Introduction to a Robust Period-independent Ground Motion Selection and Scaling Method

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SUMMARY:
A new approach for ground motion selection and scaling for structural analysis is proposed and elaborated. In the proposed approach, ground motion records are selected based on the hazard deaggregation data and then refined according to the observed trends in intensity, frequency content, and duration. Record scaling is done at a frequency bandwidth in which most of seismic energy is concentrated. Effects of the proposed method on the seismic demand parameters are studied using nonlinear time-history analyses on inelastic single-degree-of-freedom systems. The robustness of the proposed approach is demonstrated through a comparative study.

Keywords: Dominant seismic event, Ground motion parameters,

1 INTRODUCTION

Time-history analysis has become an essential tool for performance-based seismic design and evaluation. One of the important steps involved in this process is the ground motion selection and scaling which could substantially affect the outcomes and the design-rehabilitation decisions. Choosing an appropriate number of ground motion records and the selection of a practical effective method of scaling the chosen records to a given level of seismic hazard are still challenging decisions, despite the large amount of research that has been conducted on these issues. In fact, there is no general consensus in the earthquake engineering community on a proper way of handling these problems (NIST, 2012). One main reason is related to the ground motion randomness, and the complex dynamic response of structures being subjected to this aleatory input energy. The wide variability in the structural seismic demands observed when using different available selection and scaling methods is a strong motivation for developing a robust, yet simple ground motion selection and scaling approach which results in a stable central tendency and small scatter when assessing structural seismic demand. This would lead to a higher degree of confidence in demand prediction and a more definite margin of safety against unfavourable structural responses. In this study, a robust ground motion selection and scaling method is proposed and explained in detail. Statistical robustness of this method is examined throughout nonlinear time history analyses.

2 RECORD SELECTION

2.1 Dominant events

The proposed record selection method starts from deaggregation of seismic hazard for a given location. Seismic hazard deaggregation is a technique to decompose the contribution of different earthquakes to a given level of seismic intensity into clustered bins of magnitudes and site-to-source distances, with relative contribution of each bin to the intensity. This technique has been identified as a probabilistically consistent way for finding dominant earthquake magnitudes and site-to-source distances (Bazzurro and Cornell, 1999). Deaggregation of the 2% in 50 years probability of exceedance of spectral accelerations for Victoria, British Columbia, at periods of 1.0 and 2.0 second are shown in Fig. 2.1a and b.

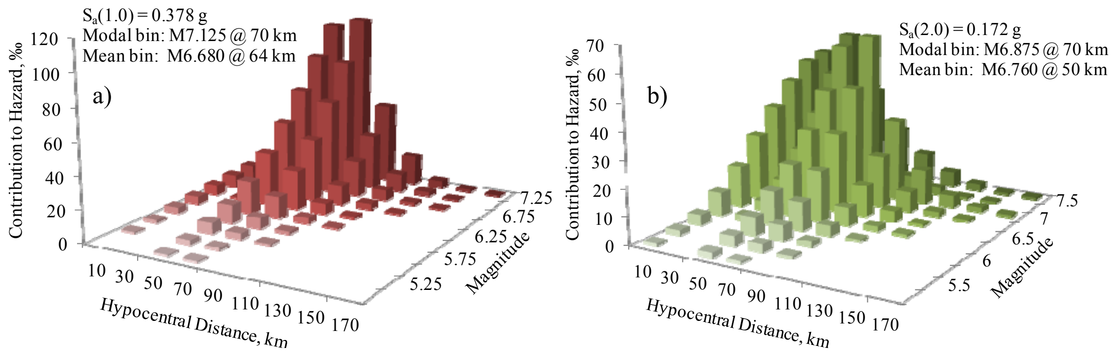

Figure 2.1. Seismic hazard deaggregation for spectral accelerations for Victoria, BC at T of: a) 1.0 s; b) 2.0 s.

In this study, hazard deaggregation results are provided by the Geological Survey of Canada (Halchuk et al., 2007). The following steps are proposed to find the dominant-event scenarios at a given location and to select the event-compatible records:

1. Event bins data (M, R and the contribution thereof) for spectral accelerations at all available periods are combined and sorted in descending order based on relative contributions;
2. For the repetitive event bins with the same M and R values, the one with the highest contribution is kept and the remaining ones are discarded;
3. Only the first *n* bins are stored, based on their relative contributions, and the remaining ones are discarded, where *n* is the number of required records for time-history analysis;
4. Dominant event bins can be either (these 4 options correspond to scenarios S1 to S4 examined in the comparative study of section 4 of this paper):
    a. the first bins with cumulative contribution exceeding 50% of the total contribution of the *n* bins with the number of records for a bin being based on the relative contribution of that bin to the total contribution of the remaining bins, i.e., more records will be selected from the bins with higher relative contributions;
    b. the first *n*/2 bins, which means that two records will be selected from each bin;
    c. all *n* bins, with only one record being selected from each bin; or
    d. the first bins up to a cumulative contribution of 50%, with equal number of records being selected from each bin, i.e., same chance for every event.

The proposed algorithm is meant to ensure that no possible event with a significant contribution will be overlooked and a wide range of frequency content, duration and intensity levels will be obtained with the selected records. The wide range ensures that inter-event variability is included in the process and potentially damaging events are considered, regardless of their magnitude and distance. Event bin-based record selection is consistent with the probabilistic approach of seismic hazard calculation, as events with the same magnitude and distance in the approach result in the same rate of exceedance for a given level of intensity.

2.2 Initial record selection

Magnitude and distance of each dominant-event bin is expanded based on the resolution of the clustered deaggregation data used. In this study, dM = 0.25 and dR = 10 km. For each bin, this expanded range of magnitude and distance was then applied as a selection criteria to all available records in the PEER database (PEER, 2011). Events were only selected from recording station located on very dense soil and soft rock sites ($360 \leq V_{s30} \leq 760$ m/s). Records with poor cut-off frequency were excluded from the initial selection process. It is believed that after a certain magnitude level, the magnitude saturation level, some ground motion characteristics such as frequency content and duration are no longer magnitude dependent (Rathje, 2004). Figure 2.2a and b show the seismological and empirical prediction models for the mean period of acceleration signal as a robust frequency content parameter (see section 2.3.2 for the definitions). These models indicate that the mean period of acceleration becomes independent of magnitude for **M** ≥ 7.25 while remains linearly distance dependent over the whole range considered. This trend was used when records for **M**7 bins were being selected. To include this effect, simply the upper bound of magnitude in the bins with the largest **M**7 could be relaxed. The process of the initial selection was performed for all event bins of the previously defined scenarios. The numbers of initially selected records for dominant-event bins were as high as 160 in the case of moderate magnitude-moderate distance bins (**M**6.125 at 50 km).

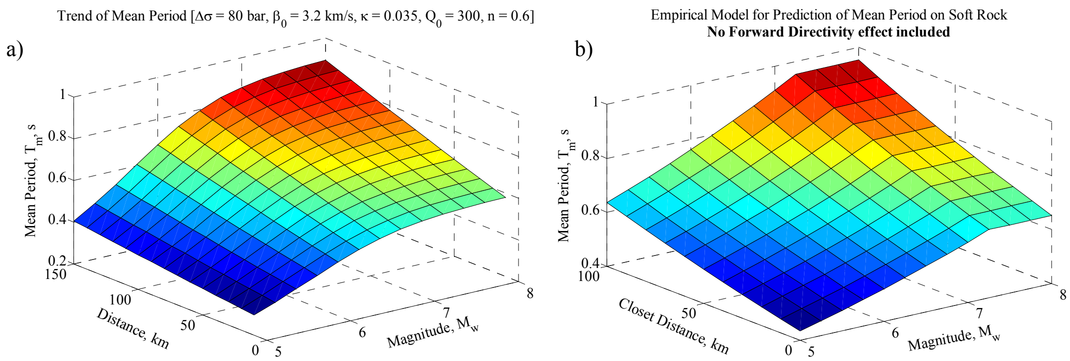

Figure 2.2. Prediction models for mean period of acceleration signal: a) seismological; b) empirical

2.3 Selection refinement

As the initial selection criteria, seismological metadata such as magnitude, distance, and soil profile are often used to find event-compatible records from the available databases such as PEER. When the search criteria are only limited to the mentioned metadata, a large number of event-compatible records may be found. Due to the required computation capacities it may not be practical to use all of these records in the time-history analysis. Therefore a systematic approach is required to refine and condense the initially selected records to the required numbers. Random refinement is often suggested in most of the ground motion selection schemes such as ATC-58 (Haselton, 2009). This may lead to a high scatter in the computed demand because all of the selected records are given the same chance, regardless of their damaging potential. Instead of random refinement, records can be ranked based on the trends in damaging potential related parameters. In this way, records close to the central tendency of damage-related parameters will be selected and the records with exceptional characteristics will be discarded. Generally, damaging potential of earthquake records are attributed to three major classes of signal parameters: 1) amplitude (intensity); 2) frequency content; and 3) strong shaking duration (Kramer, 1996).

2.3.1 Amplitude-related parameters

In general, amplitude-related parameters can be calculated from: a) time-domain signal properties such as peak ground acceleration (PGA) or velocity (PGV); or b) spectral-domain parameters such as acceleration intensity spectrum, ASI, (integral of the acceleration spectrum). A review of the most popular amplitude-related parameters was recently performed by (Ye et al., 2011). Generally, acceleration- and velocity-based parameters are well-correlated with the extent of damage in stiff and flexible structures, respectively.

2.3.2 Frequency content-related parameters

Frequency content can be specified by means of: a) time-domain parameters such as ratio of PGV to PGA; b) frequency-domain parameters such as mean period, T_m, i.e., the centre of gravity of the acceleration power spectrum between 0.25 and 20 Hz (Rathje, 2004); or c) spectral-domain parameters such as predominant spectral acceleration (T_{pSa}) or velocity (T_{pSv}), i.e., periods at peak spectral ordinates. The effectiveness of the various frequency content-related parameters is reviewed in (Kumar et al., 2011). Besides central tendency of frequencies, frequency band-width is often required to express the frequency content in a more precise way. Power spectrum moments have been shown to be effective tools to specify the frequency band-width of ground motion signals (Kramer, 1996). A study conducted on 6548 horizontal records available in the PEER database shows that the mean period has the highest average inter-correlation coefficient with the other frequency content-related parameters. It is noteworthy that this parameter has 86% linear correlation with a much simpler parameter, 2π PGV/PGA.

2.3.3 Duration-related parameters

The duration in which a time-domain intensity parameter is higher than a relative or absolute threshold (e.g., 5% of PGA) has been widely used to define most of the duration-related parameters. The reader is referred to (Hancock and Bommer, 2006) for a comprehensive review of the duration-related parameters.

In spite of the extensive research done to find the most damage-related ground motion parameters, there is no single unique parameter which could be suitable for all types of structures regardless of their dynamic properties (Ye et al., 2011). This may imply that instead of looking for the best parameter of each class, i.e., the one with the highest correlation with damage, it would be better to find the parameter which has the highest inter-correlation with the other parameters in the same class. This parameter is referred to herein as the best estimator. This ensures that when the value of the best estimator is high, other parameters are likely to be high as well. Studies have shown that among different correlation methods, rank correlation can be more appropriate when the relation between variables is nonlinear. Ranked-correlation (Spearman correlation coefficient) is defined as the linear correlation coefficient between the ranked variables:

$$\rho = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2 \sum_i (y_i - \bar{y})^2}} \tag{2.1}$$

where x_i and y_i are the rank of variables X_i and Y_i. Fig. 2.3 shows the colour-coded tables of 15 amplitude-related and 7 frequency content-related parameters calculated for a set of 160 records. In this table, darker colours represent higher values. The best estimator of amplitude and frequency content are located in the column under the name of SMV (sustained maximum velocity, the 3rd highest velocity cycle) and T_m (mean period), respectively. The table is sorted by the values in the best estimator's column in ascending order. Concentration of darker colours in the top rows of the table indicates that most of the parameters have high value when the best estimator is high.

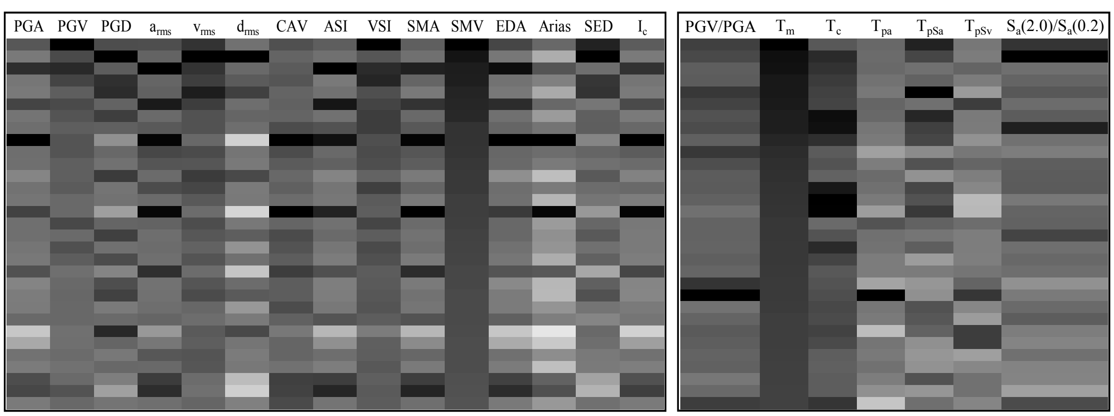

Figure 2.3. Colour-coded table of normalized amplitude- and frequency content-related parameters

The proposed record refinement procedure can be summarized in the following steps:

1. The required number of records from a given bin, *m*, is set;
2. Ground motion signal properties are calculated for the three classes of damage-related

parameters:

a. amplitude-related, including: PGA, PGV, PGD, a_{rms}, v_{rms}, d_{rms}, CAV, ASI, VSI, SMA, SMV, EDA, Arias, SED, I_c, etc;

b. frequency content-related, including: 2π PGV/PGA, T_m, T_{pSa}, T_{pSv}, $S_a(2.0)/S_a(0.2)$, etc;

c. duration-related parameters, including: $D_{5\text{-}95}$, $D_{5\text{-}75}$, $D_{Bracketed}$, $D_{Uniform}$, etc.;

3. Inter-correlation between the parameters of each class is calculated by either linear correlation coefficients (Pearson correlation) or, preferably, rank correlation coefficients (Spearman correlation);
4. For each class of signal parameters, the one with the highest average inter-correlation ratio is defined as the best estimator for that class of damage-related parameter;
5. Deviation of the best estimator of each record from the expected value of the best estimator, ϵ_i, is calculated for all of the records within the bin:

   a. when the best estimator is normally distributed: $\epsilon_i = (x_i - \mu_X)/\sigma_X$

   b. when the best estimator is log-normally distributed: $\epsilon_i = (\ln(x_i) - \mu_{\ln X})/\sigma_{\ln X}$

   where x_i is the value of the best estimator for the i^{th} record in the bin, μ_X and σ_X are the mean and standard deviation of the best estimator for all of the records respectively, and $\mu_{\ln X}$ and $\sigma_{\ln X}$ are the mean and standard deviation of the natural logarithm of the best estimator for all of the records respectively;
6. For every record, average of ϵ_i is obtained, $\epsilon_{i,avg}$;
7. Records are ranked based on the $\epsilon_{i,avg}$, with the record with the minimum $\epsilon_{i,avg}$ being ranked no. 1; and
8. Ranked records are sorted in an ascending order and the first m records are selected.

Typical outcome of this process is shown in Fig. 2.4. In this example, 160 records belonging to the bin of **M**6.125 at 50 km were refined to 10 records. The best estimators were SMV, T_{ma} and $D_{Bracketed}$. The properties of the selected records are circled in the plots on the right-hand side of the figure.

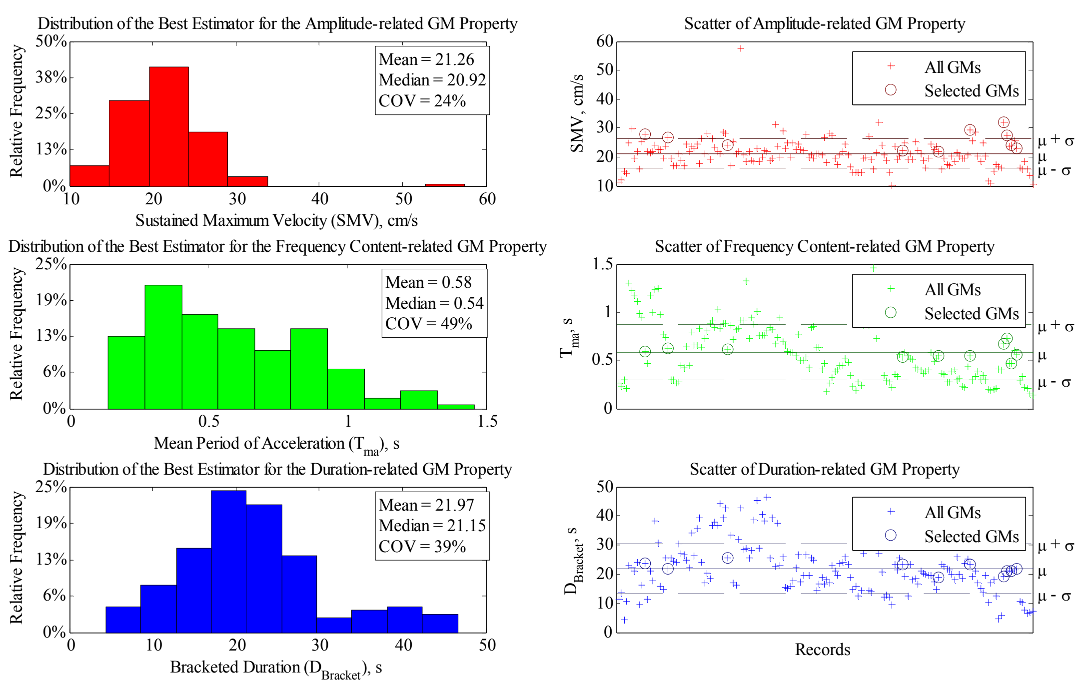

Figure 2.4. Distribution of the best estimators and outcomes of the proposed refinement process.

3 RECORD SCALING

3.1 Conventional scaling methods

Ground motion records selected for time-history analyses are often required to be scaled to a predefined level of seismic hazard or intensity. The main reason for scaling is that the exceedance rate of the intensity of an as-recorded ground motion (e.g., spectral acceleration at a given period) does not match the target hazard level. Generally, this can be done in time or frequency domain. The latter approach changes the frequency content of a given record while the former linearly adjusts the amplitude of the record without distorting its frequency content. A mathematically consistent way of time-domain scaling is to match the spectral acceleration of a record [$S_a^{\text{Record}}(T_1)$] to the ordinate of the target spectrum at the fundamental period of the building under study [$S_a^{\text{Target}}(T_1)$].

$$SF_{MFP} = S_a^{\text{Target}}(T_1) / S_a^{\text{Record}}(T_1) \tag{3.1}$$

This assures that the building will experience an acceleration level consistent with the intended level of hazard if the building remains essentially elastic and vibrates mainly in its first mode. This may not be the case in most earthquake engineering applications as this method does not guarantee a target-consistent level of hazard in the elongated periods as a result of yielding or in the higher modes of vibration. Drawbacks of the scaling at the fundamental period can be shown if a relatively high frequency record is being scaled for a flexible building or vice versa. Figure 3.1 shows a comparison between the scaled spectra of such a records and a typical target spectrum. The ground motions were selected from the same event (1994 **M**6.69 Northridge) and have been recorded on the same type of soil profile (V_{s30} = 660 and 405 m/s) at almost the same closest distance from the fault rupture (26 and 32 km). The high and low frequency records are from component 270 Monte Nido Fire Station and Playa Del Rey - Saran station, respectively. High frequency record was matched at T = 1.8 s corresponding to the fundamental period of a 9-storey buckling-restrained braced frame (BRBF) and the low frequency one is scaled at T = 0.6 s which is the fundamental period of a 3 storey BRBF. In Fig. 3.1, the taller, more flexible building will experience excessive acceleration in its higher modes (e.g., T = 0.7 or 0.4 s) whereas the shorter, stiffer building will undergo significant acceleration if its fundamental period shifts toward the longer values (e.g., T = 1.2 s). Nonlinear time-history analysis shows that this unintended level of hazard will result in an overestimated inter-storey drift ratio (4% for the flexible building and 3% for the stiff building).

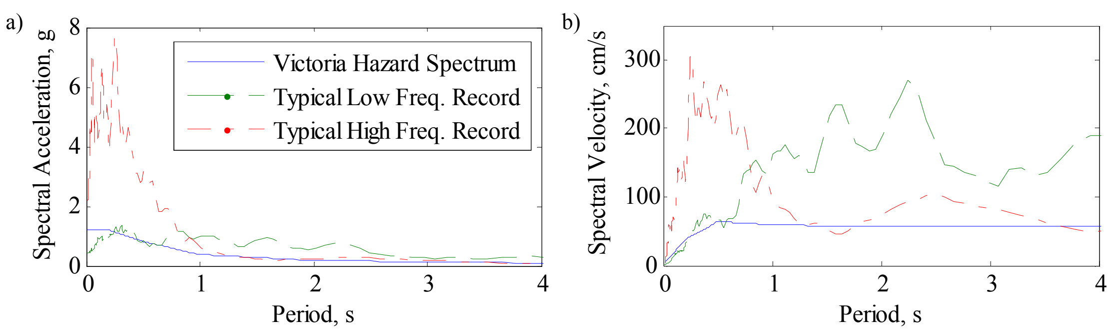

Figure 3.1. Excessive overshoot of input seismic energy when frequency content of record is neglected

This may be avoided if the scaling is done over a range of periods centred to the fundamental period of structures. Usually this range has been suggested to start from $0.2T_1$ (higher modes) and end at $1.5T_1$ (elongated period). It should be noted that the upper bound of this period range depends on the extent of damage, and the post-yield stiffness of the system. Elongated period as high as $3.0T_1$ has been reported in the literature (NIST, 2012). One can take the ratio of average spectral ordinates in this range as the scale factor as suggested in (Atkinson, 2009, Baker, 2011):

$$SF_{ASE} = \sum_{0.2T_1}^{1.5T_1} S_a^{\text{Target}}(T) \Bigg/ \sum_{0.2T_1}^{1.5T_1} S_a^{\text{Record}}(T) \tag{3.2}$$

Scale factor also can be calculated by minimizing the squared error between the target spectrum and the scaled spectrum of the record in the mentioned range $[(S_a^{\text{Target}}(T_i) - SF \times S_a^{\text{Record}}(T_i))^2]$. This scale factor can be obtained by forcing the first derivate of the squared error to be zero:

$$SF_{MSE} = \sum_{0.2T_1}^{1.5T_1} [S_a^{\text{Target}}(T)\, S_a^{\text{Record}}(T)] \Bigg/ \sum_{0.2T_1}^{1.5T_1} [S_a^{\text{Record}}(T)]^2 \tag{3.3}$$

All these types of period-dependent scaling need a predefined range of periods and none of them can completely avoid the excessive overestimation of the hazard within this range. The fundamental period is also subjected to the modelling assumptions. In addition, in the process of hazard calculation all these periods are treated independently from each other. In other words, the probability of exceeding at all these periods is not known and may not be possible to calculate. Recent studies proposed using a conditioned mean of the scaled spectra to overcome this problem (Baker, 2011). In this method ground motion prediction equation for a pair of dominant M and R is converted to a conditional mean spectrum based on the correlation between spectral acceleration ratios before and after fundamental period. In fact, this method replaces the uniform hazard spectrum with a conditioned spectrum which typically has less intensity in period range lower and higher than the fundamental period. This new spectrum is then used to find a set of records which median of their scaled spectra is close to the conditioned spectrum. This method is meant to take into account the more realistic spectral shape when records are being scaled at the fundamental period.

3.2 Least Moving Average scaling method

Alternatively, record scaling can be performed independently from the dynamic characteristics of the structures under study. This means that a unique scale factor is used for all types of structures regardless of their fundamental, higher modes, and elongated periods. In fact, early generation of scaling methods mostly relied on the normalization of record amplitudes by means of a specific level of time-domain intensity measures such as PGA or PGV (Shome and Cornell, 1998, Kurama and Farrow, 2003). However, these intensity measures can not be well-correlated with expected damage in all types of structures. As a result, damage prediction is biased to the measure of intensity used for the record normalization. In addition, site target design spectrum (e.g., uniform hazard spectrum (UHS)) can be completely neglected when these methods are used. Instead of record normalization or matching at the fundamental period (or a range), it is suggested to scale at a period range over which the spectral shape of record is similar to the target spectrum. Past studies have shown that by this method, records are matched to the target in a frequency band-width where most of seismic energy is concentrated. If the selected records cover a sufficient range of frequencies (records with different frequency content), this scaling method leads to spectral shape-consistent results. The method proposed herein is called Least Moving Average because it scans all the period ranges of the spectrum and finds a narrow period band in which the average of error between the target and record spectra is minimum. The proposed ground motion scaling approach is summarized in the following steps:

1. The 5% damped target acceleration spectrum (e.g., UHS) is defined for 200 period points, starting from 0.01 to 10 second (period intervals expand exponentially);
2. The 5% damped acceleration spectrum of record is calculated for all 200 period points, T_i;
3. Spectral error ratios, i.e., ratios between the target spectrum and the spectrum of the original records, $\left[S_a^{\text{Target}}(T_i) / S_a^{\text{Record}}(T_i)\right]$, are computed for all periods;
4. Minimum and maximum inclusive averaging periods, $T_{\min}$ and $T_{\max}$, are set (e.g., 0.1 and 10 s),
5. Averaging band is defined, either as a fixed or a variable band:

a. fixed band: averaging band width is constant for all the moving periods (e.g., ΔT = 0.25 s);

b. variable band: averaging band width increases as the moving period (T_i) moves towards longer periods; for every moving period, averaging band starts at αT_i and ends at βT_i ($\alpha = 0.5$ and $\beta = 1.5$ are suggested);

6. The first and the last moving periods, T_1 and T_{end}, are computed:

   a. for fixed band averaging: $T_1 = T_{min} + \Delta T$ and $T_{end} = T_{max} - \Delta T$,

   b. for variable band averaging: $T_1 = T_{min} / \alpha$ and $T_{end} = T_{max} / \beta$,

7. The type of average is selected; it can be either:

   a. arithmetic moving average: $\frac{1}{n_{T_i}} \sum \left[S_a^{\text{Target}}(T_i) / S_a^{\text{Record}}(T_i) \right]$;

   b. geometric moving average: $\exp\left(\frac{1}{n_{T_i}} \sum \ln\left[S_a^{\text{Target}}(T_i) / S_a^{\text{Record}}(T_i) \right] \right)$;

8. The scale factor is the minimum of the moving averages.

Results from the proposed method for typical high and low frequency records are shown in Fig. 3.2: high frequency records are matched to target at periods close to the mean period of the signal, (T_m), whereas low frequency records are matched in the low frequency region of the target spectrum.

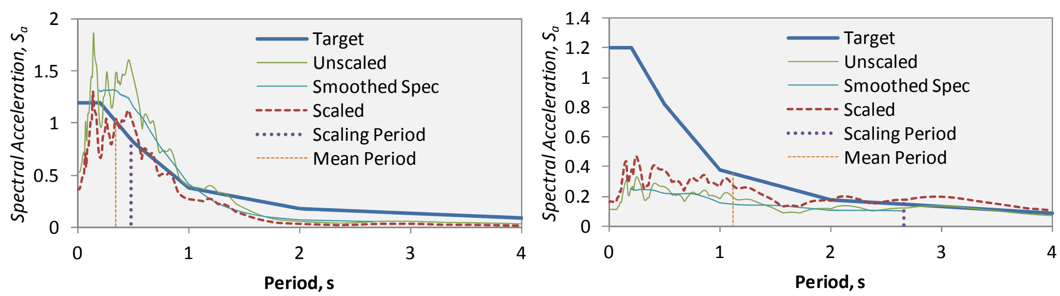

Figure 3.2. Scaling by Least Moving Average for typical: a) high frequency; b) low frequency record

4 COMPARATIVE STUDY

The effectiveness of the proposed method was examined using nonlinear time analyses performed on a series of inelastic single-degree-of-freedom (SDOF) systems. A series of dedicated computer programs for record selection and refinement, record scaling, and SDOF inelastic time-history analysis were developed. This computer code was used to evaluate the possible impacts of different dominant-event scenarios, number of records, and record scaling methods on two demand indices: 1) peak ductility; and 2) cumulative ductility. Four dominant-event scenarios (S1 to S4), three number of records (10, 20 and 40 records), and three record scaling methods were investigated.

4.1 Record scaling

Ground motion records were scaled using three methods: 1) matched at fundamental period (Eqn. 3.1); 2) least squared error (Eqn. 3.3); and 3) least moving average. The average of acceleration spectra is found to be insensitive to the dominant-event scenarios and the number of records in Fig. 4.1. The same behaviour was observed for the 'mean plus one standard deviation' of the acceleration spectra and the maximum of the acceleration spectra was only slightly affected by the dominant-event scenario in the short period range. These results indicate that the proposed selection and scaling method reduces the undesirable variations in input seismic energy that is generally observed when changing the dominant events or the number of records.

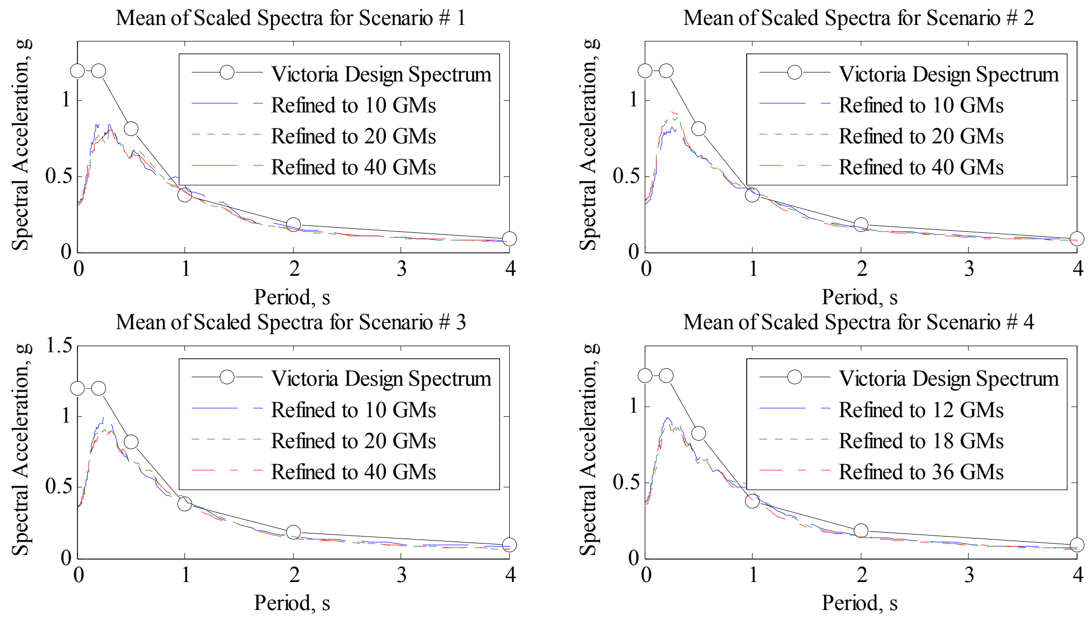

Figure 4.1. Effects of different scenarios and number of selected records on the average of scaled spectra

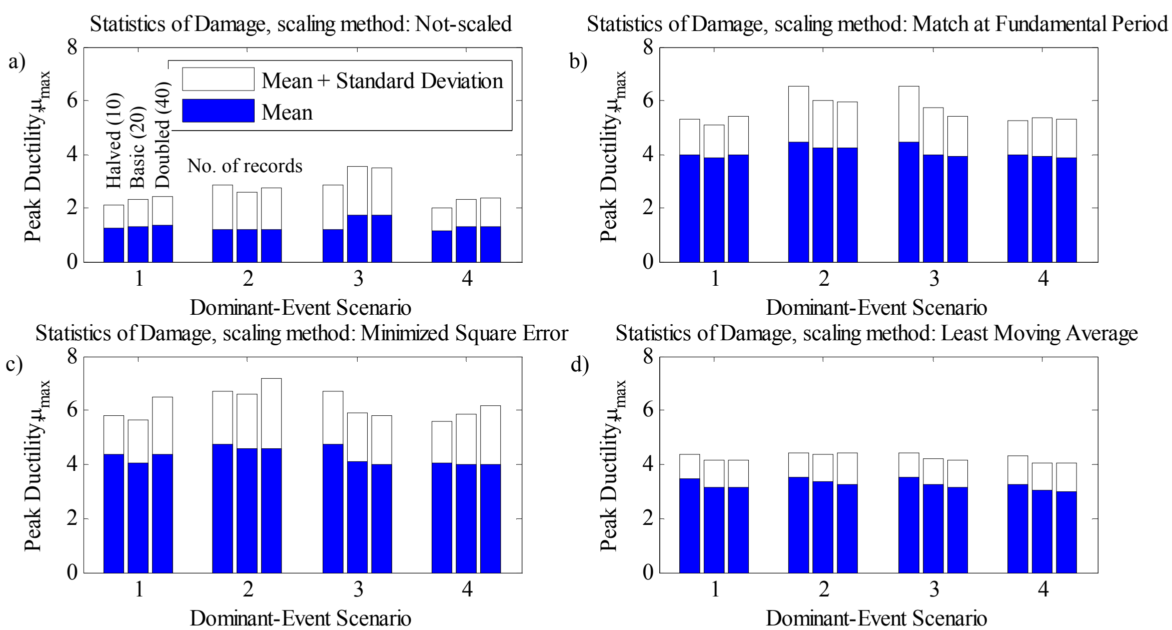

Figure 4.2. Statistics of peak ductility computed for different selection scenarios and scaling methods

4.2 SDOF analysis and results

The records selected for the previously discussed hazard scenarios were refined, scaled and applied to a series of inelastic SDOFs designed for 2% in 50 years seismic hazard in Victoria, BC. The SDOFs had periods of 0.5, 1.0, 1.5 and 2.0 s and were designed with a force reduction factor of 4.8. Inelasticity was simulated using Bouc-Wen plasticity model (Ikhouane et al., 2007). The model can reproduce smooth hysteresis response including Bauschinger effect, which is appropriate for modelling buckling-restrained braces exhibiting stable, full, and symmetric hysteresis response without stiffness and strength degradation. Bouc-Wen model parameters were calibrated throughout optimization study conducted on the results of a full-scale buckling-restrained brace test described in (Tremblay et al., 2006). Viscous damping ratio was set to 3% of the critical damping. In total, 32000 nonlinear time-history cases were carried out. Figure 4.2 shows the mean and 'mean plus one standard deviation' of the peak ductility of the analyzed SDOFs for different number of records, scaling methods, and dominant-event scenarios. It can be seen that the mean of peak ductility does not change when the basic number of records is doubled or halved. Mean is also stable regardless of the selected dominant-event scenario. The least moving average scaling method (Fig. 4.2d) shows the most robust statistics. Variability is limited (COV $\leq$ 25%) and insensitive to the dominant-event scenario and the number of record used for analysis. The same trend was observed for the cumulative ductility.

5 CONCLUSIONS

A practical and statistically robust ground motion selection and scaling method is presented and its efficiency is qualified throughout nonlinear time-history analyses. The unintended consequences of the conventional ground motion selection and scaling methods are shown. Step-by-step instructions for finding the dominant events (from hazard deaggregation data), refinement of the initially selected records, and record scaling are provided. Spectral calculations and time-history analyses showed that the proposed method leads to stable expected values of the demand with a limited variability.

ACKNOWLEDGMENT
The financial support provided by the Natural Sciences and Engineering Research Council of Canada (NSERC) for the Canadian Seismic Research Network (CSRN) is acknowledged. Scholarship awarded to the first author by Structural Steel Education Foundation (SSEF) is also acknowledged.

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