An Efficient Method for Optimal Performance-Based Design of Reinforced Concrete Structures

I. Hajirasouliha
The University of Nottingham, Nottingham, UK

M. Basafa
The University of Science and Culture, Tehran, Iran

SUMMARY:
A practical method is developed for optimum performance-based design of reinforced concrete (RC) structures subjected to seismic excitations. In this method, optimum design is obtained by redistributing material from strong to weak parts of a structure until a state of uniform damage prevails. By applying the design algorithm on 3, 5, 10 and 15 storey RC frames, the efficiency of the proposed method is demonstrated for different seismic excitations and performance targets. The results indicate that, for similar structural weight, optimum designed structures experience up to 60% less global damage compared to code-based design frames. It is shown that RC frames designed with the simulated spectrum-compatible earthquake exhibit on average 40% less structural weight compared to the conventionally designed frames. The results of this study emphasise the efficiency of the proposed design method at controlling structural performance parameters and improving the seismic behaviour of RC frames.

Keywords: Performance-based design; RC frames; Optimum seismic design; Damage index

1. INTRODUCTION

The need for finding cost-efficient and optimum structural designs has led to the development of different structural optimization methodologies. Optimum design of structures for seismic loads has been studied by many researchers over the past decades (Feng, 1997; Bhatti, 1981, Baling, 1983, Cheng, 1983; Arora, 1999). The conventional methods used in these studies are usually gradient-based solution strategies that require the satisfaction of some specific mathematical conditions. Due to the difficulty in calculating appropriate expressions for optimisation constraints, these methods cannot be practically applied for optimum design of non-linear structures subjected to seismic excitations.

The newly developed performance-based design methods (ATC-40, 1996; SEAOC, 1995; FEMA-356, 2000), which are also a good indicator of future direction for seismic design codes, tend to take into account the non-linear seismic response of structures. These methods directly address inelastic deformations to identify the levels of damage during severe seismic events. In one of the early attempts, Beck et al.(1998) developed an optimization methodology for performance-based design of simple elastic structural systems operating in an uncertain dynamic environment. Ganzerli et al. (2000) combined the contemporary concept of performance-based design with structural optimization methods. They introduced a nonlinear analysis-based approach where the performance-based constraints were applied in terms of plastic rotations of beams columns of frames, as suggested by FEMA guidelines. Zou and Chan (2005) presented a new optimisation procedure based on the "optimality criteria" concept. In this approach, by using the principle of virtual work and the Taylor series approximation, the nonlinear seismic response of an RC frame is expressed in terms of element design variables. In their proposed methodology, the inelastic drift response of the structure is determined by performing a non-linear push-over analysis, and therefore, higher mode effects are not taken into account. Fragiadakis and Papadrakakis (2008) proposed a reliability-based optimisation approach based on nonlinear response history analysis. In this method, an evolutionary optimization algorithm is used to locate the most efficient design in terms of cost and performance through generating appropriate databases of RC beam and column sections.

In the preliminary studies, Hajirasouliha et al. (2012) developed an efficient method for efficient seismic design of RC frames bnaesd on the concept of uniform damage distribution. This paper tends to adopt their proposed method for performance-based design of low-rise (3 storey) to high-rise (15 storey) RC frames under different seismic excitations. The efficiency of the new method is examined for different performance targets and seismic loading scenarios in the following sections.

2. MODELLING AND ASSUMPTION

To demonstrate the method, four RC frames with 3, 5, 10, and 15 storeys (as shown in Figure 1) were examined. The buildings were assumed to be located on a soil type C of the IBC-2009 category, with the design spectral response acceleration at short periods and 1-sec period equal to 0.88g and 0.55g, respectively. Frame members were designed to support gravity and lateral loads determined in accordance with the minimum requirements of IBC-2009 and ACI 318-08. The uniform gravity loads have been considered as 54 and 48 kN/m for interior storeys and roof, respectively. The RC frames were assumed to satisfy intermediate ductility requirements. The frames had square columns decreasing in dimensions with height. Details of the geometry of the structures are given in Figure 1. Nonlinear time-history analysis was carried out using computer program IDARC (Valles, 1996). The Properties of RC members are calculated by fibre models, and the solutions are obtained using step-by step integration of equations of motion using Newmark beta method. Rayleigh damping model with a constant damping ratio of 0.05 was assigned to the first mode and to any mode at which the cumulative mass participation exceeds 95%. Spread plasticity models were employed to model non-linear behaviour of beam and column elements.

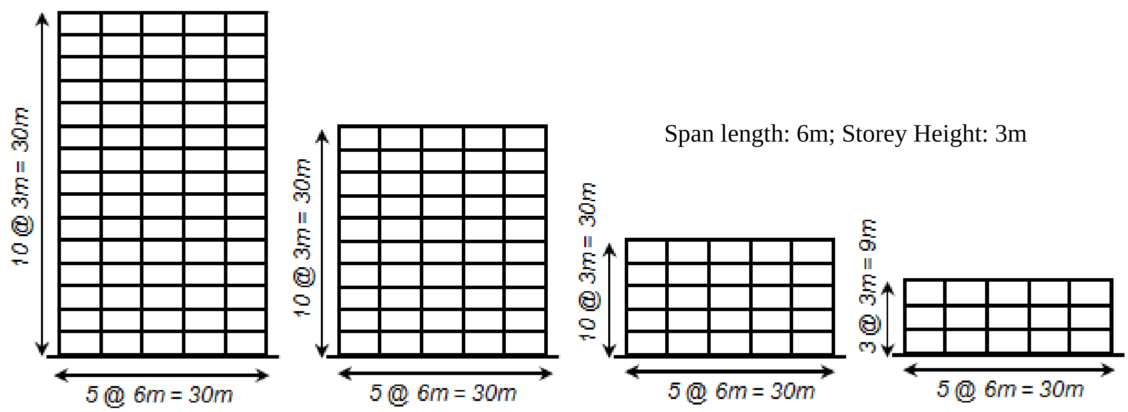

Figure 1. typical geometry of 3,5, 10 and 15 storey RC frames.

3. OPTIMUM SEISMIC DESIGN FOR SINGLE EARTHQUAKE EXCITATION

During strong earthquakes the deformation demand in code-based designed structures is not expected to be uniform (Moghaddamand Hajirasouliha, 2006). As a result, the deformation demand in some parts of the structures does not necessarily utilize the maximum level of seismic capacity. If the strength of underused elements is decreased incrementally, for a ductile structure, it is expected to eventually obtain a status of uniform deformation or damage demand. In such a case, the dissipation of seismic energy in each structural element is maximized and the material capacity is fully exploited. Therefore, in general, it can be assumed that a status of uniform damage demand is a direct consequence of the optimum use of material. Although the structure with minimum structural weight cannot necessary be shown to be the one with uniform damage demand, the proposed method has capability to decrease the required structural weight by exploiting fully the material capacity.

While conventional RC buildings are expected to remain in the elastic state during small earthquakes, they experience non-linear deformations under medium to strong earthquakes. The concrete section plays a more dominant role in providing lateral stiffness, and therefore, it is mainly responsible for controlling elastic drift under small earthquake loading. Within the nonlinear response range, the reinforcement ratio of structural elements is considered to be the main design variable, as flexural reinforcement plays a dominant role in controlling inter-storey drift and providing the required ductility. In this study, it is assumed that adequate shear confinement reinforcement is provided for each member, which is roughly proportional to the amount of flexural reinforcement. For simplicity, the compression steel ratio, ρ', is assumed to be linearly related (almost 50% for 3 &5 storey, 60% for 10 storey and 70% for 15 storey) to the tension steel reinforcement ratio, ρ, for beams and identical for columns. Consequently, the tension steel reinforcement ratio, ρ, can be considered as the major design variable in the proposed design method. The minimum and maximum ρ for columns was considered to be 1% and 4%, respectively. Based on ACI 318-08, the minimum ρ for RC beams was 0. 35% and the maximum ρ was calculated to avoid brittle failure due to concrete crushing without steel yielding. It should be noted that most beams end up with a much lower ρ than the maximum, and this should ensure adequate ductility if suitable detailing is provided. In this study, the iterative optimum design procedure developed by Hajirasouliha and Moghaddam (2009) for optimum design of shear-building (mass-spring) models is extended for seismic design of RC frame structures.

3.1. Minimum Structural Damage

In performance-based design methods, design criteria are expressed in terms of achieving specific performance targets during a design level earthquake. Performance targets could be satisfied by controlling the level of stress, displacement or structural and non-structural damage. The target of the design is to find the optimum distribution of reinforcement in a RC frame to minimize the expected structural damage for a design earthquake. The proposed method in this study can optimize the design of RC structures for different types of performance targets such as deformation, acceleration or cumulative damage. Among the different developed damage indexes in the literature, Park and Ang index (Park, 1985) is one of the most adopted models for damage analysis of RC structures. The Park and Ang damage model accounts for damage due to maximum inelastic excursions, as well as damage due to the history of deformations:

$$DI = \frac{\theta_m - \theta_r}{\theta_u - \theta_r} + \frac{\beta}{M_y \theta_u} E_h \tag{3.1}$$

Where θ_m is the is the maximum rotation attained during the loading history; θ_u is the ultimate rotation capacity of the section; θ_r is the recoverable rotation when unloading; β is the Park and Ang model constant parameter equal to 0.1 (Valles, 1996); M_y is the yield moment; and E_h is the dissipated energy in the section. The element damage is then selected as the biggest damage index of the end sections. Storey and overall damage indices are computed using the weighted average of the local element damage indices based on the dissipated hysteretic energy of components. In an attempt to reach uniform damage distribution in all structural elements, the following design procedure was employed:

1. The initial structure is designed for gravity and seismic loads based on a seismic design code, such as IBC-2009. The preliminary distribution of steel reinforcement is selected for all structural elements to obtain the most efficient initial design. The dimensions of beam and column elements are determined at this stage and remain unchanged during the design process.

2. The structure is subjected to the design seismic excitation(vary rare earthquake), and the Park and Ang damage index is calculated for all beam and column elements.

3. The Coefficient of Variation *(COV)* of damage indices for beams *(COV*$_b$) and columns *(COV*$_c$) is

calculated. If both COV_b and COV_c are small enough (e.g. less than 0.1), the structure is considered to be practically optimum. Otherwise, the design algorithm proceeds to iterations.

During the iterations, the distribution of longitudinal reinforcement in beam and column elements is modified. Longitudinal reinforcement is shifted from elements with lower damage index to the elements which experienced higher damage by using the following equations:

$$[(\rho_{beam})_i]_{n+1} = \left[\frac{(DI_b)_i}{(DI_b)_{ave}}\right]^{\alpha} [(\rho_{beam})_i]_n \tag{3.2}$$

$$[(\rho_{col})_i]_{n+1} = \left[\frac{(DI_c)_i}{(DI_c)_{ave}}\right]^{\beta} [(\rho_{col})_i]_n \tag{3.3}$$

where $[(\rho_{beam})_i]_n$ and $[(\rho_{col})_i]_n$ are the tension steel reinforcement ratio of the i^{th} beam or column element at n^{th} iteration, respectively. $(DI_b)_i$ and $(DI_b)_{ave}$ are Park and Ang damage index for the i^{th} beam and average of damage indices for all beam elements, respectively. Similarly, $(DI_c)_i$ and $(DI_c)_{ave}$ are damage indices for the i^{th} column and average of damage indices for all column elements, respectively. α and β are convergence parameters ranging from 0 to 1. In this study, convergence parameters α, β were set to be 0.1. It should be noted that using equations 2 and 3 can lead to different damage levels for beam and column elements to satisfy the strong-column/weak-beam concept.

6. The longitudinal reinforcement ratios for all beam and column elements are scaled such that the total reinforcement weight remains unchanged.

7. The new RC frame is then analyzed to ensure that it can sustain the gravity loads (i.e. design dead and live loads). If any member fails, its longitudinal reinforcement is increased accordingly to ensure the final design is capable of resisting gravity loads based on the capacity design concept. The design procedure is then repeated from step 2 until the *COV* of damage indices for both beam and column elements become small enough.

The above design algorithm has been used for more efficient seismic design of 3, 5, 10 and 15-storey RC frames (shown in Figure 1) subjected to the simulated design spectrum-compatible earthquake. The results indicate that, for similar total steel reinforcement weight, near optimum design structures always experience more uniform damage distribution and relatively less global damage index as compared with structures designed according to conventional design methods. For example, Figure 2 shows the distribution of storey damage indices for the four near optimum and conventionally designed RC frames subjected to SE_{QIBC}. The global *DI* and *COV* of storey damage indices for IBC-2009 and near optimum design models are compared in Table 1. It can be deduced that for the same structural weight, near optimum models experience up to 60% less global damage. The proposed design method is capable of preventing high local structural damage as the performance parameters (i.e. structural damage indices) are directly controlled in the proposed design procedure. It should be mentioned that in practice uniform damage distribution may not be achieved if uniformity of the section properties and minimum reinforcement requirements are considered as design constraints. However, the proposed method always leads to a more efficient design by exploiting better the capacity of structural materials.

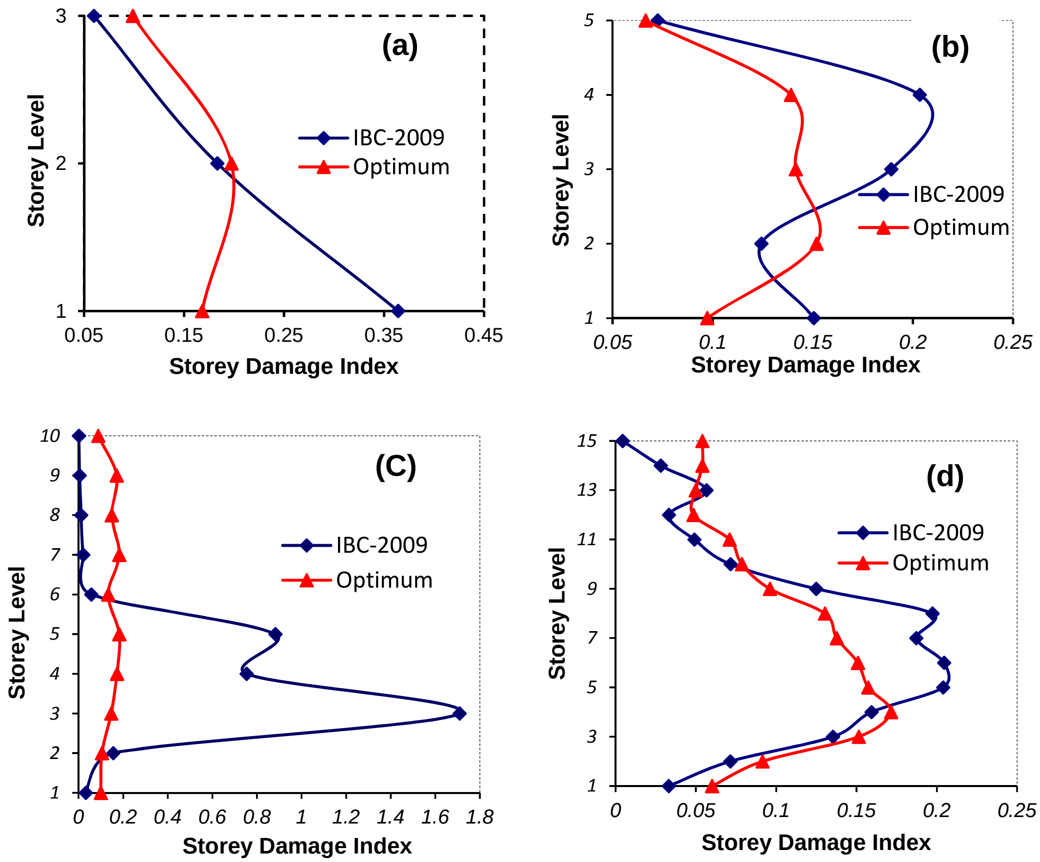

Figure 2. Storey damage distribution of IBC-2009 and near optimum design models subjected to the SEQIBC, (a): 3 storey model; (b): 5 storey model; (c): 10 storey model; (d): 15 storey model

Table 1. Global *DI* and *COV* of storey damage indices for the IBC-2009 and near optimum design models subjected to the synthetic earthquake.

MODEL	IBC-2009 Model		Optimum Model		Reduction of Global Damage Index
	Global DI	*COV* of Storey Damage Indices	Global DI	*COV* of Storey Damage Indices	
3-Storey	0.182	0.75	0.139	0.33	24%
5-Storey	0.148	0.36	0.119	0.3	20%
10-Storey	0.363	1.59	0.14	0.25	61%
15-Storey	0.104	0.7	0.098	0.45	6%

3.2. Minimum Structural Weight

The proposed more efficient design concept was also used to obtain a structure with minimum required reinforcement weight to satisfy a prescribed performance level. Performance-based design guidelines, such as FEMA 356, place limits on acceptable values of response parameters; implying that exceeding these limits is a violation of a performance level. In this study, global and local Park and Ang damage indices are considered as the failure performance criterion.

The following design algorithm was utilized to obtain the minimum weight of the structure:

1. The initial structure is designed based on a selected seismic design code, such as IBC-2009.

2. The structure is then subjected to the design seismic excitation (rare earthquake). The local Park and Ang damage index is calculated for all beam and column elements and compared with the target value. If all of the calculated damage are close enough to the performance target, the RC structure is considered to be practically optimum. Otherwise, the design algorithm is continued.

3. Whilst storeys with damage higher than the target value violate the performance objective, in the storeys with damage less than the target value the material is not fully utilized. Steel reinforcement plays a significant role in controlling damage of an RC frame within its inelastic range of behaviour. Therefore, longitudinal reinforcement is reduced or increased accordingly. To achieve this, the following equation is used in this study:

$$[(\rho_{beam})_i]_{n+1} = \left[\frac{(DI_b)_i}{(DI_b)_{target}}\right]^{\alpha} [(\rho_{beam})_i]_n \tag{3.4}$$

$$[(\rho_{col})_i]_{n+1} = \left[\frac{(DI_c)_i}{(DI_c)_{target}}\right]^{\beta} [(\rho_{col})_i]_n \tag{3.5}$$

   where $[(\rho_{beam})_i]_n$ and $[(\rho_{col})_i]_n$ are the tension steel reinforcement ratio of the i^{th} beam or column element at n^{th} iteration, respectively. $(DI_b)_i$ and $(DI_b)_{target}$ are Park and Ang damage index for the i^{th} beam and target of damage indices for all beam elements, respectively. Similarly, $(DI_c)_i$ and $(DI_c)_{target}$ are damage indices for the i^{th} column and target of damage indices for all column elements, respectively. This modified reinforcement weight is distributed to the beams and columns of that storey based on their local damage index by using equations (3.4) and (3.5). The capacity design concept can be ensured by selecting appropriate DI_{target} for each element.

4. The new RC frame is then analyzed to ensure that it can sustain the design gravity loads. If any member fails, its longitudinal reinforcement is increased accordingly. The proposed design procedure is repeated from step 2 until the *COV* of inter-storey drifts decreases to an acceptable level. As the final design frames have to resist the design gravity loads, it is not usually possible to reach a very uniform inter-storey damage distribution, especially when the effect of gravity loads is dominant.

The above algorithm has been applied for more efficient seismic design of the four RC frames shown in Figure 1. The target damage was considered to be equal to 0.03 for columns and 0.15 for beams, These values are near to maximum of damage in column and beams. Figure 3 compares the storey damage index distribution for IBC-2009 and near optimum design models subjected to the synthetic spectrum-compatible earthquake (SEQ_{IBC}). This synthetic earthquake is representative of the design spectrum, and therefore, can be utilized to evaluate the performance level of the designed RC frames. It is shown that using the proposed design method leads to a structure with a rather more uniform storey damage index distribution.

As an example, Figure 4 shows the variation of required longitudinal reinforcement weight for the 10-stroey model designed with IBC-2009 to the final design. The results show that the proposed design method practically converged to the final solution in less than 6 steps without any fluctuation. Hajirasouliha et al. (2012) showed that an acceptable convergence for RC frames is usually obtained by using α and β value 0.1 to 0.2.

The steel reinforcement ratio of beam and column elements and the total required reinforcement weight for the 3, 5, 10 and 15 storey IBC-2009 and near optimum design solutions are summarized in Tables 2 to 5. The results indicate that, for the target level of damage, using the proposed design method resulted in 33 to 40% reduction in the required longitudinal reinforcement weight. In this study, at the initial stage of the design, the dimensions of beam and column elements are determined to meet code drift limitations. This has led to RC columns with relatively large dimensions and low reinforcement ratio especially for the 10 and 15 storey frames. As the code-specified minimum reinforcement ratio is controlled at each design iteration, the reduction in the longitudinal steel reinforcement of columns is practically limited. Therefore, as shown in Tables 3 to 5, a larger reduction in the longitudinal steel reinforcement is usually obtained for the beams rather than the columns.

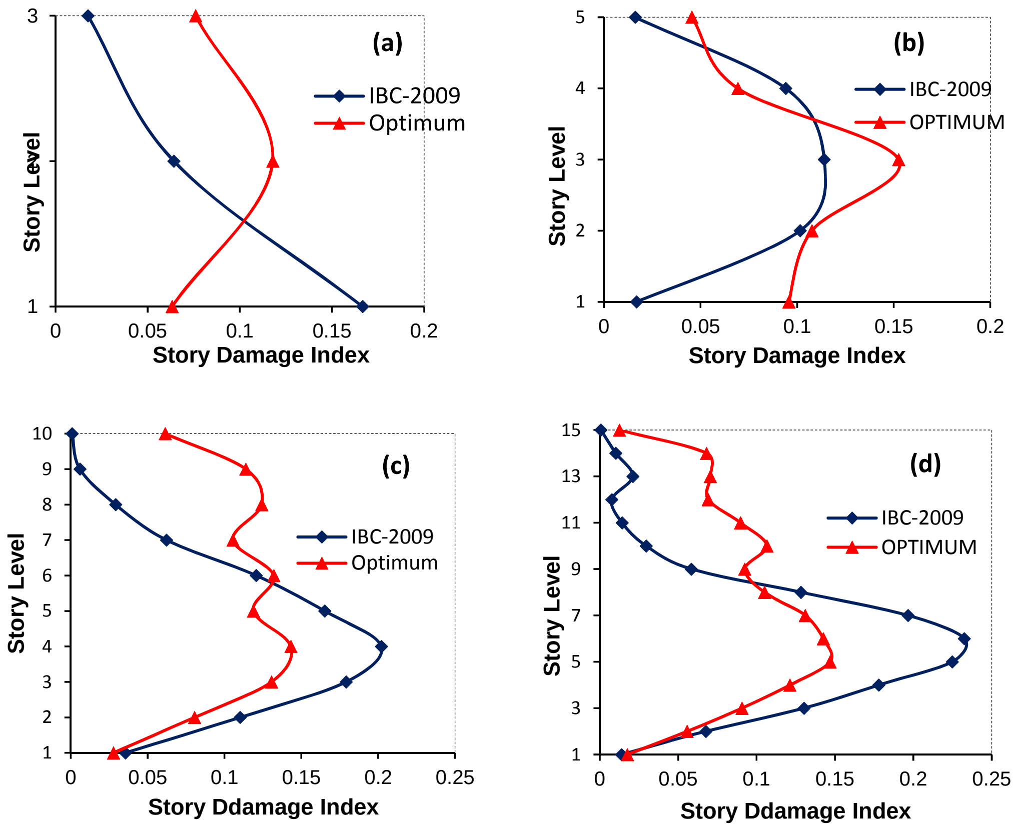

Figure 3. Storey damage distribution of IBC-2009 and near optimum design models subjected to SEQIBC, (a): 3 storey model; (b):5 storey model; (c): 10 storey model; (d): 15 storey model.

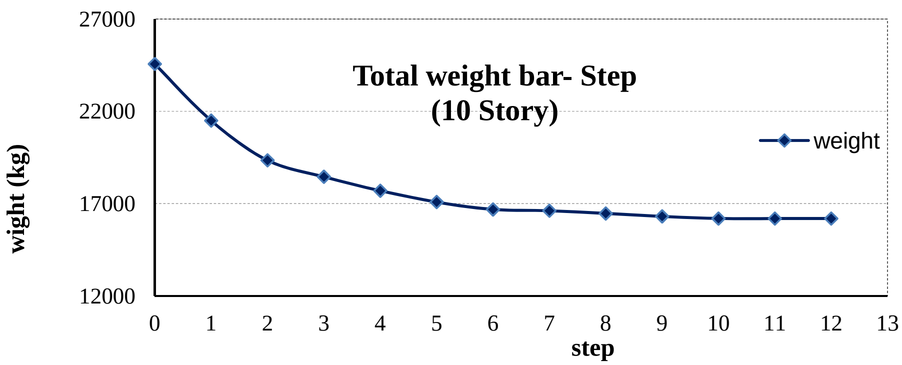

Figure4. Variation of required longitudinal reinforcement weight from IBC-2009 to final design, 10-storey frame subjected to SEQIBC.

Table 2. Comparison of steel reinforcement ratio and total required longitudinal reinforcement weight for 10-storey IBC-2009 and near optimum design models.

Story	Reinforcement Ratio (IBC-2009)				Reinforcement Ratio (Near Optimum)			
	Exterior Beams	Interior Beams	Exterior Columns	Interior Columns	Exterior Beams	Interior Beams	Exterior Columns	Interior Columns
1	1.41%	1.41%	1.83%	1.83%	0.71%	0.71%	1.11%	1.95%
2	1.41%	1.41%	1.40%	1.40%	0.71%	0.71%	1.00%	1.00%
3	1.41%	1.41%	1.62%	1.62%	0.91%	0.96%	1.00%	1.00%
4	1.41%	1.41%	1.26%	1.26%	1.02%	1.04%	1.00%	1.00%
5	1.48%	1.48%	1.26%	1.26%	0.95%	1.00%	1.00%	1.00%
6	1.42%	1.42%	1.48%	1.48%	1.13%	1.11%	1.00%	1.00%
7	1.56%	1.56%	1.76%	1.76%	1.15%	1.12%	1.00%	1.00%
8	1.73%	1.73%	1.76%	1.76%	1.24%	1.21%	1.00%	1.00%
9	1.83%	1.83%	1.86%	1.86%	1.27%	1.215	1.05%	1.52%
10	1.79%	1.79%	2.365%	2.36%	0.99%	0.78%	1.03%	1.23%
Weight	16277.8		8284.7		105265		5936	
Total Reinforcement Weight	24562.5 (kg)				16462(kg)			

Table 3. Global and local (DI) damage indices and COV for the IBC-2009 and near optimum design models subjected to the synthetic earthquake.

Story	Global *DI* in first step = 0.091 Damage (IBC-2009)				Global *DI* in final step =0 .104 Damage (Near Optimum)			
	Exterior Beams *DI*	Interior Beams *DI*	Exterior Columns *DI*	Interior Columns *DI*	Exterior Beams *DI*	Interior Beams *DI*	Exterior Columns *DI*	Interior Columns *DI*
1	0.050	0.051	0.028	0.040	0.059	0.058	0.033	0.031
2	0.085	0.085	0.030	0.037	0.104	0.108	0.022	0.020
3	0.117	0.118	0.017	0.020	0.121	0.116	0.023	0.019
4	0.130	0.131	0.010	0.016	0.122	0.120	0.013	0.010
5	0.127	0.127	0.009	0.006	0.122	0.117	0.011	0.011
6	0.098	0.100	0.013	0.012	0.127	0.129	0.012	0.014
7	0.080	0.078	0.013	0.012	0.128	0.130	0.015	0.017
8	0.060	0.056	0.012	0.013	0.136	0.141	0.024	0.031
9	0.035	0.033	0.011	0.016	0.138	0.136	0.033	0.037
10	0.022	0.021	0.013	0.010	0.129	0.137	0.034	0.025
COV	0.47		0.57		0.19		0.41	

Table 4. Comparison of total required longitudinal reinforcement weight for 3,5,10,15-storey IBC-2009 and near optimum design models.

Model	Total Reinforcement Weight(IBC-2009)	Total Reinforcement Weight(Near Optimum)	Reduction Percent
3 Storey	4700	3009	36%
5 Storey	10423	6519	37.5%
10 Storey	24563	16462	33%
15 Storey	39642	23849	39.8%

Table 5. Comparison of Global damage indices for 3,5,10,15-storey IBC-2009 and near optimum design models.

Model	Global *DI*(IBC-2009)	Global *DI*(Near Optimum)	Variation
3 Storey	0.083	0.086	4 %
5 Storey	0.069	0.094	36%
10 Storey	0.091	0.104	36%
15 Storey	0.088	0.088	0%

It should be mentioned that increasing the amount of flexural steel reinforcement may not fully control the overturning bending effects in high-rise buildings. As a result, the proposed method may not lead to a uniform damage response in high-rise frame structures in which overturning bending effects are significant, and more research is recommended for this type of building. However, the additional drift due to the axial deformation of columns at lower storeys does not usually contribute to the damage in structures. Therefore, the proposed design method can always improve the seismic behaviour of RC frames by controlling the storey damage due to the rotation of plastic hinges.

4. CONLUSIONS

In this study a more efficient performance-based design method is proposed for the seismic design of RC structures. Based on the results, the following conclusions can be drawn:

- The concept of uniform distribution of damage demands can be used efficiently to find better distribution of longitudinal reinforcement for RC structures subjected to gravity loads and seismic excitations.

- The results indicate that, for the same structural weight, a near optimum design RC frame may experience up to 60% less global damage compared to a similar code-based design frame.

- To achieve a specific performance level, the proposed design method could result in more than 40% reduction in the required longitudinal reinforcement weight.

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