Safety of Foundations Located on Slopes And Subjected to Repeated Loads

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SUMMARY:
Problem of foundations located on slopes have been the subject of many studies. However, such foundations are usually considered to undergo non-repeated non-variable loads. In many situations, foundations are under repeated variable loads either statically or dynamically. In this paper, lower bound shakedown theory is employed to evaluate the safety factor of foundations placed on the slopes subjected to repeated loads. Static and dynamic repeated loads are supposed to be applied on the foundations and safety factor are determined for each case separately. Foundations are supposed to be shallow and rigid. Soil is supposed to obey the Mohr-Coulomb yield criteria. It is shown that ignoring the repetition of loads may lead to overestimating the load capacity of foundations on slopes.

Keywords: Shakedown, Slope, Repeated load, Safety factor

1. INRODUCTION

In practice, there are many situations where pavements, foundations of retaining walls, machinery foundations and so on that have to be located on or adjacent to slopes. Most available literature concerned with the evaluation of bearing capacity of slopes for the static case (Meyerhof, 1957, 1963; Hansen, 1970; Vesic, 1973). Besides, limited works devoted to foundation on slopes subjected to dynamic loads and in particular earthquake loads. Zhu (2000) used upper bound limit analysis and earthquake reduction factor to determine bearing capacity of foundations on sloping grounds. Seismic bearing capacity of foundations on slopes was investigated by Kumar et.al. (2003) using stress characteristics method.

Some foundations such as the offshore platforms and machinery foundations bear repeated loads statically or dynamically. Foundations like that, might fail under loads much smaller than their collapse load. A trivial solution for such cases is to conduct a step-by-step load-displacement nonlinear analysis of foundation-slope system, considering all repeated loading program which obviously is time taking and uneconomical. Another alternative is shakedown analysis.

Shakedown method is a subset of limit state method. It's most important advantage over other limit state methods is the ability of considering the repeated loads. Analogues to limit analysis, shakedown theories are presented in the forms of lower bound and upper bound. Besides both lower bound and upper bound theories have been develops for static and dynamic repeated loads.

Static shakedown theory was first introduced by Melan (1938). Koiter (1956) developed upper bound shakedown theory for static loading. Dynamic shakedown was pioneered by Ceradini (1969) for structures subjected to inertial variable loading. Maier (1969), utilizing finite element method and linear programming, converted shakedown theory to an optimization problem. The objective of

Maier's solutions was to maximize a factor multiplied to all possible loadings on the structure. This factor is referred to load factor or shakedown factor of safety.

Applications of shakedown theory in geotechnical engineering mostly concerned with pavements under traffic loads (Hossain and Yu, 1996; Yu and Hossain, 1998; Shiau, 2001). Foundation shakedown of offshore platforms under vertically applied dynamic loads was studied by Haldar et al. (1990), employing upper bound shakedown theory. However, their research was concentrated on the effect of pore water generation due to shakedown of the footing-soil system. Faria (2002) investigated foundation shakedown of an offshore platform under combined static loads. Arvin et.al. (2011) studied shakedown bearing capacity of foundation on horizontal ground using lower bound shakedown method. They used a numerical method developed by Yu and Hossain (1998) for pavement static shakedown and extended later by Arvin et al. (2012) to determine the safety of slopes under repeated seismic loads, for determination of bearing capacity of foundations subjected to repeated static and dynamic loads.

The present study aims to determine the bearing capacity of foundations on slopes subjected to static and dynamic repeated loads. Lower bound shakedown method is employed herein using the method utilized by Arvin et.al. (2011).

2. SHAKEDOWN THEORY

The lower bound static shakedown theorem states that:

Shake down will occur, if a (constant) selfstress state σ^r exists such that superposition of this state and the elastic response to the given loading program at all elements and instants leads to stresses below the yield limit (Maier, 1969).

The lower bound dynamic shakedown theorem states that:

If a fictitious response $u_i^*(x,t),\ \varepsilon_{ij}^*(x,t), \sigma_{ij}^*(x,t)$ (displacement, strain and stress respectively) and a time independent residual stress field $\sigma_{ij}^r(x)$ can be found such that:

$$f\left(\sigma_{ij}^*(x,t)+\sigma_{ij}^r(x)\right)\leq 0 \tag{2.1}$$

Then, shakedown will happen at real response (Ceradini, 1980).

A fictitious response is any elastic solution of systems due to external repeated actions including external forces and strains. It is called fictitious firstly because it is purely elastic and secondly is not obtained necessarily for the real initial conditions. The real response is what actually happens for the systems in reality under variable repeated loads.

Dynamic shakedown theory can be stated in mathematical form as:

$$\lambda = \max\left\{ \alpha \left| \begin{array}{l} f\left(\alpha\sigma_{ij}^*(x,t)+\sigma_{ij}^r(x)\right)\leq 0 \\ \sigma_{ij,j}^r = non-repeated\ \ actions\ in\ domain \\ \sigma_{ij}^r.n_j = non-repeated\ actions\ on\ \ free\ boundaries \end{array} \right. \right\} \tag{2.2}$$

In this study the Eqn. 2.2 is solved by linear programming approach.

3. NUMERICAL METOHD

Based on shakedown theory, two separate steress field, namely elastic and residual stress field must be established. Then, the aforementioned stress field are involved in the optimization process to find the best (maximum) load factor λ. The numerical method employed by Arvin et.al. (2011) is utilized to optimize the results. In the following a brief description of the applied numerical method is presented.

3.1. Elastic Stress Field

A vertical static or dynamic load is supposed to be imposed on a rigid smooth strip footing resting on top of a slope. The system is discretized by triangular elements using the mesh generation ability of Plaxis software (Fig. 3.1). Load is applied on the node A in the middle of the foundation as depicted on Fig 3.1. All nodes under the foundation are assumed to move vertically with the same amount. Therefore, displacements of nodes under the footing are tied to that of node A.

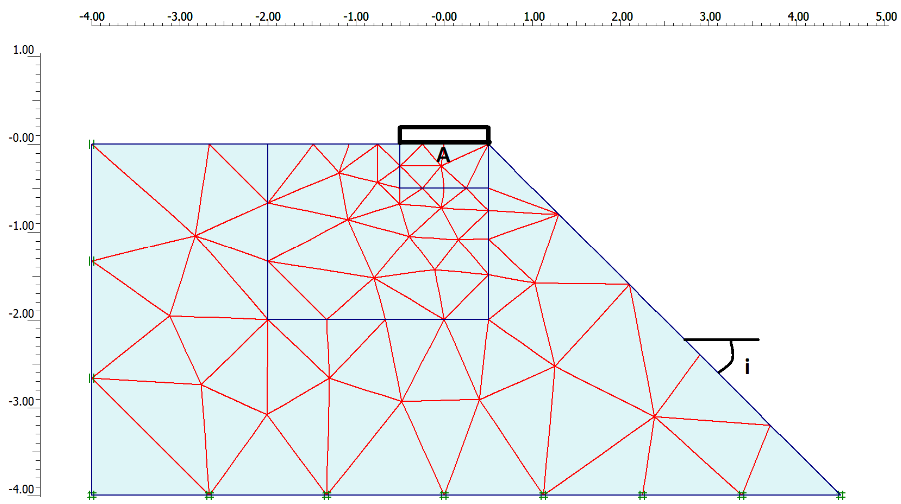

Figure 3.1. Typical footing-slope system considered in the present study

The governing equation of an elastic system under external dynamic loads is as follows.

$$M\ddot{u} + C\dot{u} + Ku = P(t) \tag{3.1}$$

Where M, C and K are mass, damping and stiffness matrices respectively. P(t) is the time dependent dynamic load which is imposed on node A (Fig. 3.1). For elastic analysis, M and C are eliminated from Eqn. 3.1 and P is considered as a constant load. Classic damping is considered. That is, damping matrix is the linear combination of mass and stiffness matrices or $C = \eta M + \zeta K$ where η and ζ are constant coefficients, obtained by considering the first and second frequency of the soil mass.

Implicit time integration method of Newmark (Bathe, 1982) was employed to find the solution of Eqn. (3.1). Finally, elastic stresses at the corner nodes of the elements were obtained and used in the optimization process.

3.2. Residual Stress Field

The same mash developed for the elastic analysis is also considered to obtain residual stress field. Residual stresses are supposed to distribute linearly across the elements. Stress discontinuity lines are between elements. Equilibrium equations in the body and on the boundaries must be satisfied for residual stresses. In addition, discontinuity conditions have to be obeyed by the stresses along the discontinuities. To find more details see Yu and Hossain, 1998 and Arvin, et.al. 2011.

3.3. Optimization

The maximum repeated load that can be applied on the foundation is a portion of available load domain and referred to as the load coefficient λ. The aim is to find the maximum value of λ under some constraint. The constraints consist of equality constraints and non-equality constraints. Equality constraints are composed of equilibrium of residual stresses in domain and on the free boundaries and stress discontinuity constraints. Residual stresses and combination of residual and elastic stresses everywhere and at any time must be inside the yield surface. Using the piecewise linearized Mohr-coulomb yield criteria developed by Sloan (1998), required inequality constraints are obtained.

Since objective function and constraints are of linear type, linear programming approach (simplex method) is employed to solve the problem. At the end of optimization process, load factor and residual stress field are determined. For further details detail see Yu and Hossain, 1998 and Arvin, et.al. 2011.

4. RESULTS

In order study the effects of ground inclination and load repetition on bearing capacity of strip foundations some illustrative examples were considered.

4.1. Illustrative Examples

A rigid strip foundation is considered to be placed on the tip of a slope. The foundation width and slope height were taken as 1m and 4m respectively (Fig. 3.1). Two different slope inclination angles i=45° and i=60° were considered to study the effects of ground inclination on the foundation bearing capacity. Load is applied on the center point of the foundation as a unit linear load for static analysis and as a linear unit sin load for dynamic analysis. Plaxis standard boundary conditions were imposed on the boundaries. Side boundaries were placed far enough so that the effects of boundaries on the results became negligible. Poisons ratio was assumed to be 0.333 for all examples resolved in the present study.

4.2. Statically Applied Load

Standard form of bearing capacity equation consists of cohesion factor N_c, embedment factor N_q and weight factor N_γ. To investigate the load repetition on static bearing capacity of foundations on slopes, a unit weight was employed in the center of the footing. Then numerical approach described in section 3 was performed to find λ value. Obviously for a weightless cohesive soil, the obtained λ value is directly equal to N_c. Using aforementioned strategy, N_c values were obtained for i=60° and for different internal friction angle by shakedown analysis. N_c values in Elastic limit and in plastic limit were also obtained for the same footing-slope system. The results were shown in Fig. 4.1.

As Fig. 4.1 shows, shakedown limit is less than collapse limit and bigger than elastic limit. It shows clearly that foundation under repeated load cannot be treated like those under monotonic loads. Besides, N_c increases with increase in internal friction angle.

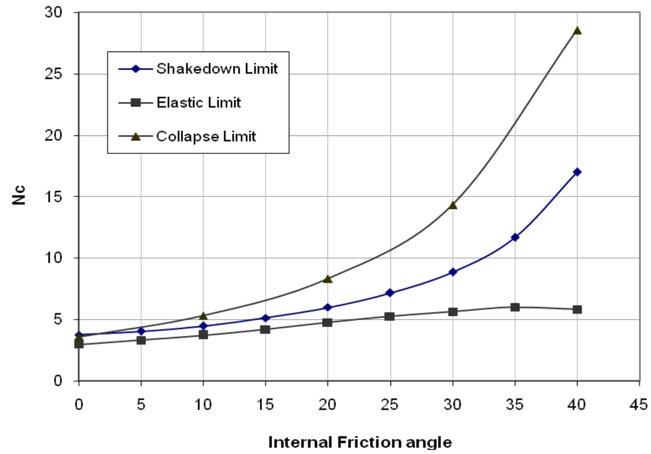

Figure 4.1. N_c values vs different φ for footing on slope with i=60°

In Fig. 4.1, elastic limit was obtained by elimination of residual stress field in the optimization process, while collapse limit are illustration of Hansen (1971) results.
Effects of slope inclination angle on shakedown bearing capacity of foundations were examined by calculating N_c values for i=45°, i=60° versus different values of φ. Results of the present study were presented along with the values of N_c for i=0° derived from Arvin. et.al. (2011) in Fig. 4.2.

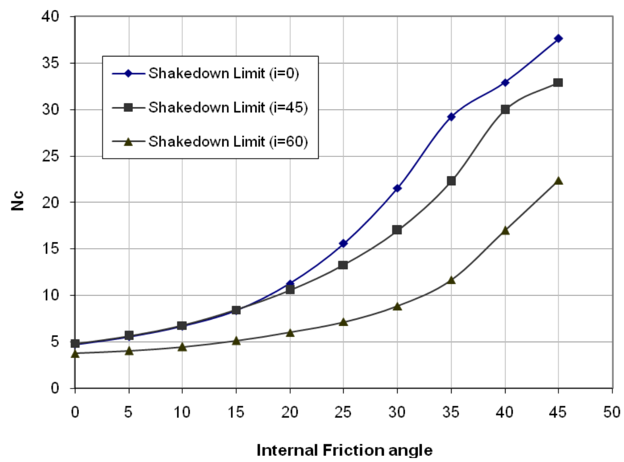

Figure 4.2. N_c values vs different φ for footing on slopes with different inclination angles

Results show that raise in slope inclination angle leads to reduction of footing bearing capacity.

4.2. Dynamically Applied Load

According to shakedown criterion, the footing is safe if it finally settles to elastic state against repeatitive prescribed dynamic load. In general situations, when time history of dynamic load repetition is unknown, dynamic load may be conceived as an inertial load repeated virtually in time. An imaginary sufficient time between two subsequent loadings is assumed during which, motion caused by the previous loading cease to develop due to material damping (Fig. 4.3)

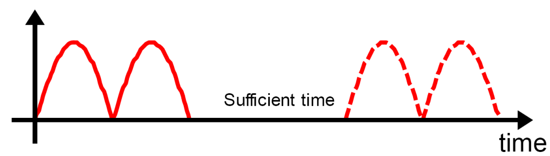

Figure 4.3. Two successive half-sine loads (solid line) and imaginary reloading (dashed line) considered to evaluate dynamic shakedown limit load of footing in this study (Arvin. et.al., 2011)

For the present study, as shown in Fig. 4.3, vertical dynamic loading is considered as a number of successive half-sine loads, applied on the center of footing. The peak value of load is equal to unity. Number and period of the load for each loading can be different and depend on the situation.

Results of dynamic shakedown analysis were presented against parameter T_s/T_m where T_s is the dominant period of slope and T_m is the mean period of dynamic loading. To produce different values of T_s/T_m for a specified footing-slope system (in terms of geometry and material properties), variety of loads with different T_m were considered.

In order to verify the effects of ground inclination on the results, dynamic shakedown bearing capacity of footings were calculated for i=45°, φ=10°, n=2 , γ=20 KN/m^3 and DR=5%, where n is the number of successive half-sin load and DR is damping ratio. Results are presented as λP/c versus T_s/T_m in Fig. 4.4. Results for i=0° were extracted from Arvin. et.al. (2011).

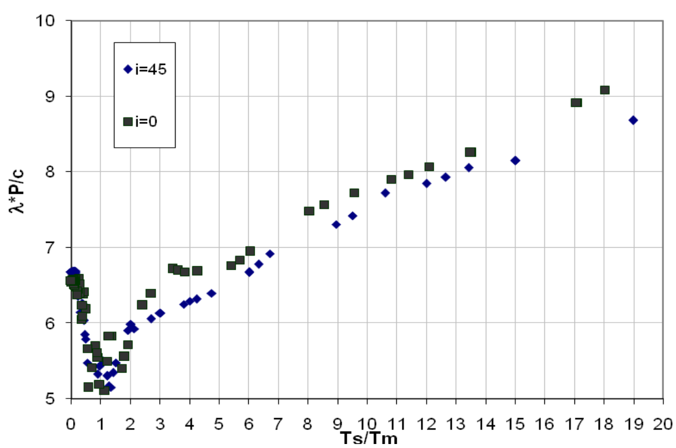

Figure 4.4. λP/c values vs T_s/T_m for footing on slopes with i=45° and i=0°, n=2 and soil properties DR=5%, γ=20 KN/m^3 and φ=10°

As Fig. 4.4 shows, unlike static shakedown bearing capacity which is independent of soil and load dynamic properties, the shakedown dynamic bearing capacity is greatly influenced by T_s/T_m. Bearing capacity first experience a reduction when T_s/T_m increases and then rises upward. The minimum value of λP/c occurs at about T_s/T_m=1 where resonant happens. Besides, greater slope inclination angle results in reduction of dynamic shakedown limit of footings.

4. CONCLUSIONS

Effect of repetition of static and dynamic loads on bearing capacity of rigid smooth footings resting on top of a slope were investigated using lower bound static and dynamic shakedown analysis. Shakedown theory was employed as an optimization process in the form of linear programming.

The following are the main conclusions made from the present study:

1- Static shakedown limit of footings on slopes lies between its elastic limit and its collapse limit.

2- Static shakedown limit of footings decrease with increase of slope inclination angle.

3- Static shakedown limit of footings on slopes is not influenced by dynamic properties of loads and subsoil.

4- Results show that dynamic shakedown bearing capacity of foundations on slopes first decrease with T_s/T_m and then ascend upward so that the minimum value of bearing capacity occurs at $T_s/T_m=1$ where resonance happens.

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