A Design Procedure for Concrete Rectangular Liquid Storage Tanks Using Generalized SDOF System

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SUMMARY:

In this paper, a design procedure based on the structural model using the generalized single degree of freedom (SDOF) system is proposed for concrete rectangular liquid containing structures (LCS). The proposed model considers the effect of flexibility of tank wall on hydrodynamic pressures and uses the consistent mass approach in dynamic analysis. The contribution of higher modes to the dynamic response of LCS is included in the proposed model. The square root of sum of square (SRSS) method is proposed for the combination of the first two modes. Three tanks classified as shallow, medium and tall are used for verification. The response spectra for three suites of time history representing low, moderate and high earthquake zones are used for generalized SDOF system. The results based on the lumped mass, and the distributed mass approach, as well as those obtained using the sequential method and the generalized SDOF systems are compared. It is concluded that the results based on the generalized SDOF system have good agreement with those used the distributed mass and the sequential analysis models. The proposed design procedure using the generalized SDOF system can be simply used in seismic design of LCS.

Keywords: Reinforced concrete; liquid containing; rectangular tank; seismic; dynamic analysis

1. INTRODUCTION

Liquid containing structures (LCS) as part of environmental engineering facilities are primarily used for water and sewage treatment plants and other industrial wastes. Normally, they are constructed of reinforced concrete in the form of rectangular or circular configurations. Currently there are few codes and standards available for seismic design of LCS in North America. In almost all of codes and standards, Housner's model (Housner, 1963) has been adopted for dynamic analysis of LCS. This model approximates the effect of hydrodynamic pressure for a two fold-symmetric-fluid container subjected to horizontal acceleration as shown in Figure 1. The hydrodynamic pressures induced by earthquakes are separated into two parts of impulsive and convective components which are approximated by the lumped added masses. The added mass in terms of impulsive pressure is assumed rigidly connected to the tank wall and the added mass in terms of convective pressure is assumed connected to the tank wall using flexible springs to simulate the effect of sloshing motion. In this model, the boundary condition in the calculation of hydrodynamic pressures is treated as rigid.

Yang (1976) and Veletsos (1984) studied the effect of the wall flexibility on the magnitude and distribution of hydrodynamic pressures and associated tank forces. It was found that for tanks with realistic flexibility, the impulsive forces are considerably higher than those in rigid wall. Haroun (1984) presented a very detailed method of analysis in the typical system of loadings for rectangular tanks. However, the formula of hydrodynamic pressures only considered the rigid wall condition. Park et al. (1990) used the boundary and finite elements to study the dynamic behaviour of rectangular tanks. Subsequently, Kim et al., (1996) presented an analytical method for calculation of hydrodynamic pressures based on three-dimensional analysis of tanks. It is worth noting that only few studies on the seismic response of flexible rectangular tanks are currently available and their results



are not in a form suitable for direct use in design as mentioned in Eurocode 8 (CEN, 2006).

Figure 1 Housner's model

Chen and Kianoush (2005) developed a procedure referred to as the sequential method for computing hydrodynamic pressures based on a two-dimensional model for rectangular tanks in which the effect of flexibility of tank wall was taken into consideration. The sequential method is a coupling technique in which the two fields of fluid and structure are coupled by applying results from the first analysis as loads or boundary conditions for the second analysis. Later Kianoush et al. (2006) and Ghaemian et al. (2005) applied the staggered method to solve the coupled liquid storage tank problems in three-dimensional space. The staggered method is a partitioned solution procedure that can be organized in terms of sequential execution of a single-field analyzer. The scheme of staggered method is to find the displacement and hydrodynamic pressure at time i. Compared to Housner's model, these results show that in most cases the lumped mass approach leads to overly conservative results. Also, the effect of flexibility of tank wall on hydrodynamic pressure should be considered in dynamic analysis.

Chen and Kianoush (2009) proposed a generalized single degree of freedom (SDOF) system for dynamic analysis of LCS. The consistent mass approach and the effect of flexibility of tank wall on hydrodynamic pressures were considered. The prescribed vibration shape functions representing the mode shapes for the cantilever wall boundary condition were validated. The effective heights for liquid containing system and the effect of higher modes on dynamic response of LCS were studied. Chen and Kianoush (2010) further investigated the effects of height of liquid and width of tank on dynamic response of LCS. The liquid level varied from the empty $H_L=0$ to the full tank $H_L=H_W$. Also, instead of using the ratio of width of tank to the liquid height, L_x/H_L , the ratio L_x/H_W and the ratio of height of liquid to the wall height, H_L/H_W were used as a characteristic parameter for tank size and liquid level in tank respectively in the study of dynamic response of LCS. The added mass of liquid due to impulsive hydrodynamic pressure and the effective height in relationship with the ratios of L_x/H_W and H_L/H_W were presented which can be used in seismic design of LCS.

As part of continuous research effort to apply the generalized SDOF system in dynamic analysis of LCS, a design procedure is proposed in this paper. The proposed structural model can overcome the deficiencies in the current design codes and standards. The theories and procedures for dynamic analysis of LCS are summarized. Three tanks classified as shallow, medium and tall are used for verification. The response spectra for three suites of time history representing low, moderate and high earthquake zones are used for generalized SDOF system. The results based on the lumped mass, and the distributed mass approach, as well as those obtained using the sequential method and the generalized SDOF systems are compared.

2. GENERALIZED SDOF SYSTEM FOR DYNAMIC ANALYSIS OF LCS

2.1 Analysis Model

The generalized SDOF system is a simplified system using only one variable for dynamic analysis of distributed mass and stiffness characteristics for a predetermined mode shape. Normally, the amplitude of vibration is the only variable or degree of freedom (DOF) used in dynamic analysis and varies with time. In this paper, the generalized SDOF system is applied in the dynamic analysis of concrete rectangular LCS subjected to earthquakes.

It is assumed that the width of tank is sufficiently large so that the unit width of tank can represent the tank wall. The fluid filled in the tank is of height, H_L above the base. It is assumed that the liquid storage tank is fixed to the rigid foundation. A Cartesian coordinate system (x, y) is used with the origin located at the center of the tank base.

In this study, the walls for the concrete rectangular LCS are considered as fixed at bottom and free at top. Figure 2 shows a cantilever wall with the distributed mass m(y) and stiffness EI(y) per unit height subjected to earthquake ground motion $u_g(t)$.



Figure 2 Concrete rectangular tank in generalized SDOF system

It is worth noting that the configuration of concrete rectangular tanks may vary. In this study, only the top open rectangular tank is considered in the analysis for simplicity. However, the design procedure based on the generalized SDOF system can be applied to any configuration of concrete rectangular tanks with some modifications.

2.2 Equation of Motion

The equation of motion for the generalized SDOF system in the dynamic analysis of LCS is that:

$$\widetilde{m}\cdot \ddot{u} + \widetilde{c}\cdot \dot{u} + \widetilde{k}\cdot u = \widetilde{p} \tag{1}$$

where \widetilde{m} , \widetilde{c} , \widetilde{k} , \widetilde{p} are defined as the generalized system of mass, damping, stiffness and force respectively as shown below:

$$\widetilde{m} = \int_{0}^{H_{W}} m(y) \cdot [\psi(y)]^{2} \cdot dy + \widetilde{m}_{L}$$
⁽²⁾

$$\widetilde{k} = \int_{0}^{H_{W}} EI(y) \cdot [\widetilde{\psi}(y)]^{2} \cdot dy$$
(3)

$$\widetilde{p} = \ddot{u}_g(t) \cdot \left[\int_{0}^{H_w} m(y) \cdot \psi(y) \cdot dy + m_L\right]$$
(4)

where $\psi(y)$ is the assumed shape function, and \tilde{m}_L and m_L are the generalized and effective added mass of liquid due to impulsive hydrodynamic pressure.

The direct coupling method is used in the dynamic analysis. The interaction between liquid and tank wall is solved directly in the equation of motion using the added mass method.

The equation of motion for coupling the structure and the contained liquid subjected to earthquakes is obtained by substituting the Eqs.2 to 4 into Eq.1. Then by dividing both sides of equation by \tilde{m} , the following relationship is obtained:

$$\ddot{u} + 2 \cdot \zeta \cdot \omega_n \cdot \dot{u} + \omega_n^2 \cdot u = -\hat{q} \cdot u_g(t) \tag{5}$$

where $\omega_n = \sqrt{\tilde{k}/\tilde{m}}$ are the circular frequencies of liquid containing structure system and \hat{q} is the factor of external applied load that is:

$$\hat{q} = \hat{p} / \widetilde{m} = \frac{m_W + m_L}{\widetilde{m}_W + \widetilde{m}_L} \tag{6}$$

If an estimated damping ratio ζ is assumed, all the unknown parameters i.e. u, \dot{u} and \ddot{u} can be determined by an assumed shape function. Therefore, the infinite degrees of freedom of liquid containing structure can be simplified to a generalized SDOF system.

2.3 Hydrodynamic Pressure

The fluid in the tank is considered to be ideal, which is incompressible, inviscid, and with a mass density ρ_l . The response of the body of fluid to an earthquake can be obtained using the velocity potential method as presented in the previous study (Chen and Kianoush, 2005). Only the impulsive component is considered in this study.

The hydrodynamic pressure was solved using the separation of variables method which satisfies the boundary conditions. The hydrodynamic pressure distribution on the flexible wall condition can be expressed as follows:

$$p = \sum_{i=1}^{\infty} \frac{2 \cdot \rho_i \cdot \tanh(\lambda_i \cdot L_x)}{\lambda_i \cdot H_L} \cdot \cos(\lambda_i \cdot y) \cdot \int_{0}^{H_L} \cos(\lambda_i \cdot y) \cdot \ddot{u}(t) dy$$
(7)

where $\lambda_i = (2i-1)\pi/2H_{L_c}$ As the series in the above equation convergence very fast, only the first term of the series is used for practical applications.

2.4 Shape Functions

The general beam vibrating function can be used as an admissible shape function to approximate the vibration mode. For simplicity, the prescribed vibration shape function representing the first mode shape for the cantilever wall boundary condition was proposed for the dynamic analysis of LCS. This is defined as shape function SF1 as follows:

SF1(y) =
$$\frac{3}{2} \frac{y^2}{H_W^2} - \frac{1}{2} \frac{y^3}{H_W^3}$$
 (8)

For the second mode, the general form can be expressed as follows (Paz, 1997):

SF2=(
$$\cosh(4.694\frac{y}{H_W})$$
- $\cos(4.694\frac{y}{H_W})$)-1.018(($\sinh(4.694\frac{y}{H_W})$ + $\sin(4.694\frac{y}{H_W})$) (9)

The validity of the shape functions SF1 and SF2 was verified and discussed in the previous study (Chen and Kianoush, 2009). It is worth noting the prescribed shape functions used in this study are based on the cantilever wall boundary condition. For any other configuration of concrete rectangular tanks, proper mode shape functions should be used for approximation of vibration modes when using the generalized SDOF system.

3. CALCULATION PROCEDURE FOR THE PROPOSED MODEL

In this study, a design procedure based on the structural model using the generalized SDOF systems for seismic design of concrete rectangular LCS is proposed. The procedure is developed considering the consistent mass and the effect of flexibility of tank wall based on the theories discussed previously. The conceptual procedure for this methodology is similar to that using Housner's model. However, the generalized and effective added mass of liquid due to impulsive hydrodynamic pressure and the corresponding effective heights are introduced in this proposed model.

It is worth noting that ACI 350.3 (2006) outlines the calculation procedure for dynamic analysis of concrete rectangular LCS. Housner's model is adopted and the lumped added mass and the rigid wall boundary condition are considered in the practice.

The calculation procedure using the generalized SDOF system for seismic design of concrete rectangular LCS is summarized as follows:

(1) Calculate the generalized and effective inertial mass of tank wall, \widetilde{m}_{w} and m_{w}

The previous studies (Chen and Kianoush, 2009 and 2010) show that for the cantilever wall condition, the generalized inertial mass of tank wall \tilde{m}_{w} is 25% of total mass of tank wall and the effective inertial mass of tank wall m_{w} is about 39% and 22% of total mass of tank wall for the first and second mode shapes respectively. These ratios can be used to calculate the generalized and effective inertial mass of tank wall, \tilde{m}_{w} and m_{w} .

(2) Calculate the generalized and effective added mass of liquid due to impulsive hydrodynamic pressure, \tilde{m}_L and m_L .

Similar to Housner's model, the hydrodynamic pressure is incorporated into the coupling analysis through the added mass, when using the generalized SDOF system. The ratios of the general and effective added mass of liquid due to impulsive hydrodynamic pressure to the total mass of liquid in the containment, i.e. \tilde{m}_L / M_L and m_L / M_L , or alternatively the ratios of the generalized and effective added mass of liquid due to impulsive hydrodynamic pressure to that of rigid wall condition, i.e. $f_{mass} = m / M_{L-rigid}$ and $\tilde{f}_{mass} = \tilde{m} / \tilde{M}_{L-rigid}$ can be used in design application. The details are discussed in the previous studies (Chen and Kianoush, 2009 and 2010).

In order to consider the variable liquid level in tanks and the configuration of tanks, it is recommended

to use H_L/H_W and L_X/H_W in design application (Chen and Kianoush, 2010). The values of added mass of liquid due to impulsive pressure as function of H_L/H_W and L_X/H_W are tabulated in the reference (Chen, 2010).

(3) Calculate the generalized stiffness of tank wall

The stiffness of tank wall can be calculated using Eq.3 defined previously.

(4) Calculate the natural period of vibration

The natural period of vibration T_i , including the effect of the tank wall and the impulsive hydrodynamic pressure component can be calculated as follows:

$$T_i = 2\pi \sqrt{\frac{\widetilde{m}_W + \widetilde{m}_L}{\widetilde{k}}}$$
(10)

(5) Obtain the mapped maximum earthquake response spectral acceleration A_a

The mapped maximum earthquake response spectral acceleration can be obtained using the applicable seismic ground motion maps in the codes.

(6) Calculate the maximum displacement at top of tank wall

$$u_{\max} = \frac{\hat{q}}{\omega_i^2} \cdot A_a \tag{11}$$

(7) Calculate the base shear using the following relationship

$$V_B = \hat{p} \cdot \hat{q} \cdot A_a \tag{12}$$

where \hat{q} and \hat{p} are defined in the Eq.6.

(8) Calculate the effective heights of wall and effective added mass of liquid due to impulsive hydrodynamic pressure

In the current design practice, the inertial mass of concrete wall and the added mass of liquid due to hydrodynamic pressure are lumped at defined effective heights based on Hounser's model. Similarly the effective heights at which the effective inertial mass of tank wall and the effective added mass of liquid due to hydrodynamic pressure is applied, i.e. h_w and h_i , can be calculated based on the generalized SDOF system (Chen and Kianoush, 2009 and 2010).

It is worth noting that considering the flexibility of tank wall, the effective height h at which the overall lateral dynamic force is applied is higher than that obtained from the rigid wall condition.

For cantilever wall condition, the effective height of wall is $0.75H_w$ and $0.21H_w$ for the first and the second modes respectively.

The effective height of liquid containing structure h can be calculated using Eq.13.

$$h = \frac{m_w \cdot h_w + m_L \cdot h_i}{m_w + m_L} \tag{13}$$

(9) Calculate the base bending moment

$$M_B = V_B h \tag{14}$$

(10) Calculate the vertical distribution of the impulsive hydrodynamic component and the impulsive hydrodynamic force

$$P_i = m_L \cdot \hat{q} \cdot A_a \tag{15}$$

(11) Include the effect of the second mode in dynamic response of LCS

The calculation procedure for the second mode is the same as that for the first mode. The inertial mass of tank wall, the added mass of liquid due to hydrodynamic pressure and the corresponding effective highs are calculated using the design parameter as discussed before. The overall dynamic response of LCS can be calculated using the SRSS method to combine the dynamic response of the first two modes. More details are discussed in the references (Chen and Kianoush, 2009 and 2010).

4. ANALYSIS AND RESULTS

In this study, three different tanks are used for dynamic analysis using the generalized SDOF system based on a two-dimensional model. The height of tank wall, H_w , the height of liquid filled inside the tank, H_l , and the corresponding thickness of tank wall, t_w , for the three different tanks are as follows:

(1) Tank 1 (shallow tank): $H_w = 3.0 \text{ m}$, $H_l = 2.7 \text{ m}$, $t_w = 0.3 \text{ m}$ (2) Tank 2 (medium tank): $H_w = 6.0 \text{ m}$, $H_l = 5.5 \text{ m}$, $t_w = 0.6 \text{ m}$ (3) Tank 3 (tall tank): $H_w = 9.0 \text{ m}$, $H_l = 8.1 \text{ m}$, $t_w = 0.9 \text{ m}$

Other dimensions and properties of the tanks are as follows: $L_x = 15 \text{ m}, \rho_w = 2300 \text{ kg/m}^3, \rho_l = 1000 \text{ kg/m}^3, E = 26440 \text{ MPa}, \nu = 0.17$

In the previous study (Chen and Kianoush, 2004), three different tank models, i.e. lumped mass, distributed mass and sequential analysis models were considered for dynamic time-history analysis. In the lumped model, the impulsive mass of liquid is determined using the procedure described by Housner (Housner, 1963). In the distributed mass model, both the impulsive mass of liquid and the mass of wall are distributed over the height of wall. In this case, the impulsive mass is determined assuming a rigid wall condition. In the sequential analysis model, the hydrodynamic impulsive pressure is determined considering the wall flexibility and the sequential analysis procedure is used.

The time history of ground motions which are as the same as the ground motions used in the previous study (Chen and Kianoush, 2004) are used. Three suites of time history in locations of Boston (BO32), Seattle (SE23) and Los Angeles (LA25) corresponding to seismic zones 2, 3 and 4 respectively are used in the dynamic analysis. The response spectra for the specific ground motions are developed and used for the generalized SDOF system.

The comparison of top displacement, base shear and base moment between the model using the generalized SDOF system and those used in the previous study (Chen and Kianoush, 2004) are shown in Figures 3 to 5. It is worth noting that time history analysis was used in the lumped mass, distributed mass and sequential analysis models. Therefore, the variable response spectral values due to the specific ground motions may significantly affect on the dynamic response of liquid storage tanks. For the generalized SDOF system, however the response spectrum method is used. The variable response spectral values due to ground motions are averaged and represented by a response spectrum. The seismic data for response spectrum in a specific seismic zone can be found in the building codes and easily applied in seismic design.

















Figure 3 Tank 1 ($H_W = 3m$, $H_L = 2.7m$)







Figure 4 Tank 2 ($H_W = 6m$, $H_L = 5.5m$)



(c) Base Moment

Figure 5 Tank 3 ($H_W = 9m$, $H_L = 8.1m$)

In the lumped mass model, the dynamic response of liquid containing structures show significant difference such as base shear as compared with those calculated using the other models. This is due to the dynamic properties of lumped mass model which can not accurately represent liquid containing structures. As a result, the variable response spectral corresponding with the fundamental natural frequencies result in significantly different values in dynamic response of liquid storage tanks as compared to the results calculated using the other models.

In the distributed mass model, the mass distribution is similar to that used in the generalized SDOF system. Therefore, the dynamic response based on the distributed mass model using the time history analysis is consistent with that based on the generalized SDOF model using the response spectrum method. It is worth noting that the dynamic response obtained using the generalized SDOF system is higher than that obtained using the distributed mass model due to the consideration of wall flexibility.

The sequential analysis model is assumed to be the most accurate model for dynamic analysis of liquid containing structures. However, the effect of ground motions is still sensitive to the time history analysis due to sequential transferring of data between two domains. Figures 3 to 5 show slight difference in the dynamic response of tanks for different seismic zones between the sequential and distributed mass models using the time history analysis. However, the results of analysis using the generalize SDOF system show good agreement with those based on the distributed mass and sequential analysis models.

5. CONCLUSIONS

In this paper, a design procedure based on structural model using the generalized SDOF system is proposed for seismic design of LCS. The procedures for dynamic analysis of LCS are summarized. The proposed model can consider the consistent mass and the effect of flexibility of tank wall in design. The conceptual procedure for this methodology is similar to that of Housner's model adopted in the current design codes and standards. However, the generalized and effective added mass of liquid due to impulsive hydrodynamic pressure and the corresponding effective heights are introduced in the proposed model.

Three tanks classified as shallow, medium and tall tanks are used for verification. The response spectra for three suites of time history representing low, moderate and high earthquake zones are used for generalized SDOF system. The results based on the lumped mass, the distributed mass, as well as those based on the sequential analysis and the generalized SDOF systems are compared. It is concluded that the results based on the generalized SDOF system have good agreement with those used the distributed mass and the sequential analysis models. The proposed design procedure using the generalized SDOF system can be simply used in seismic design of LCS.

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