Design Method for RC Building Structure Controlled by Elasto-Plastic Dampers Using Performance Curve

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SUMMARY:

This paper proposes a design method of elasto-plastic dampers for seismic control of reinforced concrete building structures. The dynamic property parameters of the single degree of freedom passive control system are described, and the parametric study is carried out. The performance curves which describe the relationship between dampers, structural component, and seismic input intuitively are developed; the performance curve can be used to determine the necessary damper quantity for given target structural performance. A damper distribution method for MDOF structures is also proposed, which is based on the turning of the equivalent stiffness of structure. The time history analysis on a 7-story RC building with elasto-plastic dampers designed by the proposed method verified its accuracy and applicability.

Keywords: Elasto-plastic damper, RC structure, seismic control, passive control design

1. INTRODUCTION

Energy dissipation technology for seismic protection of building structures has got active practice and significant advance in recent years. As one of major types of energy dissipation device, elasto-plastic dampers including steel damper and friction damper are being widely used in both new construction and retrofitting of various types of structures, including reinforced concreted (RC) building structures. Compared with other types of dampers, elasto-plastic damper has relatively high energy dissipation capability and less dependence on the usage environment. The elasto-plastic damper has become one kind of most popular energy dissipation devices in structural engineering.

On the other hand, due to the complex hysteretic characteristics of RC structural members, there is still no applicable method to make efficient damper design with comprehensive consideration of hysteresis of both main structure and damper. The time history analysis is able to accomplish this work and produces accurate result, but repeated analysis for determining damper quantity is time-consuming. It is highly desirable to develop a comprehensive design method that clearly indicates effects as well as necessary capacities of the dampers for the target performance of RC structures. Prior to this research, the writers have extended design method for elastic structures (JSSI 2005, 2007) as well as elasto-plastic structures including steel structure and timber structure (Kasai and Pu 2010, Kasai et al. 2010). And so far, only the visco-elastic damper was considered in these methods for elasto-plastic structures.

By extending the previous methods, this study proposes a damper design method for RC structure added with elasto-plastic dampers. The passively controlled structure designed by present method develops identical story drift in each story as prescribed. The performance curves for single degree of freedom system and a damper distribution rule for multi-degree of freedom system are proposed.

2. DYNAMIC PROPERTIES OF SYSTEM 2.1. Stiffness degrading hysteretic model for RC frame

The stiffness degrading Takeda model (Takeda et al. 1972) is employed to reproduce the hysteretic behaviour of RC structures. As shown in Figure 1, Takeda model has tri-linear skeleton curve. Displacement u_c , defines happening of cracking of RC, and displacement u_y , defines yielding of reinforced bar. The structure behaviors elastically, if $u_{\text{max}} < u_c$. Figure 1(a) shows the hysteresis and skeleton curve for $u_c < u_{\text{max}} < u_y$; Figure 1(b) shows the hysteresis for $u_y < u_{\text{max}}$.



Figure 1. Stiffness degrading model

Figure 2. Stationary state hysteresis of system

The elastic stiffness, post-crack stiffness and post-yield stiffness are represented by K_f , $\alpha_1 K_f$, and $\alpha_2 K_f$, respectively. Hereinafter, ductility ratios μ and μ_c are used respectively to denote the maximum and cracking displacements normalized by u_{ν} , as given in Eq. (2.1a, b).

$$\mu = u_{\text{max}} / u_v, \quad \mu_c = u_c / u_v$$
 (2.1a,b)

The stationary state hysteresis curves of Takeda model are governed by its loading and unloading rules which are dependent on maximum displacement. Unloading stiffness κ_f^u is given by Eq. (2.2a) and (2.2b), depending on the ductility ratio μ .

$$K_{f}^{u} = K_{f} \frac{2\mu_{c} + \alpha_{1}(\mu - \mu_{c})}{\mu + \mu_{c}} \qquad \mu_{c} < \mu \le 1$$
(2.2a)

$$K_f^u = K_f \frac{2\mu_c + \alpha_1(1 - \mu_c)}{(1 + \mu_c)\mu^{0.4}} \qquad 1 < \mu$$
(2.2b)

Structural loading stiffness is maximum deformation-oriented. The reloading curve can be constructed by connecting the point with zero force and the point reached in the previous cycle, if that point lies on the skeleton curve or on a line aimed at a point on the skeleton curve. The more details about this model can be found in Takeda et al. (1970).

The damper considered in this paper is a combination of an elasto-plastic energy dissipating element and an elastic supporting such as brace. Because both elasto-plastic energy dissipating element and brace are displacement-dependent, for purpose of convenience, this combination assembly is called as elasto-plastic damper in this paper. Obviously, as long as the elastic stiffness of damper K_d and ductility ratio of damper μ_d are determined, for given stiffness of brace, the necessary stiffness and ductility ratio for energy dissipating element can be calculated readily. Because energy dissipating element and brace are connected in series, the yielding forces for damper and energy dissipating element are equal, and the elastic stiffness and ductility ratios for them become large or small simultaneously. Figure 2 shows the hysteresis of structural system and its elements.

2.2 Dynamic properties of system

The ratio of damper stiffness to the elastic stiffness of RC frame is defined as damper stiffness ratio r_d . The elastic stiffness of system is denoted by K, the ratio of K to K_f can be calculated by Eq. (2.3)

$$K/K_{f} = (K_{f} + K_{d})/K_{f} = 1 + r_{d}$$
(2.3)

Use T_{f_i} h_f to denote the natural period, damping ratio of RC frame, the natural period T_0 and damping ratio h_0 of the elastic system with added elasto-plastic damper can be expressed as:

$$T_0 / T_f = 1 / \sqrt{1 + r_d}$$
, $h_0 / h_f = 1 / \sqrt{1 + r_d}$ (2.4)

The equivalent stiffness of RC frame $K_{eq,f}$ is defined as the secant stiffness of RC frame at the maximum deformation, and expressed as Eq. (2.5),

$$K_{eq,f} = pK_f \tag{2.5}$$

Where p is the ratio of the equivalent stiffness to the initial elastic stiffness of RC frame, and expressed as the function of ductility ratio μ , as given in Eq. (2.6).

$$p = \frac{\mu_c + \alpha_1(\mu - \mu_c)}{\mu} , \ \mu_c < \mu \le 1$$
 (2.6a)

$$p = \frac{\mu_c + \alpha_1(1 - \mu_c) + \alpha_2(\mu - 1)}{\mu} , \mu > 1$$
 (2.6b)

The equivalent stiffness K_{eq} and equivalent period T_{eq} of the system are given as follows.

$$K_{eq} = K_{eq,d} + K_{eq,f} \tag{2.7}$$

$$K_{eq,d} = K_d / \mu_d \tag{2.8}$$

$$T_{eq} / T_f = \frac{1}{\sqrt{r_d / \mu_d + p}}$$
(2.9)

Dissipated energy E_p in one stationary state cycle is given by Eq. (2.10a) and Eq. (2.10b), respectively.

$$E_p = 2K_f \mu^2 u_y^2 \frac{p\mu_c(1-p)}{\mu_c + p\mu}, \ \mu_c \le \mu \le 1$$
(2.10a)

$$E_p = 2K_f \mu^2 u_y^2 \left[p - \frac{p^2 (1 + \mu_c) \mu^\lambda}{2\mu_c + \alpha_1 (1 - \mu_c)} \right], \mu > 1$$
(2.10b)

The energy dissipated by damper in one stationary state cycle, E_d , is easily obtained as Eq. (2.11).

$$E_d = 4K_d (\mu_d - 1)u_{dy}^2$$
(2.11)

The strain energy possessed by system can be calculated based on the equivalent stiffness of the system, and it includes strain energy of the frame E_{sf} and of damper E_{sd} .

$$E_{sf} + E_{sd} = \frac{1}{2} K_f (p + r_d / \mu_d) \mu^2 u_y^2$$
(2.12)

Therefore, the damping ratio contributed by plastic deformation of RC frame is obtained as follows.

$$h_p = \frac{1}{4\pi} \frac{E_p}{E_{sf} + E_{sd}} = \frac{1}{\pi} \frac{p\mu_c(1-p)}{(p+r_d/\mu_d)(\mu_c + p\mu)}, \ \mu_c < \mu \le 1$$
(2.13a)

$$h_p = \frac{1}{\pi (p + r_d / \mu_d)} \left[p - \frac{p^2 (1 + \mu_c) \mu^\lambda}{2\mu_c + \alpha_1 (1 - \mu_c)} \right], \mu > 1$$
(2.13b)

The damping ratio contributed by damper is,

$$h_d = \frac{1}{4\pi} \frac{E_d}{E_{sf} + E_{sd}} = \frac{2r_d(\mu_d - 1)}{\pi \mu_d^2 (p + r_d / \mu_d)}$$
(2.14)

The above describes the equations of energy dissipation and the damping ratios for system and its constitution elements in stationary state. Figure 3 plots the curves for T_{eq}/T_f versus damping ratio h_{eq} , where the ductility ratio of damper is set to be 1, 2, 3, 4, and 5 and damper stiffness ratio set to be 0.05, 0.1, 0.2, 0.35, and 0.5. Generally, as K_d/K_f increases, the T_{eq} decreases; as ductility ratio of damper becomes larger, h_{eq} increases significantly. On the other hand, for small ductility ratio of damper, as the quantity of damper is increased, the decrease of T_{eq} is more significant than the increase of h_{eq} .



Figure 3. Damping ratio versus equivalent period

In order to use the elastic response spectrum, the elasto-plastic damping system is converted to an equivalent elastic system. The equivalent stiffness has been defined in Eq. (2.7). The damping ratio for system in stationary state has been defined; however, the response of system subjected to earthquake excitation is of great randomness, the direct use of the previously defined damping ratio lacks of reasonability. Hereinafter, the average damping proposed by Newmark and Rosenblueth (1971) is introduced. Averaged damping ratio is the average of the damping ratios of the stationary state cycles with amplitude ranging from 0 to the maximum. With the initial damping ratio included, the total equivalent damping ratio is calculated by Eq. (2.15).

$$h_{eq} = h_f + \frac{1}{\mu} \int_0^\mu (h_p(\mu') + h_d(\mu')) d\mu'$$
(2.15)

3. DAMPER DESIGN BASED ON PERFORMANCE CURVE FOR SDOF SYSTEM 3.1 Response reduction of passive system

The evaluation method of response reduction of passive control building has been proposed based on the elastic response spectrum previously. In this section, the design method for RC structure added with elasto-plastic damper is constructed by extending the previous method. The displacement spectrum, pseudo-velocity spectrum, pseudo-acceleration spectrum are defined as S_d , S_{pv} and S_{pa} , respectively. From any one of the three spectra, the other two spectra can be calculated in terms of Eq. (3.1).

$$S_d(T_f, h_f) = \frac{T_f}{2\pi} S_{pv}(T_f, h_f) = \left(\frac{T_f}{2\pi}\right)^2 S_{pa}(T_f, h_f)$$
(3.1)

The response reduction is the combination effect of decreased vibration period and increased damping effect by adding damper into structure. We define the ratios of the displacement u, force F of passive system to the displacement and force of original elastic frame u_{el} , F_{el} as displacement reduction ratio R_{d} , pseudo-acceleration reduction ratio R_{pa} , respectively.

$$R_{d} = \frac{S_{d}(T_{eq}, h_{eq})}{S_{d}(T_{f}, h_{f})} = D_{h} \frac{T_{eq}}{T_{f}} \frac{\overline{S}_{pv}(T_{eq}, h_{f})}{S_{pv}(T_{f}, h_{f})}$$
(3.2a)

$$R_{pa} = \frac{S_{pa}(T_{eq}, h_{eq})}{S_{pa}(T_f, h_f)} = D_h \frac{T_f}{T_{eq}} \frac{\overline{S}_{pv}(T_{eq}, h_f)}{S_{pv}(T_f, h_f)}$$
(3.2b)

where $\overline{S}_{pv}(T_{eq}, h_f)$ is the averaged pseudo-velocity spectrum over period range from T_0 to T_{eq} (Kasai et al. 2005, 2009). The reason to use averaged pseudo-velocity spectrum is in that the spectrum is smoothed and the possible difference between the spectrum value at T_f and T_{eq} can be reduced in such a way.

Assume the pseudo-acceleration spectrum in short-period range and pseudo-velocity spectrum in medium- to long-period range are identical, respectively, the response reduction of system can be rewritten as

$$R_d = D_h \frac{T_{eq}(T_{eq} + T_0)}{2T_f^2}, \quad R_{pa} = R_d \left(\frac{T_f}{T_{eq}}\right)^2$$
 (S_{pa}=Const.) (3.3a,b)

$$R_d = D_h \frac{T_{eq}}{T_f}, \quad R_{pa} = R_d \left(\frac{T_f}{T_{eq}}\right)^2 \qquad (S_{pv} = \text{Const.})$$
 (3.4a,b)

Given parameters of main frame h_f , μ_c , α_1 , α_2 , μ_f , and parameters of damper μ_d , K_d/K_f , calculate T_0 / T_f , T_{eq}/T_f , and h_{eq} , and then obtain R_d and R_{pa} by using Eqs. (3.3) and (3.4). The plot for R_d - R_{pa} estimated for varied parameters is defined as "performance curve".

Assume $\mu_c=0.1$, $\alpha_1=0.22$, $\alpha_2=0.05$ and $h_f=0.02$, the performance curves for $\mu=0.7$ and 2 are plotted in Figure 4, where constant pseudo-acceleration spectrum S_{pa} , and constant pseudo- acceleration spectrum S_{pa} are considered separately. The ductility ratio for damper $\mu_d=1\sim8$ and damper stiffness ratio $K_d/K_f=0\sim2$ are considered. The constant *a* in D_h is 75.

Figure 4 show the following tendency. The use of large stiffness of damper and low yield strength can make the damper yield early, and produce large ductility ratio of damper. Such a design can reduce the displacement as well as shear force. This means the friction damper with high initial stiffness is very

effective. On the other hand, if damper has high yield strength, the ductility ratio becomes small. In such a case, increasing stiffness ratio has little effect to reduce the displacement, and increase of shear force should be pay attention to. Comparatively, the response reduction effect in more significant in S_{pa} = constant than that for S_{pv} = constant.



Figure 4 Passive control performance curves

3.2 Passive control design based on performance curve

The already-known include the initial period T_{f_2} initial damping ratio h_{f_2} all structural parameters for RC frame, brace stiffness ratio K_b/K_{f_2} target ductility ratio μ . Based on the performance curve, the passive control design to determine the quantity of damper can be performed as following.

(1). Make performance curves for target ductility ratio μ .

- (2). Calculate responses of elastic frame u_{el} , F_{el} .
- (3). Calculate the response reductions R_d , R_{pa} .

(4). Determine the damper stiffness ratio K_d/K_f , with an assumed damper ductility ratio μ_d .

(5). Get the pseudo-acceleration reduction R_{pa} , and subsequently get maximum shear force of system.

If both the target maximum force and maximum displacement are satisfied, the design is finished.

4. DAMPER DISTRIBUTION RULE FOR MDOF SYSTEM 4.1 Design procedure

The writers have developed a series of design methods for elastic and elasto-plastic frames with various types of dampers, and validated them by extensive numerical tests considering arbitrary parametric distributions. The present study extends the previous methods to RC building structures which have more complex hysteretic properties.

The general design procedure is shown as follows.

- Step 1. Establish the required basic design data, including mass m_i and story height h_i of each story of building, and determine the target drift angle, design spectrum and earthquake motions for dynamic analysis.
- Step 2. Establish the constitutive model for each story based on the pushover analysis, and construct a MDOF shear beam model for building.
- Step 3. Reduce the degree of freedoms to get an equivalent SDOF model at target drift level.
- Step 4. Get the necessary damper quantity for SDOF system through performance curves.
- Step 5. Distribute damper into each story of MDOF system (Section 5.3).
- Step 6. Examine the performance of passive control system by time history analysis.

4.2 Conversion of MDOF frame to SDOF frame

The conversion of MDOF structural frame into an equivalent SDOF frame is divided into three cases according to the target drift angle θ_t . Firstly, the elastic stiffness of the equivalent SDOF system is

estimated by,

$$K_{f} = \sum_{i=1}^{N} K_{fi} h_{i}^{2} / H_{eff}^{2}, \quad H_{eff} = \sum_{i=1}^{N} m_{i} H_{i}^{2} / \sum_{i=1}^{N} m_{i} H_{i}$$
(4.1a,b)

where H_{eff} is the effective height of the RC building, and straight-line mode giving uniform story drift angle is assumed, as often considered by the writers (Kasai et al. 2006, 2010).

The storage and dissipated energies are respectively assumed equal between equivalent SDOF system and MDOF system, namely,

$$\frac{1}{2} p K_f (H_{eff} \theta_t)^2 = \sum_{i=1}^N E_{si} , \ q K_f (H_{eff} \theta_t)^2 = \sum_{i=1}^N E_{pi}$$
(4.2a,b)

In case of $\mu_i > 1$, namely, yielding happens in one or more stories at target drift angle θ , it is assumed similarly that $\mu > 1$ holds true for SDOF system. In addition to the previous two constraint conditions, μ_c and α_2 of SDOF system are assumed to be the average value of μ_{ci} , α_{2i} of each story level of MDOF frame (Eq. (4.3)).

$$\alpha_2 = \frac{1}{N} \sum_{i=1}^{N} \alpha_{2i} , \quad \mu_c = \frac{1}{N} \sum_{i=1}^{N} \mu_{ci}$$
(4.3a,b)

In this way, an equivalent SDOF system is determined. It is noted that the equivalent SDOF system depends on the target displacement level, because the equivalent SDOF is made based on the equivalence of damping and period SDOF and MDOF systems at specified performance level.

4.3 Damper distribution rule for MDOF system

The damper quantity obtained in equivalent SDOF system is distributed to each story of MDOF system by using the following two constraint conditions.

(a) The equivalent damping ratios of SDOF and MDOF passive control systems are equal.

(b) The equivalent stiffness of passive control MDOF system at target drift angle is proportional to the story shear force Q_i acting on each story.

Equations (4.4) and (4.5) are constructed based on the above conditions.

$$h = \frac{E_p + E_d}{4\pi (E_{sf} + E_{sd})} = \frac{\sum_{i=1}^N (E_{pi} + E_{di})}{4\pi \sum_{i=1}^N (E_{sfi} + E_{sdi})}$$
(4.4)

$$\theta = \frac{Q}{h_i(p_i K_{fi} + K_{di} / \mu)} = \frac{\sum_{i=1}^N Q_i h_i}{\sum_{i=1}^N h_i^2(p_i K_{fi} + K_{di} / \mu)}$$
(4.5)

The stiffness of elasto-plastic damper distributed to *i*nto story K_{di} is obtained as Eq. (4.6).

$$K_{di} = \frac{\mu_d Q_i}{\theta^2 h_i} \cdot \frac{8(\mu_d - 1)\sum E_{sfi} - \sum E_{pfi}\mu_d}{2(2\mu_d - \mu_d\pi h - 2)\sum Q_i h_i} - \mu_d K_{fi} p_i$$
(4.6)

5. DESIGN EXAMPLE

Time history analysis is performed to verify the accuracy of present design method. The x-direction of a 7-story RC building considered in Kasai et al. (2011) is used as example. The total height of the

building is 28.8m, and the fundamental period in x-direction is 1.43s. In Kasai et al. (2011), the pushover curves have been converted into the tri-linear skeleton curves, of which parameters are shown in Table 1.

Story	Wi	hi	K _{fi}	$a_{1i}K_{fi}$	a _{2i} K _{fi}	u _{ci}	u _{yi}	F _{ci}	F _{yi}
No.	(kN)	(cm)	(kN/cm)	(kN/cm)	(kN/cm)	(cm)	(cm)	(kN)	(kN)
7	5351	500	730	680	37	1.00	3.15	733	2200
6	10417	420	2380	1400	119	0.84	3.61	1947	5840
5	10084	360	3270	2030	164	0.83	3.42	2667	8000
4	10221	360	3660	2450	183	0.86	3.38	3139	9419
3	10427	360	3950	2800	198	0.86	3.35	3467	10400
2	9369	420	3820	2590	191	1.01	3.86	3776	11328
1	11152	460	6580	3290	329	0.74	3.54	4686	14059

Table 1. Parameters of Takeda model for for 7 story RC building

Table 2. Parameters of Takeda model for equivalent SDOF frame

	$H_{eff}(cm)$	K_{f} (kN/cm)	a ₁	a ₂	μ_{c}	μ	h _p
θ=1/100	1889	1141	0.58	0.05	0.25	1.11	0.043
θ=1/75	1889	1141	0.57	0.05	0.25	1.48	0.081

Four artificial earthquake waves including BCJ-L2, Hachinone, JMA Kobe, Tohoku University are used in design. These waves show almost constant pseudo-acceleration response spectrum between 0.16-0.64s, and constant pseudo-velocity spectrum over 0.64s. The response spectrum of 2% damping have constant spectrum of S_{pa} =1434cm/s² and S_{pv} =143cm/s in corresponding range (Figure 5). The constant *a* in D_h is set to be 25 for JMA Kobe wave, and 75 for other 3 waves.

Through time history analysis, it is found that the 7-story RC building produces very large story drift of about 1/41 in 2^{nd} and 3^{rd} story when subjected to the design earthquake waves. In the passive control design, the target drift angle is set to be 1/75 and 1/100, which lead to ductility ratio of main frame of about 1.5 and 1.0 in each story. The parameters for the equivalent SDOF frame are shown in Table 2.

The target ductility ratio for damper is considered to be 5. In terms of the proposed method, the damper stiffness ratios are designed as shown in Table 3 and Table 4. Figures 6(a) and 6(b) show the maximum story drift of RC building for θ =1/75 design and θ =1/100 design, respectively. The response for building with dampers is shown by solid line; the response for building without dampers is shown by solid line; the response for building without dampers is shown by broken line. Before dampers are added, the building produces irregular story drift over the height, and very large story drift in 2nd and 3rd story. By adding the damper design by the proposed method inproportionally, the story drift angles are almost identical and satisfy the target value.

θ=1/100	a=75	•	3 2	a=25		
Story No.	K _{di} (kN/cm)	K _{di} /K _{fi}	F _{di} (kN)	K _{di} (kN/cm)	K _{di} /K _{fi}	F _{di} (kN)
7	1040	1.42	1040	1446	1.98	1446
6	1036	0.44	870	2083	0.88	1749
5	1846	0.56	1329	3581	1.10	2578
4	2730	0.75	1966	4903	1.34	3530
3	3480	0.88	2506	6027	1.53	4340
2	3210	0.84	2696	5631	1.48	4730
1	561	0.09	516	2977	0.45	2739

Table 3. Damper design results for MDOF system (θ =1/100)

θ=1/75	a=75 (kN/cm)			a=25			
Story No.	K _{di} (kN/cm)	K _{di} /K _{fi}	F _{di} (kN)	K _{di} (kN/cm)	K_{di}/K_{fi}	F _{di} (kN)	
7	635	0.87	847	876	1.20	1168	
6	361	0.15	405	972	0.41	1088	
5	725	0.22	696	1728	0.53	1659	
4	1235	0.34	1185	2483	0.68	2384	
3	1663	0.42	1597	3117	0.79	2992	
2	1490	0.39	1668	2863	0.75	3206	
1	0	0.00	0	674	0.10	827	

Table 4. Damper design results for MDOF system (θ =1/75)



Figure 5. Response spectra of ground motions





6. CONCLUSIONS

This paper proposed a passive control design method for RC structure added with elasto-plastic damper. The dynamic properties of the system constituted by the main frame and the added component in parallel are discussed, the variation of equivalent period and damping were described.

The dampers are designed by tuning the equivalent stiffness of the system so that an optimum equivalent stiffness distribution can be achieved at target drift by adding elasto-plastic dampers, while the earthquake energies are dissipated by plastic deformation of dampers as well. The time history analysis on structures with irregular distribution of structural parameters was performed, and good accuracy of the proposed method was verified.

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