Probabilistic vulnerability assessment of a reinforced Concrete structure by using a 3-D model

Y.F. Vargas, A.H. Barbat, L.G. Pujades
Universidad Politécnica de Cataluña

J.E. Hurtado
Universidad Nacional de Colombia

SUMMARY:
In the seismic vulnerability and fragility assessment of buildings, the variables involved, such as the mechanical properties of the materials, the distribution of loads and the seismic action, among others, should be considered random as they really are. Many researches have considered these uncertainties by means of the Monte Carlo method and the stochastic dynamics analysis. Nevertheless, in order to simplify the procedure, the building is usually approximated by a 2-D model. It is well known that, under this simplified hypothesis, the effects of the flexural interaction and torsion on the structural elements are lost. To overcome this limitation, in this paper, the vulnerability of a reinforced concrete building is evaluated by considering a 3-D model while the uncertainties are considered by means of Monte Carlo simulation. The results obtained by using incremental dynamic analysis are compared with those provided a simplified 2-D model and compared from a probabilistic point of view..

Keywords: Vulnerability assessment, incremental dynamic analysis, reinforced concrete building

1. INTRODUCTION

In many studies, the seismic risk of structures is evaluated either by using incremental static analysis or by means of nonlinear dynamic analysis performed in an incremental way (Borzi et al. 2007; Barbat et al. 2008; Lantada et al. 2009; Pujades et al. 2011). It is well known that there are huge uncertainties in the variables involved in such structural analyses, mainly the mechanical properties and the seismic actions, which have to be treated as random. These uncertainties may lead to an underestimation or an overestimation of the actual seismic risk of the structure. Due to the current capacity of the computers, a great number of structural analyses can be performed in order to study the behavior of buildings from a probabilistic standpoint within the framework of Monte Carlo simulation. This study focuses on the nonlinear seismic response of reinforced concrete buildings with a view of estimating their seismic damage considering uncertainties in the seismic action, in the mechanical properties of the materials and in the live loads applied to the structure. On the other hand, in order to simplify the structural analyses, mainly due to the symmetry of the structure, in previous studies the building model is usually reduced to a 2-D model (Vargas 2011). This simplified hypothesis does not take into account the effects on the structural elements of the flexural interaction and torsion. In order to overcome this limitation, a full 3-D model of a reinforced concrete building is considered in this paper, and a metric of its seismic risk is calculated. To do that, the incremental dynamic analysis, proposed by Vamvatsikos & Cornel (2002), is considered in this paper and the results are compared with those obtained by means of a simplified 2-D model.

2. DESCRIPTION OF THE STUDIED BUILDING

The reinforced concrete building selected for this study has been designed according to the prescriptions of Eurocode 8 for reinforced concrete structures. The building has been located in a city of Lorca, Spain, which recently was affected by a moderate earthquake of magnitude 5.1, but which caused 9 casualties and great economic losses. The building is regular in plan and in elevation, with 4 spans of 6 m in x and y directions, and the story height is of 3.65 m. In this article, two structural models are considered, one 3-D and the second 2-D (see Figure 1)

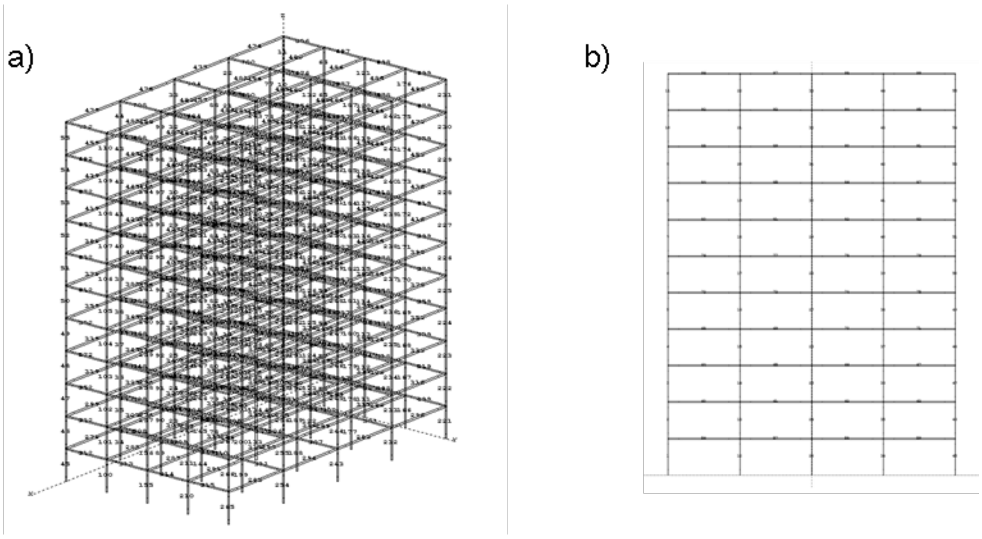

Figure 1. 3D and 2-D models of the analyzed building

As mentioned above, for the 3-D model of the building, the yielding surfaces representing the structural elements take into account the interaction between flexural and axial forces. Therefore, the yielding interaction surface proposed by Tseng and Penzien (1973) is considered for the columns. In the case of the beams, only the flexural interaction is considered by means of the following equation:

$$\left(\frac{Mz}{MBz}\right)^{\alpha} + \left(\frac{My}{MBy}\right)^{\alpha} = 1 \tag{1.1}$$

where *Mz* is the bending moment for z-z axis, *MBz* is the corresponding yield bending moment, *My* is the bending moment for y-y axis, *MBy* is the corresponding yield bending moment and α is the flexural interaction factor; if α equals 2, the surface is elliptical and, if α equals 0, the surface is a rectangle. For the 2-D model building, in the case of columns the yielding surfaces are given by the bending moment-axial interaction diagram and in the case of beams by the bending moment-rotation diagram. For all the structural analyses wich we performed, the constitutive law of the structural elements has been considered elastoplastic without hardening or softening. Computer programs have been developed in order to calculate the mentioned variables which are necessary when defining the behavior of the elements of the model used in the structural analyses. In this article, all the structural analyses have been performed by using the RUAUMOKO computer code (Carr 2000).

3. NONLINEAR DYNAMIC ANALYSIS

3.1. Incremental dynamic analysis

A nonlinear dynamic analysis, NLDA, performed for a given accelerogram, provides the time history response of a building and, then, the maximum response variables of the structure like the displacement at the roof, the global damage index according to a certain criterion, etc. can be calculated. Scaling the accelerogram with a given increment of the PGA, for values starting from a lower limit (which includes the elastic range) until reaching an upper one, corresponding to the building collapse and performing for each increment a NLDA, a curve relating the PGAs to the maximum roof displacement is obtained, which is usually called dynamic pushover curve. When a curve relates the PGAs to the global damage index of a structure, it is denoted as damage curve. When, instead of a single accelerogram, several of them are used to perform nonlinear dynamic analyses, and statistics are made with the obtained results, we are faced with an incremental dynamic analysis, IDA (Vamvatsikos and Cornel 2002). Summarizing, IDA allows obtaining the nonlinear dynamic response of a structure, for a group of earthquakes which are scaled to different levels of intensity of the seismic action; in this article, the Peak Ground Acceleration, PGA, has been considered to characterize the intensity of the earthquake. Besides, uncertainties in the structural properties and in the seismic action have been included in the performed analysis (Vamvatsikos and Fragiadakis 2010).

3.2. Damage indices

Several seismic damage indices have been proposed for evaluating the damaged state of the members of reinforced concrete structures starting from a post-process of the nonlinear dynamic response. Some of them are described in the following. A first simple method calculates the damage index as the ratio of the maximum ductility achieved during the seismic action to the ultimate ductility at element level and this is addressed herein as the ductility based damage index

$$DI_{\mathrm{E}} = \frac{\mu_m}{\mu_u} \tag{3.1}$$

where μ_m and μ_u are the maximum and ultimate ductilities, respectively, and the subscript E stays for element level damage index. Banon and Veneziano (1982) proposed a damage index using a nonlinear equation considering the maximum and yielding ductility, the dissipated hysteretic energy, the yielding action and a numerator corresponding to monotonic loading.

$$DI_{\mathrm{E}} = \sqrt{\frac{\left(\frac{\mu_m}{\mu_y} - 1\right)^2 + \left(1.1\left(\frac{2E_h}{F_y \mu_y}\right)^{0.38}\right)^2}{\text{Numerator for monotonic load}}} \tag{3.2}$$

where μ_y is the yielding ductility, Eh is the hysteretic energy dissipated and Fy is the yielding action. The damage index of Park and Ang (1985) is the sum of the maximum ductility divided by the ultimate ductility, that is, the ductility based damage index, with a term related to the dissipated energy. The corresponding equation is:

$$DI_{\mathrm{E}} = \frac{\mu_m}{\mu_u} + \frac{\beta E_h}{F_y \mu_u \delta_y} \tag{3.3}$$

where β is a non-negative parameter which represent the effect of cyclic loading on structural damage and δ_y is the yield displacement. Roufaiel and Meyer (1987) proposed a damage index considering the maximum, the yielding and the ultimate ductility and, besides, the maximum and yield actions

$$DI_{\mathrm{E}} = \frac{\frac{\mu_m}{F_m} - \frac{\mu_y}{F_y}}{\frac{\mu_u}{F_u} - \frac{\mu_y}{F_y}} \tag{3.4}$$

where F_m and F_u are the maximum and the ultimate actions, respectively. Bracci et al. (1989) proposed a damage index as the ratio of the work done at the maximum ductility to the work done at the ultimate ductility

$$DI_{\mathrm{E}} = \frac{E_m}{E_u} \tag{3.5}$$

where E_m and E_u are the work done at the maximum ductility and the work done at the ultimate ductility, respectively. Cosenza et al. (1993) proposed a damage index as the ratio of the maximum ductility minus one to the ultimate ductility minus one

$$DI_{\mathrm{E}} = \frac{\mu_m - 1}{\mu_u - 1} \tag{3.6}$$

In all the cases, the global damage index of the structure, *DI*, is a weighted mean of the member damages, in which the weights are the ratio of the hysteretic energy dissipated by each element to the total hysteretic energy dissipated by the structure (Park and Ang (1985).

$$DI = \sum_i \lambda_i DI_{\mathrm{E}} \tag{3.7}$$

where *DI* is the dynamic analysis based global damage index of the structure, λ_i is the ratio of the dissipated hysteretic energy of an element E to the dissipated hysteretic energy of the entire structure.

3.3. Seismic demand

The Lorca earthquake of 11th of May 2011, mentioned above, is considered as seismic hazard in this paper. Its horizontal components and the corresponding response spectra are shown in Figure 2. The uncertainties in the seismic hazard are considered by rotating the horizontal components of the record by an angle θ (Beyer & Bommer 2007)

$$\begin{pmatrix} a_{x(\theta)}(t) \\ a_{y(\theta)}(t) \end{pmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{pmatrix} a_x(t) \\ a_y(t) \end{pmatrix} \tag{3.8}$$

where $a_{x(\theta)}(t)$ and $a_{y(\theta)}(t)$ are the horizontal components of the accelerogram when rotated anti-clockwise by an angle θ, while $a_x(t)$ and $a_y(t)$ are the original components of the record. In order to compare the results obtained by means of 2-D and 3-D models of the building, nonlinear dynamic analyses are performed by rotating the horizontal components by varying the angle θ from 0° to 180° at increments of 1°. The original horizontal acceleration components are scaled by 1.5, what it is necessary in order to obtain relevant damage indices because the building was designed to remain within the elastic rang for the original record.

3.4. Results of the dynamic analyses

180 nonlinear dynamic analyses have been performed by using the 3-D model and the rotated signals applied in the *x* and *y* axis. In the case of the 2-D model, it was necessary to perform two groups of 180 nonlinear dynamic analyses by using a representative frames in the *x* and *y* directions and the corresponding rotated signal. In this case, due to the fact that the building is perfectly symmetric, it was necessary to perform only 180 nonlinear dynamic analyses considering *x* or *y* rotated signals and shifting the results by 90° to obtain the response in the orthogonal direction. Figure 3 shows the maximum displacements obtained with the 2-D model in the *x* and *y* direction; they are compared with those obtained at the time in which the orthogonal displacement is maximum.

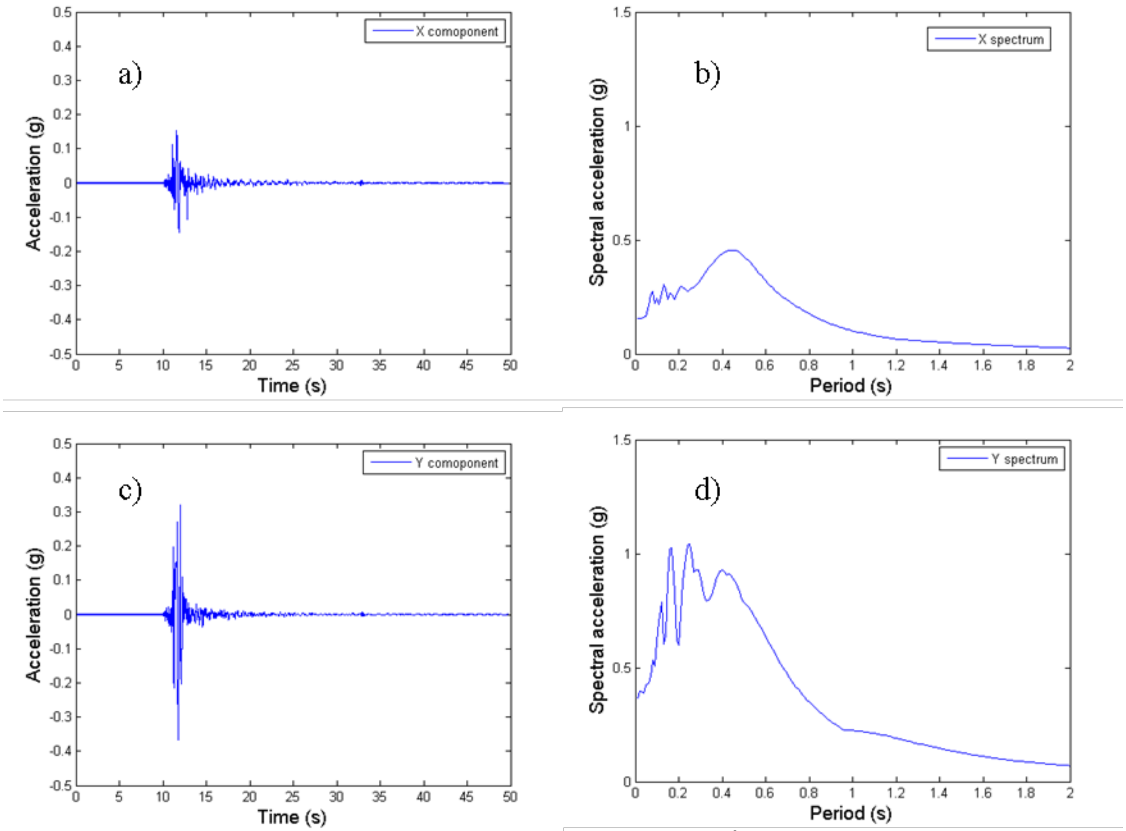

Figure 1. Acceleration records of the Lorca, Spain, earthquake of 11th of May 2011 and the corresponding response spectra. a) and b) E-W direction; c) and d) N-S direction.

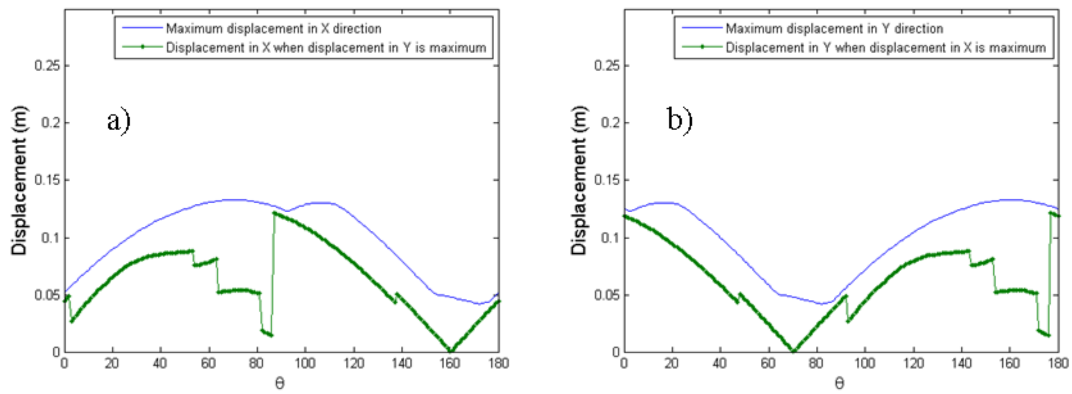

Figure 2. Results obtained with the 2-D model. a) Maximum displacement at the roof in x direction b) Maximum displacement at the roof in y direction

The total displacement at the roof level of the building is calculated as the square root of the sum of the square displacements in both directions, and performing the combination of the maximum dynamic displacement, it means independently of time, and by performing the combination of the displacement at the time instant when it is maximum in one of both direction. Figure 4a shows the high difference between the mentioned approaches. It also shows the total displacement, calculated by means of the square mean combination, obtained with the 3-D model. In this case, the maximum total displacement must be calculated by combining the *x* and *y* displacements at the same time. From these curves one can conclude that, for calculating the maximum displacement at the roof level starting from a 2-D model, the best approach is to combine the maximum displacements independent of time. Notwithstanding, the results are not conservative when 2-D model is used. This fact can be seen in Figure 4b which shows the ratio results obtained with the 2-D model to the results obtained with the 3-D model, expressed as a percentage.

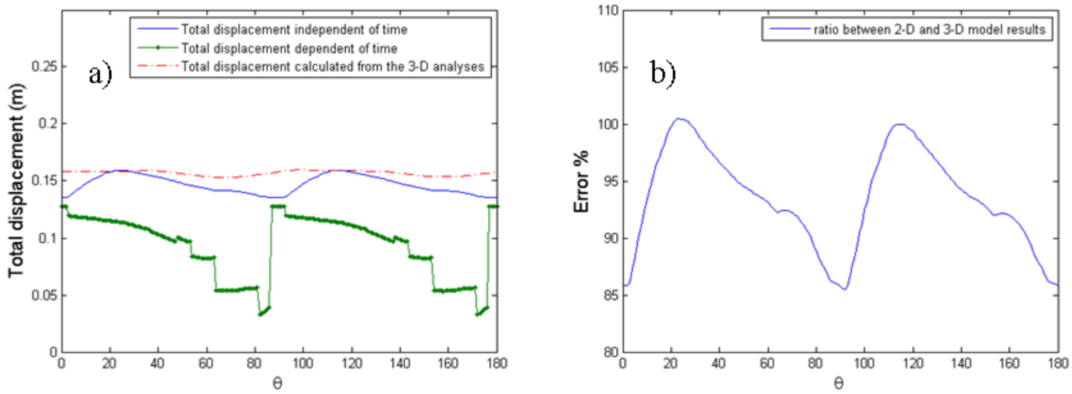

Figure 3. a) The total displacement obtained with the 2-D and 3-D models. b) Ratio of the 2-D results to the 3-D results, expressed as a percentage

The damage indices mentioned in section 3-1 are also calculated as a function of the rotation angle of the earthquake θ and the results are again compared for the 2-D and 3-D models. In this case, it is necessary to propose a measure for combining the damage indices calculated with the 2-D model in both directions. To do that, in this article, the global damage indices are the mean of damage indices obtained in both directions. Figure 5 shows the results comparison for the different global damage indices. Again, the 2-D model does not estimate with sufficient precision the damage indices calculated with the 3-D model. For instance, in the case of deformation and Park & Ang damage indices, the results are very conservative and for the remaining damage indices the results are conservative for certain angles and for others they are not. This lack of precision can increase for higher PGAs. If the earthquake is scaled by 2 instead of 1.5, the differences are higher for certain damage indices and angles; in some cases they are conservative and in others not. Figure 6 shows this comparison.

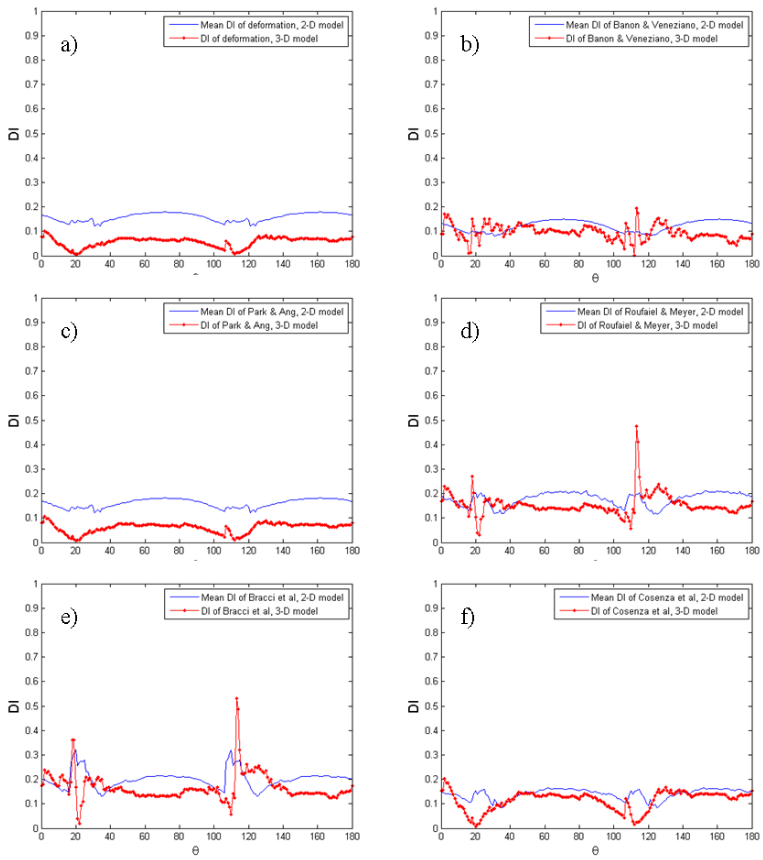

Figure 4. Comparison of the global damage indices represented in function of the rotation angle of the earthquake for a PGA scale factor of 1.5.

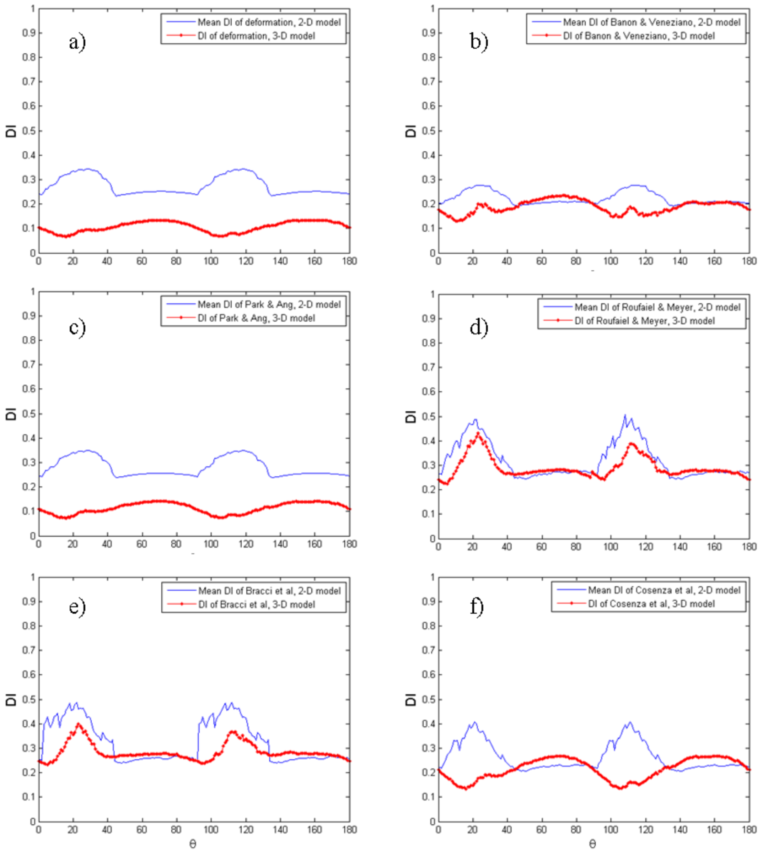

Figure 5. Comparison of the global damage indices represented in function of the rotation angle of the earthquake for a PGA scale factor of 2.

4. PROBABILISTIC ASSESSMENT OF THE DAMAGE

Due to the fact that the 2-D model is not able to estimate with sufficient precision the results obtained with the 3-D, which are expected to be the more realistic, the probabilistic assessment is performed by using the 3-D model. The mechanical properties of concrete and steel are the values commonly used in the design of such buildings. Design standards require characteristic strength values for the materials obtained during the quality control process, from compression and tension tests in concrete and steel samples, respectively. Starting from these tests, the concrete compressive strength, *fc*, and the elastic modulus of the steel, *Es*, can be modeled as random variables. Table 1 shows the mean, μ, the standard deviation, σ, and the coefficient of variation *cov* of these random variables and we assume that they follow a normal distribution.

Table 4.1. Characteristics of the input random variables μ, σ and *cov* represent the mean, the standard deviation and the coefficient of variation of the input random variables

	μ	σ	*cov*
fc (kN)	3E04	3E03	0.1
Es (kPa)	2E08	2E07	0.1

In order to consider the uncertainties related to the structural characteristics, we used the Monte Carlo method. It is well known that the spatial variability of the structural elements characteristics greatly influences on the results (Franchin et al. 2010) Therefore, this spatial variability is considered by generating, for all the columns of the same story of the building, one random sample for the compressive strength of concrete, *fc*, and, for each column of the same story, one random sample of the elastic modulus of the steel, *Es*. The same criterion is used for generating random samples for the characteristics of the materials of the beams of this story. It is important to note that the samples corresponding to each story are independent. This consideration is based on the fact that, usually, the structural elements of the same storey are made of the same concrete but the properties of the reinforcement can be supposed as independent from rebar to rebar. Other random variable considered is the live load. In order to considerer its variability, random samples are generated and applied in each node whose value is weighed by the area afferent to the node. The mean value and the standard deviation of this random variable are calculated starting from the values given by the Eurocode 8 (CEN 2004). The uncertainties in the seismic hazard are considered by rotating the horizontal components of the accelerogram by an angle θ, assuming a uniform distribution ranging from 0° to 180°. This consideration is based on the fact that the urban planning of the cities does not consider the seismic hazard. Therefore, the results obtained roughly provide the information of the randomness of a building typology because the building to building variation of the structural characteristics within a structural class is not considered herein. Further discussion of this issue can be found in Crowley et al. Afterwards, random samples of the mechanical properties of the materials, live loads and the angle θ are generated and nonlinear dynamic analyses, NLDA, are performed. This procedure is repeated for different PGAs ranging from 0.015 g to 1.4 g at intervals of 0.015 g. The Latin Hypercube method is used for generating random samples and for combining these randomly with the accelerograms. Figure 7 shows the mean damage indices as a function of the PGA and one can see in terms of the standard deviation the high dispersion obtained after considering the implied random variables.

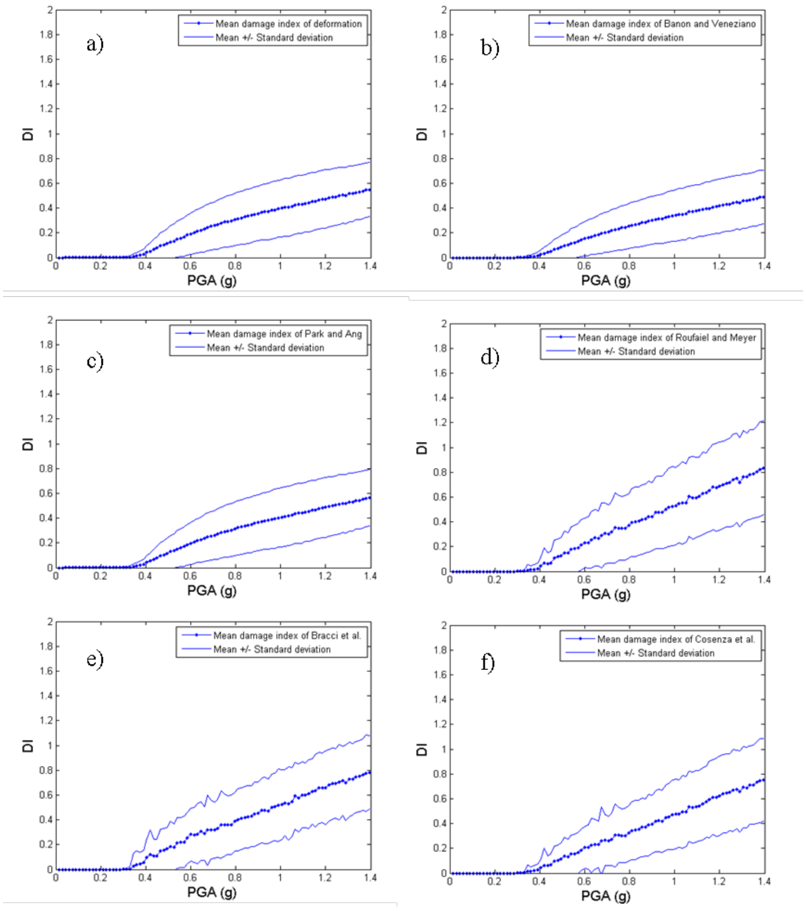

Figure 7. The considered global damage indices represented as functions of the PGA. The figures show the mean values and the standard deviation

5. CONCLUSIONS

The widely used 2-D models for evaluating the seismic behaviour of buildings can be inaccurate even in the case of symmetric structures. One of the most relevant conclusions of this work is that the parameters influencing upon the seismic damage curves of the structures must be considered as random. It can be seen how uncertainties in these parameters produces significant uncertainties in the seismic response. Simplified deterministic procedures based on characteristic values usually lead to conservative results but some abridged assumptions on the definition of the seismic actions can lead also to underestimate the real damage that can occur in a structure. Further investigation should be made in order to include the axial and flexural interaction and the torsional effects.

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REFERENCES

Banon H., Veneziano D. (1982). Seismic safety of reinforced concrete members and structures. *Earthquake Engineering and Structural Dynamics* 10:2, 179-193.

Barbat A.H., Pujades L.G., Lantada N., Moreno R. (2008). Seismic damage evaluation in urban areas using the capacity spectrum method: application to Barcelona. *Soil Dynamics and Earthquake Engineering* 28:10-11, 851–865.

Bracci J.M., Reinhorn A.M., Mander J.B., Kunnath S.K. (1989). *Deterministic for seismic damage evaluation of reinforced concrete structures*. NCEER-89-0033, National Center for Earthquake Engineering Research, State University of New York at Buffalo.

Borzi B., Phino R. and Crowley H. (2008). Simplified Pushover analysis for large-scale assessment of RC buildings. *Engineering Structures* 30, 804-820.

Carr A.J. *Ruaumoko-Inelastic Dynamic Analisys Program*. Department of Civil Engineering, University of Canterbury, Christchurch, New Zealand 2000.

CEN. *Euro*code 8 (2004). Design of structures for earthquake resistance. Part 1: General rules, seismic actions and rules for building. European committee for standardization.

Cosenza E., Manfredi G., Ramasco R. (1993). The use of damage functionals in earthquake engineering: A comparison between different methods. *Earthquake Engineering and Structural Dynamics*. 22:10, 855-868.

Crowley H., Bommer J., Pinho R., Bird J.F. (2005). The impact of epistemic uncertainty on an earthquake loss model. *Earthquake Engineering and Structural Dynamics* 34:14, 1635-1685.

Franchin P., Pinto P., Pathmanathan R. (2010). Confidence factor? *Journal of Earthquake Engineering* 14:7, 989-1007

Katrin B., Bommer J. (2007). Selection and scaling of real accelerograms for bi-directional loading: A review of current practices and code provisions. *Journal of Earthquake Engineering* 11, 13-45.

Lantada N., Pujades L.G. and Barbat A.H. (2009). Vulnerability index and capacity spectrum based methods for urban seismic risk evaluation. A comparison. *Natural* Hazards 51:3, 501-524.

Park Y-J, Ang A.H-S. (1985). Mechanistic seismic damage Model for Reinforced Concrete. *Journal of Structural Engineering ASCE*; 111:4, 722-739.

Park Y-J., Ang A.H-S Kwei-Wen Y. (1985). Seismic damage analysis of reinforced concrete buildings. *Journal of Structural Engineering ASCE* 111:4, 740-757.

Pujades L.G., Barbat A.H., González-Drigo R., Avila J., Lagomarsino S. (2012). Seismic performance of a block of buildings representative of the typical construction in the Eixample district in Barcelona (Spain). *Bulletin of Earthquake Engineering* 10:1, 331–349.

Roufaiel M.S.L, Meyer C. (1987). Analytical model of hysteretic behaviour of RC frames. *Journal of Structural Engineering ASCE* 113:3, 429-457.

Tseng W.S. and Penzien J. (1973). Analytical investigation of the seismic response of long multiple span highway bridges, UCB/EERC-73/12, Earthquake engineering research center, University of California, Berkley, CA.

Vamvatsikos D., Cornell C.A. (2002). The Incremental Dynamic Analysis. *Earthquake Engineering and Structural Dynamics* 31:3, 491-514.

Vamvatsikos D., Fragiadakis M. (2010). Incremental dynamic analysis for estimating seismic performance sensitivity and uncertainty. *Earthquake Engineering and Structural Dynamics* 39:2, 141-163.

Vargas Y.F., Pujades L.G., Barbat A.H. and Hurtado J.E. (2011). Pushover analysis and incremental dynamic analysis. A probabilistic comparison. *Proceeding 3rd International Conference on Computational Methods in Structural Dynamics and Earthquake Engineering*, Corfu, Greece.