CDSM) Mobilram attachment (left) and CDSM column arrangement and test pile placement (right)

2.2. Experimental Procedure

Dynamic and quasi-static testing protocols were selected for each pile to evaluate the pile at varying levels of loading. Because each test had a degree of disturbance, the level of loading was increased gradually. Generally, a minimum of three cycles of the same force or displacement amplitude and frequency were applied to the pile cap before using the next load pattern. Initially, small vibration tests were conducted with force control enabled on the actuator. Displacement control was used for a majority of the tests in the dynamic loading protocol.

Field evaluation started with the dynamic testing of the test pile in improved soil (TPI). After completion, the cantilever of the reaction frame was removed and placed on the other side to perform dynamic testing of the test pile in unimproved soil (TPU). Generally, the test setup and loading were completed in one day for each test pile. Quasi-static testing was performed after dynamic testing with the loading protocol shown in Fig. 2.3.



Figure 2.3. Loading schedule for test pile in improved soil (TPI) and test pile in unimproved soil (TPU)

3. FINITE ELEMENT ANALYSIS

An open-source software framework, called the Open System for Earthquake Engineering Simulation (OpenSees), employs the finite element method (FEM) to model and analyze structural and geotechnical systems (Mazzoni et al., 2006 and http://opensees.berkeley.edu/). Mesh design using this interface is a time consuming process, especially for three dimensional (3D) models involving soilfoundation interaction. To simplify this process, a graphical user interface (GUI), called OpenSeesPL (Lu et al., 2006 and http://cyclic.ucsd.edu/openseespl/), specifically developed to model a 3D "soil island" with the inclusion of a single pile, was used to generate the soil and pile mesh for the models developed in this paper. Although OpenSeesPL is a comprehensive tool for the development of a number of geometries and configurations, it does not allow for excavation around the pile or the separation between the pile and soil (gapping). Therefore, a MATLAB script was written to change the Tcl developed by OpenSeesPL to include these characteristics and give an accurate representation of the piles in the field experiments.

The mesh used for all models is shown in Fig. 3.1. Soil elements were removed around the pile near the ground surface to replicate the excavation in the field. To improve computational efficiency, only half of the soil-pile system was considered. The length of the soil island is 10.3 meters parallel to the direction of loading, 5.15 meters transverse to the direction of loading, and 7.62 meters in height. The width and depth of the excavation is 1.22 meters and 1.1 meters, respectively. The pile, modelled with one dimensional (1D) beam elements, extends 1.93 meters above the base of the excavation and 5.30 meters into the soil domain. The behaviour of the pile is modelled with a fibre section where each fibre has a nonlinear uniaxial material simulate the behaviour of the steel in the pile and protective angles. Again, only half of the pile section is included due to the symmetry of the model.

Soil behaviour is simulated using constitutive models. For the clay material, the response exhibits plasticity in the deviatoric stress-strain relationship. The volumetric stress-strain relationship is linearelastic and independent of the deviatoric response. The sand material, however, exhibits response characteristics of pressure sensitive material (e.g., the volumetric contraction or dilation induced by shear stresses in the soil). Initially, the model was calibrated by determining a uniform cohesive strength and shear modulus for the unimproved soil such that the analysis reflects the lateral forcedisplacement behaviour of TPU pile in the field. A more rigorous approach to defining soil parameters will be conducted following the completion of detailed lab results for the soil in the field. Recommended soil parameters for stiff sand (Lu et al., 2006) were used for the sand layer just above bedrock.



Figure 3.1. Finite element mesh of a single pile in soil domain with excavation around the pile

The interface between the pile and soil is formed by defining a constitutive frictional material. The contact material between 1D beam elements and 3D soil solids, implemented into OpenSees (Petek 2006 and http://opensees.berkeley.edu/wiki/index.php/BeamContact3D) allow for frictional slip, separation between the pile and soil, and sticking. The theory behind this model can be found in Wriggers (2002). Contact elements, which utilize this material model, define master nodes and slave nodes of the impacting bodies. Two master nodes were assigned to the end nodes of the 1D beam elements have a width equal to the radius of the pile and are assumed initially in contact with the adjacent soil nodes. Once gravity is applied to the soil, the soil elements and nodes within the region of the pile are removed, leaving a hole equal to the diameter of the pile as shown in Fig. 3.1.

Each model consists of 3,450 nodes, 2,492 soil elements, and 34 beam elements. A mesh configuration was chosen such that varying soil improvement dimensions can be achieved by simply assigning improved soil properties to specific elements without changing the geometry of the model. Static and cyclic loads were applied to the top of the pile to determine the lateral resistance of a single pile to static forces or forces produced by the shaking of a structure during an earthquake. A displacement controlled analysis matching the testing protocol performed in the field was conducted and the results of the FEM model were compared to the field experiments.

4. RESULTS

The force-displacement responses for both test piles are shown in Fig. 4.1. TPU experienced large displacements of up to ±0.4 m with little resistance to lateral forces due to the presence of soft clay surrounding the pile. A pushover analysis in the FEM model matches the envelope for the force-displacement response of the pile head reasonably well. TPI reached its lateral capacity at a displacement of 0.1 m, at which point the critical region at the base of the pile above the improved ground experienced buckling and fractured due to low cycle fatigue. The hysteretic curves for each cycle of equal head displacement magnitude from the two test types plotted nearly on top of the other, suggesting that the effects of load history on the force-displacement response were less significant in TPI as compared to TPU. In addition, it can be seen that the calibrated parameters for the FEM model, based on the field results from TPU, work well for characterizing the response of TPI. However, buckling of the sidewall in the pile was not considered in the FEM model, and therefore, this failure mode was not captured. Compared to TPU, TPI in the improved ground increased its lateral strength by 42%.



Figure 4.1. Force-displacement of pile head compared to finite element (FE) pushover analysis for piles in unimproved (left) and improved soil (right)

TPU experienced little inelastic action compared to TPI. Fig. 4.2 shows strain variation along the length of both test piles for a head displacement of 100 mm. Maximum strains in TPU at this stage were well below the yield strain (ε_y) of 0.0023 although the change in strain along the full length of the pile occurred. Strains in TPI reached the ε_y when subjected to a 100 mm of lateral displacement at the pile head. These strains concentrate just underneath the ground surface and reduced to zero strain within the improved soil volume. Although the soil was improved over 2.9 m, the effective improvement depth was found to be approximately 1.3 m below the ground surface as can be seen in the strain gage profiles in Fig. 4.2.



Figure 4.2. Strain profiles for test piles in improved and unimproved soil at the first occurrence of 100 mm pile head displacement

Soft soil surrounding TPU had undergone significant plastic deformations causing separation or gapping between the pile and the soil during unloading. The amount of gapping depends on many factors including, but not limited to, soil stiffness, soil strength, pile displacement, pile diameter, and load history. Displacements of the pile along the un-embedded length were recorded in three locations using string potentiometers. The maximum displacement of the pile at the ground surface (d_p) was extrapolated from these measurements. In addition, the width of gap (d_{e}) was manually measured using a scale at the ground surface after the pile was brought back to zero displacement following each critical load cycle. Fig. 4.3 shows the ratio of d_g to d_p , herein referred to as the gapping ratio, for various cycles and head displacement magnitudes for TPU. For ratios close to one, the gap is equal to the maximum pile displacement at the ground surface. For gapping ratios greater than one, for instance in Fig. 4.3 when head displacements are less than or equal to 100 mm, the gap is larger than the pile displacement at the ground surface. This is because the soil surround the pile was disturbed by the previous dynamic testing which allowed the pile head to displace up to 113 mm before start of the quasi-static testing. Beyond a head displacement of 113 mm, the pile is pushed further into the virgin soil and gapping ratios are reduced and stayed relatively constant at about 80% then increase gradually to 100% at higher head displacements. Therefore, it can be said that for the pile and soil conditions exhibited at the test site, the soil will rebound less than 20% of the maximum pile displacement at the ground surface. Gapping for TPI was significantly less at about 3 mm for all quasi-static tests.



Figure 4.3. Gap ratio (d_g/d_p) at various cycles and nominal head displacement magnitudes for TPU

In calibrating the FEM model, soil parameters were changed to match the envelope for the forcedisplacement response of TPU. The cyclic response for TPU was more challenging to capture as can be seen in Fig. 4.4. The first cycle of FEM model is with no disturbance to the soil prior to loading. In the field, there was a ± 25 mm displacement before the first cycle of the ± 50 mm test shown in Fig. 4.4. After the first cycle, a gap formed between the pile and the soil. The soil was reengaged during the second cycle and the stiffness of the system increased sooner in the FEM model than in the field. This discrepancy is due to the soil in the field undergoing larger plastic deformations and thus larger gapping. An attempt was made to increase the gapping in the FEM model by increasing the shear modulus and decreasing the cohesive strength of the clay. However, the FEM model could not converge to a solution. In addition, the shear modulus and cohesive strength for the soil had to be changed to values that would be unrealistic for soft clay. An investigation is underway to try to improve our method of modelling pile foundations in soft clay. A 20-node brick element based on the total Lagrangian formulation will be used for this purpose. This element, available in OpenSees, does not support the small deformation assumption like the brick elements used in this study. Therefore, deformations should be more accurate and is the first step in improving the cyclic response of the FEM model.



Figure 4.4. Cyclic response of FEM model after two cycles at ±50 mm compared to field data for ±50 mm displacement, zoomed in to show the pull direction for TPU

5. CONCLUSIONS

Two full-scale test piles were driven into improved and unimproved soft soil to observe pile response to simulated earthquake loads. Significant gapping between the pile and soil with little resistance to lateral load was observed for the pile in unimproved soil. The pile in improved soil reached its lateral capacity at a displacement of 0.1 m, at which point the critical region at the base of the pile above the improved ground experienced buckling and fractured due to low cycle fatigue. Compared to the pile in the unimproved ground, the improved ground increased the system strength by 42%. It was demonstrated that OpenSees can accurately predict the force-displacement envelope for laterally loaded piles in improved and unimproved soft clay. However, additional research is required to improve simulations of the cyclic response for piles undergoing large deformations. A 20-node brick element, developed using the total Lagrangian formulation, will be used as opposed to the standard 8-node brick element which uses the small-deformations assumption.

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