An Optimal Distribution of Stiffness over the Height of Buildings and its Influence on the Degree and and Distribution of Earthquake induced Damages

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SUMMARY:
Based on Housner's assumption, the seismic input energy to a building, with low damping ratio and long period, is related mainly to total mass of the system. Thus, the seismic input energy is mainly related to the ground motion features, not to the period of SDOF or distribution of stiffness in MDOF systems. This study analytically reveals the range of validity of these assumptions in linear systems and finds an optimal stiffness distribution over the height of high-rise shear linear buildings to minimize the seismic input energy. First, it is shown that, the input energy is a function of stiffness and for low to moderate height shear type structures, the optimal distribution of stiffness obeys a parabolic form. Then it is shown that such a stiffness distribution causes minimal hysteretic energy in regular nonlinear moment resisting steel frames and has a significant effect on the reduction of studied damage indices

Keywords: Seismic input energy, input energy spectrum, damage indices, stiffness distribution

1. INTRODUCTION

Housner was first who used concept of input energy as seismic design criteria (Housner, 1956). Two main conclusions of his paper are of special concern in this study:
1- The seismic energy input to a SDOF structure with specified damping, looking from "spectral" or "average" standpoint, is basically constant and independent of its period, especially for low damping ratios.
2- Seismic energy input to a MDOF system basically depends to its "total mass" and therefore it is equal to energy input to an equivalent SDOF system with the same mass and main period of vibration.

Based on Housner's work, Akiyama (1985) developed input energy spectrum for different site soils. The spectrums were basically constant with respect to the period of vibration except for periods smaller than predominant periods of the ground. As Housner, Akiyama tried to simplify seismic design of structures by supposing and demonstrating that "input energy to structures is related mainly to earthquake excitation but scarcely to structural features". Most of researchers adapted this assumption and equation proposed by Kuwamura and Galambos (1989), Fajfar *et al* (1989), Uang and Bertero (1990), and Kuwamura *et al* (1994) for establishing the earthquake input energy are based on the ground motion characteristics only.

Parallel to these works in estimating the energy "demand", other researchers focused on the mechanism of dissipation of the input energy in structural elements by the hysteretic action and Ang and Park (1985) related their damage index to the energy dissipation via hysteretic loops. So it arises an important question: “Is it possible to minimize the seismic input energy to structures by a specific design pattern?”

Various aspects of seismic input energy and its calculation, like absolute and relative energies, time interval for integration of related equations and so forth, have been discussed in the literature

(Kuwamura and Galambos (1989), Uang and Bertero (1990), Khashaee *et al* (2003)). In this paper some basic assumptions and definitions which are widely used in the literature are adopted, as follow:
1- Relative, rather than absolute input energy is studied (the difference between relative and absolute energies vanishes if they evaluated at the end of ground motion duration).
2- Input energy input energy is defined as the energy imposed to the structure by strong ground motion from the beginning (t=0) of motion to the end of it ($t=t_0$).

It has demonstrated that the maximum input energy may be attained not necessarily at the end of motion (Uang and Bertero, 1990). However, as mentioned earlier, the input energy of the system at the ending moment of ground motion is considered as seismic input energy. So the seismic input energy to a SDOF system with mass m, frequency ω and damping ratio ζ is defined mathematically as:

$$E_1(m,\omega,\zeta) = -\int_0^{t_0} m\ddot{y}_g\,\dot{y}dt \qquad (1.1)$$

Where $\ddot{y}_g$ and $\dot{y}$ are respectively ground acceleration and system relative velocity. For a system with unit mass, Eqn. 1.1 can be written as:

$$E_1(1,\omega,\zeta) = -\int_0^{t_0} \ddot{y}_g\,\dot{y}dt \qquad (1.2)$$

It is helpful to use an equivalent velocity V_E (Akiyama.1985), defined based on the input energy, as:

$$V_E = \sqrt{\frac{2E_1}{m}} \qquad (1.3)$$

Where E_1 is input energy to the SDOF.

2. INPUT ENERGY SPECTRA

Based on the definition of the input energy, given in the previous section, some input energy spectra have been obtained using ten typical earthquake ground motion records shown in table 2.1. All of these records have been extracted from PEER STRONG MOTION DATABASE.

Table 2.1. List of records used to obtain input energy spectra

Name	Location	Date	Duration (Sec.)	PGA (g)
Chi-Chi, W	TAIWAN	09/20/99	89.99	0.9675
Duzci, 90	TURKEY	11/12/99	55.89	0.8224
El Centro, 180	U. S. A.	5/19/40	39.99	0.3129
Gazli, 90	UZBEKISTAN	5/17/76	16.20	0.7175
Kobe, 00	JAPAN	01/16/95	47.98	0.8213
Landers, 00	U. S. A.	6/28/92	41.91	0.7848
Loma Prieta, 00	U. S. A.	10/18/89	39.94	0.6437
Manjil, Long.	IRAN	06/20/90	53.50	0.5146
Northridge, 142	U. S. A.	1/17/94	39.99	0.6125
Tabas, Tr.	IRAN	09/16/78	32.82	0.8518

All records have been normalized to 1.0g. Fig. 2.1(a) and 2.1(b) shows equivalent velocity V_E spectra versus period of vibration respectively for ζ = 0 and 0.05. Design input energy spectrum (DIES), proposed by Akiyama (1985), is also shown in the Figures.

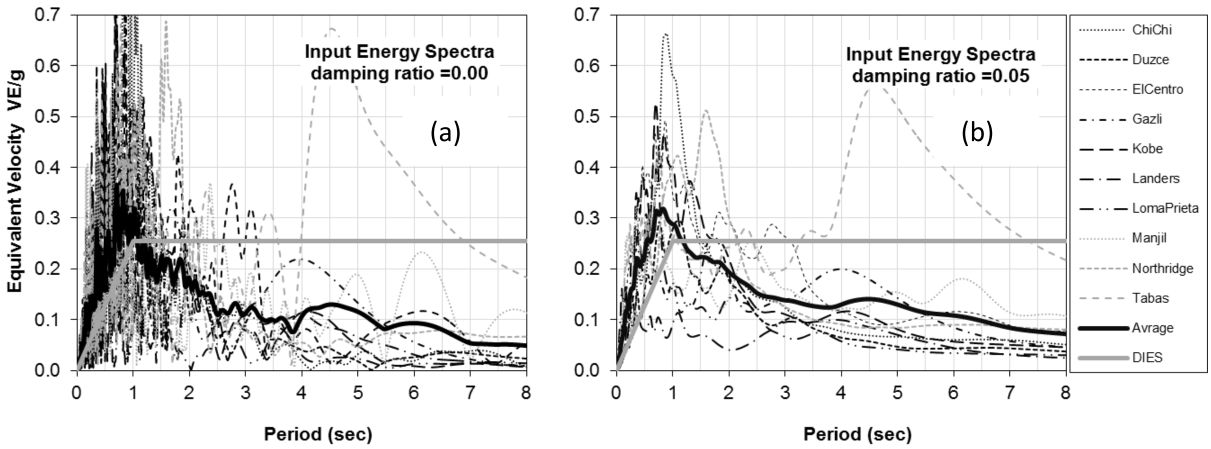

Figure 2.1. Input Energy spectra for (a) ζ=0%, (b) ζ=5%,

It should be mentioned that the shown DIES values are for very stiff site soil and damping ratio of %10. It is worthy to know that most of the newly proposed design elastic input energy spectra by researchers have the same spectral shape and character as shown by the average spectrum in Fig. 2.1 (Ye L. *et al*, 2009, Godrati Amiri G. *et al*, 2008, Venavent A. *et al*, 2002, Decanini Luis D. and Mollaioli F. 1998).

3. EQUATION OF MOTION AND INPUT ENERGY TO MULTI-STORY BUILDINGS

A schematic illustration of the simplified model of multi-story buildings, considered in this study, is shown in Fig. 3.1.
The equation of motion of the system shown in Fig. 3.1 can be written as:

$$[M]\{\ddot{y}\}+[C]\{\dot{y}\}+[K]\{y\}=-\ddot{y}_g[M]\{r\} \tag{3.1}$$

Or

$$[M][\Phi]\{\ddot{z}\}+[C][\Phi]\{\dot{z}\}+[K][\Phi]\{z\}=-\ddot{y}_g[M]\{r\} \tag{3.2}$$

Where

$$\{y\}=[\Phi]\{z\}$$

$[\Phi]^T[M][\Phi]=[I]$ ($[\Phi]$ is orthonormalized and $[I]$ is the unit matrix.)

$[\Phi]^T[C][\Phi]=[c]$ ([c] is a diagonal matrix with elements $c_{ii}=2\xi_i\omega_i\approx\text{const.}$)

$$[\Phi]^T[K][\Phi]=[\omega^2]$$

$$\{r\}=\{1\}$$

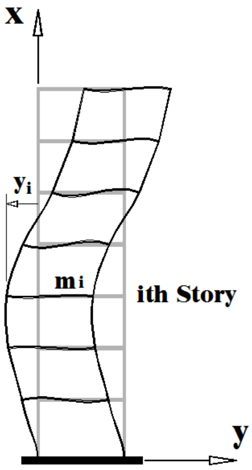

Figure 3.1. The structural model used in this study

It is obvious that the input energy to this system can be written as:

$$E = -\int_0^{y_0} \ddot{y}_g \{dy\}^T [M]\{r\} = -\int_0^{t_0} \ddot{y}_g \{\dot{y}\}^T [M]\{r\}\,dt = -\int_0^{t_0} \ddot{y}_g \left([\Phi]\{\dot{z}\}\right)^T [M]\{r\}\,dt \tag{3.3}$$

Thus:

$$E = -\int_0^{t_0} \ddot{y}_g \{\dot{z}\}^T [\Phi]^T [M]\{r\}\,dt = \sum_{i=1}^{n}\left(-\int_0^{t_0} \ddot{y}_g \dot{z}_i \{\varphi_i\}^T [M]\{r\}\,dt\right) \tag{3.4}$$

Now, decupling Eqn. 3.2 results in:

$$[\Phi]^T [M][\Phi]\{\ddot{z}\} + [\Phi]^T [C][\Phi]\{\dot{z}\} + [\Phi]^T [K][\Phi]\{z\} = -\ddot{y}_g [\Phi]^T [M]\{r\} \tag{3.5}$$

$$[I]\{\ddot{z}\} + [c]\{\dot{z}\} + \left[\omega^2\right]\{z\} = -\ddot{y}_g [\Phi]^T [M]\{r\} \tag{3.6}$$

$$\ddot{z}_i + 2\omega_i \xi_i \dot{z}_i + \omega_i^2 z_i = -\ddot{y}_g \{\varphi_i\}^T [M]\{r\} \tag{3.7}$$

Eqn. 3.7 can be interpreted as equation of motion of a SDOF system with unit mass subjected to ground acceleration $\ddot{y}_g$, magnified by $\{\varphi_i\}^T [M]\{r\}$. It is evident that magnifying the excitation by $\{\varphi_i\}^T [M]\{r\}$ leads to magnification of the input energy by $\left(\{\varphi_i\}^T [M]\{r\}\right)^2$, thus considering Eqn. 1.2 the input energy to the system shown by Eqn. 3.7 may be written as:

$$-\int_0^{t_0} \ddot{y}_g \dot{z}_i \{\varphi_i\}^T [M]\{r\}\,dt = E_1(1,\omega_i,\zeta_i)\left(\{\varphi_i\}^T [M]\{r\}\right)^2 \tag{3.8}$$

By comparing Eqn. 3.4 and Eqn. 3.8 the following Equation can be written:

$$E = \sum_{i=1}^{n} E_1(1,\omega_i,\zeta_i)\left(\{\varphi_i\}^T[M]\{r\}\right)^2 \tag{3.9}$$

But, as indicated previously, in a wide range of relatively long to very long periods $E_1(1,\omega_i,\zeta_i)$ has no notable variation and may be taken as constant, and thus Eqn. 3.9 can be written as:

$$E = E_1(1,\omega_1,x_1)\sum_{i=1}^{n}\left(\{\varphi_i\}^T[M]\{r\}\right)^2 \tag{3.10}$$

Based on Eqn. 3.10 it can be claimed that, considering E as a constant value (as assumed by Akiyama), the summation term in this Equation must be a constant. In fact, it can be said that as the mode shapes of any linear system are "bases" of a "vector space" V**,** each vector, including $\{1\}$, in this space can be written as a linear combination of "bases", which is:

$$\forall\{v\}\in V\ \exists a_i \in R \mid \sum a_i\{\varphi_i\} = [\Phi]\{a\} = \{v\}$$

Thus, if $\{v\} = \{1\}$ then $[\Phi]\{a\} = \{1\}$ and one can write:

$$[\Phi]^T[M][\Phi]\{a\} = [\Phi]^T[M]\{1\} \tag{3.11}$$

$$a_i = \{\varphi_i\}^T[M]\{1\} \quad \text{And} \quad \{a\} = [\Phi]^T[M]\{1\} \tag{3.12}$$

Now one can write the following equation:

$$\{a\}^T\{a\} = \left([\Phi]^T[M]\{1\}\right)^T\left([\Phi]^T[M]\{1\}\right) = \{1\}^T[M]^T[\Phi][\Phi]^T[M]\{1\} \tag{3.13}$$

But, $[\Phi][\Phi]^T[M]$ in the right hand side of Eqn. 3.13 must be a unit matrix because $[\Phi]^T[M][\Phi] = [I]$, and by pre-multiplication of both sides by $[\Phi]$ one can obtain the desired result. Thus, Eqn. 3.13 may be rewritten as:

$$\{a\}^T\{a\} = \sum_{i=1}^{n} a_i^2 = \{1\}^T[M]\{1\} = \sum_{i=1}^{n} m_{ii} = m_{total} = \text{total mass of MDOF structure} \tag{3.14}$$

And finally, Eqn. 3.10 can be written as:

$$E = m_{total}E_1(1,\omega_1,\xi_1) = E_1(m_{total},\omega_1,\xi_1) \tag{3.15}$$

Eqn.3.15 means that the seismic input energy to a MDOF system is the same as input energy to a SDOF system with same mass, main frequency and damping, provided that the following conditions are met:

1- Input energy is calculated at a specified instant of a record for all modes, say at the end of record.
2- Input energy spectra are constant all over the wide range of periods.

As it can be seen from Fig. 2.1, the constancy of input energy spectra is a simplifying assumption only. In fact, the input energy as expressed by Eqn. 3.9 depends on the shape of the spectrum, and therefore, on the structural features. This fact demonstrates the possibility of existence of an optimal distribution of stiffness to minimize seismic input energy.

4. OPTIMAL DISTRIBUTION OF STIFFNESS

If distribution of mass and damping of the system are assumed to be constant, then distribution of stiffness will be the only effective structural property of linear MDOF systems. For planar structures the distribution of stiffness can be defined by an N×1 vector as $\mathrm{k} = \{\mathrm{k_i}\}$, and for any given input energy spectrum, the seismic input energy to a MDOF system (En) will be a function of the stiffness distribution vector only:

$$\mathrm{En} = \mathrm{En}(\mathrm{k}) \tag{4.1}$$

Now the problem is finding the optimum distribution of stiffness k such that En's value becomes minimal. The main constrain for the stories' stiffness values is raised from the maximum acceptable story drift. Knowing the response spectrum, mass and damping of the system, maximum drift is a function of stiffness distribution only:

$$\mathrm{maxDrift} = \mathrm{f(k)} \tag{4.2}$$

En is the objective function, k is the design variable and $\mathrm{maxDrif}t = \mathrm{f(k)} \le \mathrm{d_{all}}$ is the inequality constraint ($\mathrm{d_{all}}$ is the maximum allowable story drift). The optimization problem solved using a program, developed in the MATLAB environment (Yang W. *et al*, 2005).**V**arious structures with different numbers of degrees of freedom (5 to 40 stores shear buildings) were studied (Haddad Sh. F. 2011). In all studied structures, an interesting solution showed up, based on which, the stiffness distribution must be so that, all stories reach the maximum allowable drift limit, and making the modal displacement of the structure a linear form. The distribution for shear structures with low to moderate height (up to 20 stories), in which first mode governs their modal behavior, best fitted by a parabolic one.

5. OPTIMAL STIFFNESS DISTRIBUTION AND THE HYSTERETIC ENERGY

It was demonstrated that stiffness distribution which causes "equality of maximum drift ratios" in all stories, implies a minimal seismic input energy (the obtained stiffness distribution referred as the *optimal stiffness distribution).* Is this conclusion beneficial as a strategy in structural design? To answer the question, three planar special moment resisting steel frames 12, 16, 20 stories (Fig. 5.1) each with three stiffness distribution pattern (uniform, linear and "optimal") were analysed and designed according to ASCE/SET 7-10 and AISC 2005.

Response spectrum procedure used in structural analysis. Unique conditions were satisfied for each frame (maximum allowable drift and demand to capacity ratios D/C of members).To facilitate the compression of results; a story index is defined as $SI = \dfrac{n-1}{N-1}$, where n is the nth story and N is numbers of whole stories of structures (Khashaee P. 2005). Thus SI=0 means the first story and SI=1 means the most upper story.

Fig. 5.2 shows drift ratio distribution. It is seen in the figure that the optimal stiffness distribution causes the uniformity of drift distribution over the height of frames except at two lower and two upper stories of buildings because of code recommended restriction in sever and suddenly change of stiffness.

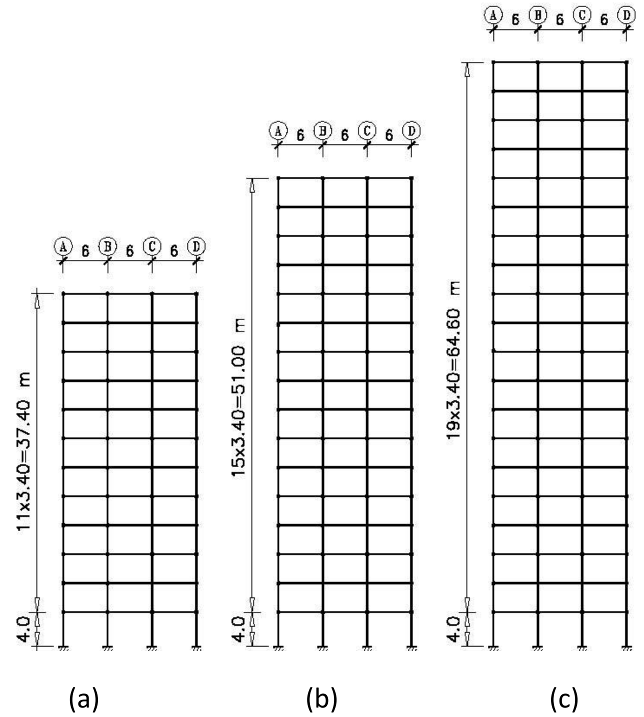

Figure 5.1. Studied frames (a) 12 stories, (b) 16 stories (c) 20 stories

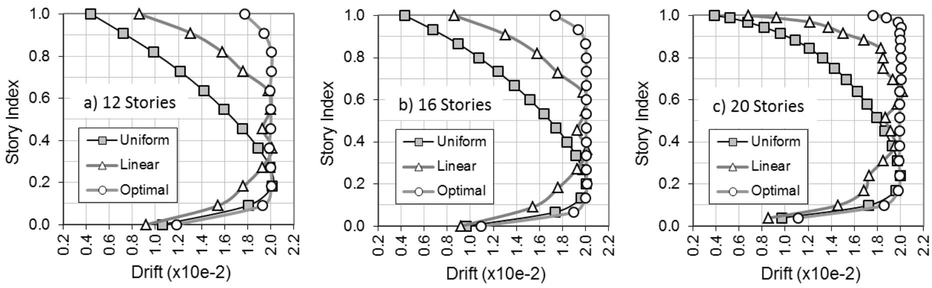

Figure 5.2. Drift ratio distribution for studied frames: (a) 12 stories, (b) 16 stories (c) 20 stories

Resultant stiffness distribution is shown in Fig. 5.3. It is seen in the figure that a step by step reduction in stiffness is intended for linear distribution of stiffness and that the *optimal distribution* is approximately parabolic; although the studied buildings were not shear type frames.

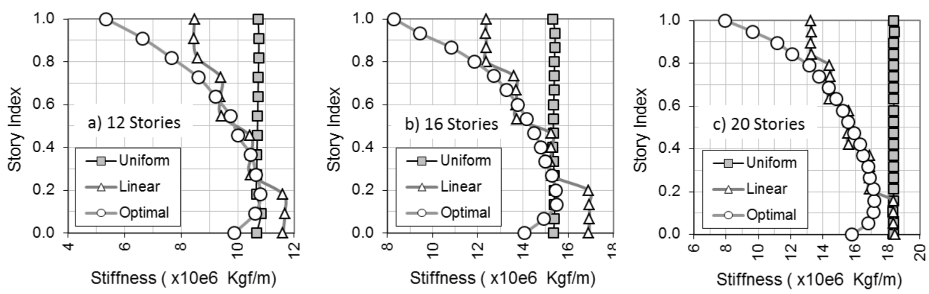

Figure 5.3. Stiffness distribution for studied frames: (a) 12 stories, (b) 16 stories (c) 20 stories

Demand to capacity ratios of all beams of frames are 1 and of columns 1.3; although this is not practically attainable but conceptually is possible. Thus all frames are in the same condition from structural design viewpoint unless in stiffness distribution. To study the effect of this difference, nonlinear analysis was employed. Eight records were selected based on period of structures (Mousavi M., Ashtiany G.M. and Azarbakht A. 2011) from Ashtiany's proposed M6.5 databank and scaled according to ASCE/SEI7-10 for each building. Plastic hinges, according to FEMA356, were used to model the nonlinear behavior of members.

5.1. Hysteretic dissipated energy

Fig. 5.4 shows the total dissipated energy via hysteretic behavior of members. It is seen that in all buildings, frames with the optimal distribution of stiffness, has minimum value of dissipated hysteretic energy.

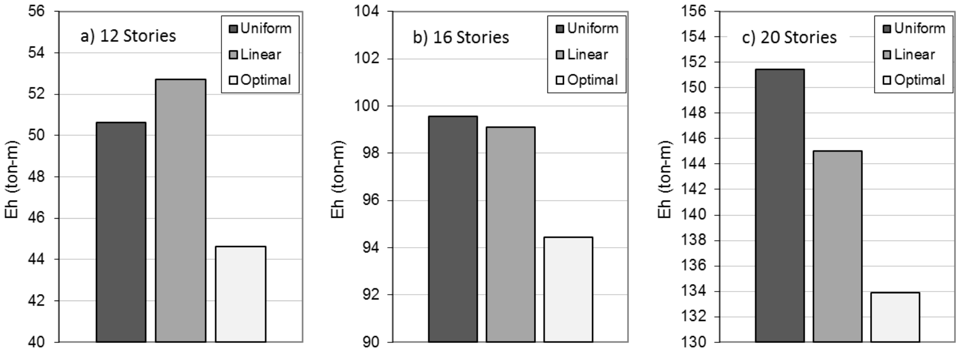

Figure 5.4. Hysteretic dissipated energy in studied frames: (a) 12 stories, (b) 16 stories (c) 20 stories

This reduction in dissipated hysteretic energy gained without employing any seismic controlling.

5.2. Modified Ang-Park damage index

Modified Ang-Park damage index (Kunnath et al. 1992) is defined as Eqn. 5.1:

$$DI = \frac{\theta_m - \theta_y}{\theta_u - \theta_y} + \frac{\beta}{m_y \theta_u} E_h \tag{5.1}$$

Where θ_m is the maximum rotation attained during the loading history; θ_u is the ultimate rotation capacity of the section; θ_y is the recoverable rotation when unloading; m_y is the yield moment; and E_h is the dissipated energy in the section.

Fig. 5.5 shows the distribution of modified Ang-Park damage index over the height of studied buildings. It is seen that in all buildings, frames with the optimal distribution of stiffness, has minimum value of the damage index. Since the features of frames (D/C and max. drift ratio) are similar from code-designing view point, the observed differences is addressed to the difference in the pattern of stiffness distribution.

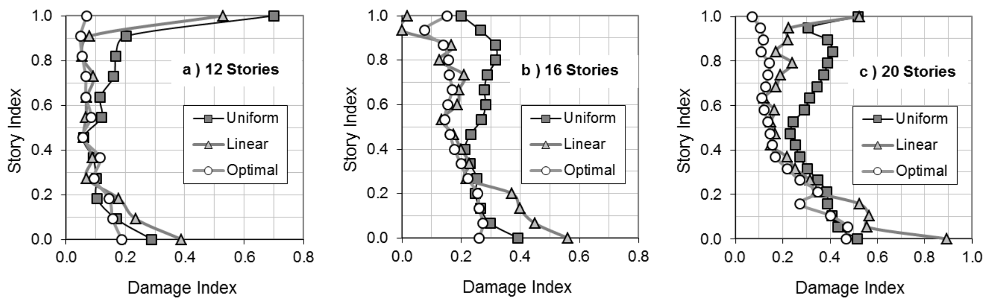

Figure 5.5 Ang and Park damage index distribution: (a) 12 stories, (b) 16 stories (c) 20 stories

Table 5.1 summarizes the overall damage index of the frames. It is seen that the overall damage index for the optimally distributed stiffness frame, considerably is less than two other frame types. Thus it is concluded that the stiffness distribution has a significant effect on the reduction of the studied damage index.

Table 5.1. Overall damage index

Frame type	Modified Ang-Park Index		
	Uniform	Linear	Optimal
12 Story	0.2953	0.3005	0.154
16 Story	0.3245	0.3479	0.2417
20 Story	0.4469	0.5391	0.3534

6. CONCLUSIONS

1- It is possible to find an optimal distribution of stiffness over the height of a multi-story building to minimize the seismic input energy for the structure.
2- For low to moderate height (up to 20 stories) shear structures one possible optimal distribution over the height, in linear range, is parabolic. This distribution implies linearity of modal displacement of the structure, and the equality of drifts in all stories.
3- The obtained optimal distribution of stiffness, for the studied moment resisting steel buildings, causes the minimal dissipated hysteretic energy as the base shear; but it results grater drift ratios of upper stories.
4- The obtained optimal distribution of stiffness, for the studied moment resisting steel buildings, reduces the story and overall modified Ang-Park damage induces by a considerable amount.

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