Reliability Assessment of Water Distribution Networks Considering Mechanical-Hydraulic Behavior Using Informational Entropy

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SUMMARY:
This paper presents a new weighted entropy-based measure for assessing reliability of water distribution networks. The proposed methodology considers both of mechanical uncertainties (probability of pipe failure) and uncertainties due to hydraulic parameters (flow in pipe) simultaneously. In this methodology a penalty function is defined for different links of the network based on their probability of failure in the specified hazard scenario. This penalty function is inserted in the hydraulic entropy function of the network in an appropriate manner so that the effect of mechanical behaviour of links is taken to account in the network's entropy. In this manner the amount of supply loss due to absence of each link in the network is incorporated in the corresponding penalty function. Using some sample networks, it is shown that on the basis of the proposed entropy-based index the optimum hydraulic layout for designing a new system, or for finding the best mitigation plan against different natural hazards like earthquake can be chosen.

Keywords: Lifeline, Mechanical-Hydraulic Reliability, Network's Redundancy, Informational Entropy

1. INTRODUCTION

A water distribution system is a network of source nodes, pipes, demand nodes and other hydraulic components such as pumps, valves and tanks. The objectives of the water distribution system are to supply water at a sufficient pressure level and quantity to all its users and provide water for the purpose of fighting fires. Quantification of water distribution networks' reliability, as a lifeline system whose failure causes serious consequences in the social and economical environment, has been considered as a one of the most important research topics in the risk management in the past decades. Reliability of a water distribution network can be defined as the probability that the given demand nodes in the system receive sufficient supply with satisfactory pressure head. There are numerous measures of reliability for water distribution networks proposed by various researchers. But one of the reasons that reliability has not yet become a common phase in design practice is its complexity (Moghtaderi-Zadeh & Kiureghian, (1983); Cullinane, (1985); Wagner et al., (1988); Quimpo & Shamsi, (1991); Li et al., (1993); Wu et al., (1993); Kansal et al., (1995); Selçuk & Yücemen, (1999); Tanyimboh et al., (1999); Ostfeld, (2001)).

Redundancy, on the other hand, in a water distribution network implies the reserve capacity of network and also the demand nodes have alternative supply paths in the event that links go out of service (Awumah et al., (1990, 1991)). Redundancy, which is related to reliability, is an aspect of the overall system performance that is often neglected. A truly redundant network is inherently very reliable. Seismic performance of lifeline networks during the past earthquakes have turned out that a single redundancy provides a tremendous increase in reliability. In other words, networks with some amount of redundancy have higher ability to respond to partial failure in the network (Javanbarg, 2006). Thus, Redundancy can be considered as a surrogate measure for the reliability of water distribution networks. Awumah and his colleagues were the earliest researchers in this field but later Tanyimboh and Templeman (1993a, 1993b) established a better definition of the entropy function for water distribution networks. The thorough definition of the entropy function is presented in the next section, together with a discussion on its interpretation.

Tanyimboh and Templeman also developed a non-iterative algorithm to find the maximum-entropy flow distribution for single-source networks. The only known data are assumed to be the network topology, the flow directions in every pipe, and the supplies and demands at every node. As the parameters such as pipe length, diameter and roughness are not known, there will be an almost infinite number of possible flow distributions, unless the network is of the branching-tree type. This non-iterative algorithm is formulated using the path entropy concept and Laplace's principle of insufficient reason. They also tried to extend the single-source algorithm to cover multiple-source networks, through the use of the super-source concept. But this extended version of the single-source algorithm for multiple-source networks was shown to be inconsistent in a discussion paper by Walters. Based on the single-source algorithm, Yassin-Kassab et al. (1999) presented a non-iterative algorithm for calculating the maximum-entropy flow distribution in multiple source networks.

Tanyimboh and Templeman (2000) investigated the relationship between the entropy and reliability of water distribution network. This seems to support the hypothesis that water distribution networks that are designed to carry the maximum entropy flows will be highly reliable. Further studies by Tanyimboh and Sheahan (2002) explored the possibility of optimizing the layout of water distribution systems by using a minimum-cost/maximum-entropy design concept. The research on entropy flows in water distribution network is advancing to the stage where applications are possible, but the actual interpretation of the meaning of network entropy has never been fully elucidated.

Ang and Jowitt (2003, 2005a) investigated the meaning of network entropy with the use of a simple water distribution network. The investigation concentrated on the relationship between the total power dissipated by the water distribution network and the numerical value of the network entropy. In another article by them (Ang and Jowitt (2005b)), an alternative method to calculate the network entropy of water distribution systems was presented, which gives new insights into the meaning of network entropy. This alternative method, termed the Path Entropy Method (PEM), offers a simpler explanation to the entropy of branching-tree networks and the maximum entropy of water distribution networks. The formulation of the PEM was based on the fact that the entropy of the water distribution network arises because of the different paths available to a water molecule to move from a super-source to a super-sink. In the next section, a brief explanation of the PEM is presented.

On the other hand, for water distribution systems connection to a source is not only necessary, but a sufficient condition to ensure that a given node is functional. That is why hydraulic calculation has to be included in determining mechanical-hydraulic reliability. Previously defined redundancy indices for water distribution networks in the literature are generally based on only hydraulic or mechanical characteristics of the network and they do not simultaneously consider both characteristics in their calculations. This is while the network's risk is highly affected by both of these characteristics.

In this regards, the aim of this paper is to explore deficiencies of previous definitions for the entropy of water distribution networks and presenting a new weighted entropy-based measure for assessing reliability of water distribution networks considering both of aforementioned characteristics of a system.

2. ENTROPY FUNCTION FOR WATER DISTRIBUTION NETWORKS

One of the most appropriate entropy functions for water distribution networks defineed by Tanyimboh and Templeman (1993). The formulation of the entropy function mainly relied on Shannon's measure of uncertainty, which is the underlying principle of information theory. They assumed that the available information on the water distribution network were the topological layout, the supply and demand at all nodes, and the flow direction in each pipe member. The data on flow direction in each pipe is critical, as there will be a maximum-entropy flow distribution for every set of flow directions.

The missing data are the length, diameter, and roughness of all pipes. Unless the network has a branching structure, there will be a very large number of feasible flow patterns. They define network entropy function as:

$$\frac{S}{K} = S = S_0 + \sum_{n=1}^{N} P_n S_n \tag{2.1}$$

where S is the entropy defined by Shannon and N is the total number of nodes and K is the Boltzman constant which is usually set to unity and it will be shown in this paper that this can be true only in special cases.

The entropy of the external inflows, S_0 is represented by:

$$S_0 = -\sum_{i \in I} P_{0i} \ln P_{0i} \tag{2.2}$$

where I is the set of all source nodes,

$$P_{0i} = \frac{q_{0i}}{T_0} \tag{2.3}$$

where q_{0i} is the external inflow at source node i and T_0 is the total supply or demand. The second term in the entropy function consists of the outflow entropy at each node S_n weighted by the ratio P_n of the total inflow of each node to the total inflow of the whole network. An important point in the definition of outflow is that it is inclusive of any demand at the node.

$$P_n = \frac{T_n}{T_0} \tag{2.4}$$

where T_n is the total outflow at node n.

$$S_n = -\sum_{nj \in ND_n} P_{nj} \ln P_{nj} \tag{2.5}$$

where ND_n is the set of all outflows from node n, and

$$P_{nj} = \frac{q_{nj}}{T_n} \tag{2.6}$$

where q_{nj} is the flow from node n to node j.

Entropy function, given by Eqn. 2.1 shows that the entropy of a water distribution network has two components. The first part is the amount of entropy in the external inflows and the second part consists of the weighted entropy values at every demand node. Informational entropy measures the amount of uncertainty in a situation or system. For a water distribution network, the uncertainty can be imagined from the viewpoint of a water molecule. Using the simple example water distribution network, shown in Fig. 2.1, the concept of entropy function is investigated. The water distribution network, shown in Fig. 2.1, has a single source node and three demand nodes and the maximum entropy flows are assigned to the pipes. The entropy of the external inflows S_0 is the uncertainty faced by a water molecule moving from the super-source to the individual supply nodes. For all nodes, the entropy would be non-zero only if there are two or more paths for the water molecule to take at each node. However, the entropy S_n calculated for each node n would have to take account of the probability for the water molecule arriving at that node, which is expressed by the P_n term in Eqn. 2.1. Details of entropy calculation of the sample network with its tree diagram are shown in Fig. 2.2.

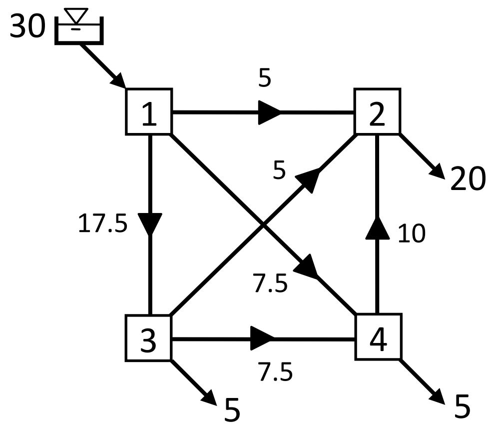
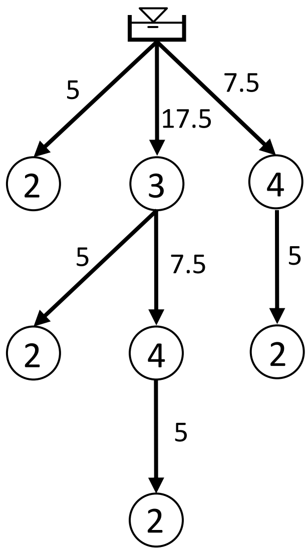


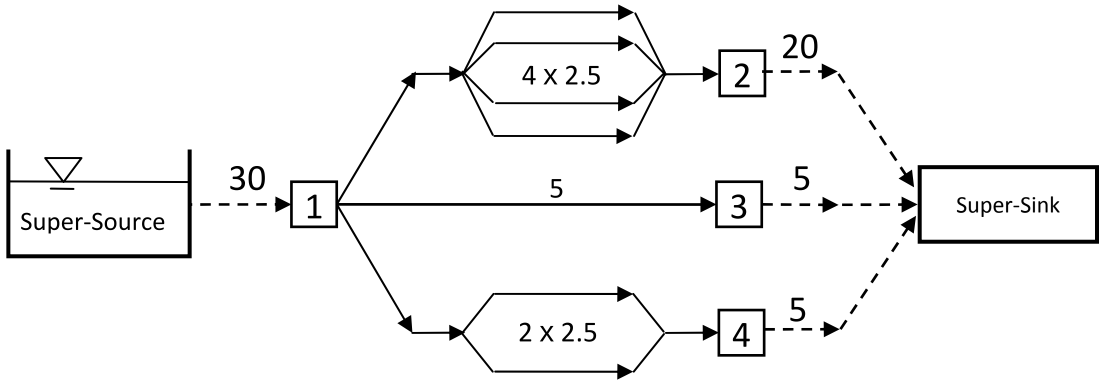
Figure 2.1. Sample fully–connected network with maximum network entropy based on Eqn. 2.1.



$$S=-\frac{30}{30}\times\left[\frac{5}{30}\ln\left(\frac{5}{30}\right)+\frac{17.5}{30}\ln\left(\frac{17.5}{30}\right)+\frac{7.5}{30}\ln\left(\frac{7.5}{30}\right)\right]$$
$$-\frac{17.5}{30}\times\left[2\times\frac{5}{30}\ln\left(\frac{5}{30}\right)+\frac{7.5}{30}\ln\left(\frac{7.5}{30}\right)\right]$$
$$-\frac{15}{30}\times\left[\frac{5}{30}\ln\left(\frac{5}{30}\right)+\frac{10}{30}\ln\left(\frac{10}{30}\right)\right]=1.9073$$

Figure 2.2. Tree diagram of sample network, shown in Fig. 2.1, with entropy calculation

From the above-mentioned definition, it is clear that the entropy of a water distribution network can be represented by the number of paths available to a water molecule moving from the super-source to the super-sink (Fig. 2.2). Based on this observation, an alternative way of calculating network entropy is the path entropy method (PEM) (Ang and Jowitt, 2005). The PEM diagram shows the number of paths from the super-source to the super-sink and the amount of flow in each path. Development of the PEM diagram includes two main steps. The first step is to establish the number of paths from the source nodes to every demand node and draw the PEM diagram with all the nodes and links. The second step involves determining the flow carried by each link, which is performed by an inspection of the flow rates in all of the network links. Once the PEM diagram is developed, the calculation of the network entropy is relatively straightforward, as compared to the network entropy equations by Tanyimboh and Templeman (1993). However, it must be noted that the less complicated entropy calculation is a result of the efforts spent in organizing the data into a PEM diagram. The true strength of the PEM lies in its ability to give new insights into the meaning of the network entropy, such as the entropy of branching-tree networks and the maximum-entropy flows of a single-source network with given flow directions, which will be discussed in the next section. PEM diagram of the sample network and its entropy calculation are shown in Fig. 2.3.



$$S = -4 \times \frac{5}{30} \ln\left(\frac{5}{30}\right) - 1 \times \frac{5}{30} \ln\left(\frac{5}{30}\right) - 2 \times \frac{2.5}{30} \ln\left(\frac{2.5}{30}\right) = 1.9073$$

Figure 2.3. PEM diagram of sample network with entropy calculation

3. DISCUSSION ON THE PREVIOUS DEFENITION OF ENTROPY FUNCTION

In a discussion paper by Walters (1995), it was stated that any tree that connects the demand nodes will have the same minimum entropy value, or, in other words, all branching-tree networks will have the same minimum entropy value. Afterward Ang and Jowitt showed this fact using path entropy method. For branching-tree networks with a single source, there is only one path from the source node to every demand node. From an informational point of view, all the different layouts of branching-tree networks, related to the sample network shown in Fig. 2.1, which are shown in Fig. 3.1., have essentially the same entropy. Thus, for a branching-tree network, the entropy is an invariant measure.

The PEM diagram for the branching-tree sample network is shown in Fig. 3.2, which can be used for representing any of the different layouts. A water molecule moving from the super-source to the super-sink is only uncertain about the demand node it would arrive. The question of different paths to the same demand node does not exist in a branching-tree network. Thus, Tanyimboh and Templeman's definition of entropy function cannot investigate any differences between branching-tree networks with the same number of source and demand nodes, like networks shown in Fig. 3.1 which all of them have same PEM diagram as shown in Fig. 3.2. But with a cursory look at these networks, it can be easily seen that some of these networks are more sensitive than the others to the damage of its links. For instance, if the link 1-3 in the networks (c), (d) and (e) in Fig. 3.1 gets damaged due to any hazards like earthquake, the amount of loss will be 30, 10 and 5, respectively. The amount of loss entirely depends on the layout of the network in which demand nodes are connected to source node in their series or parallel configuration. In other words, redundancy of the network affects the amount of loss and therefore on the reliability of network, but the definition of entropy function by Tanyimboh and Templeman cannot detect properly the effect of redundancy.

To overcome this problem, Emamjomeh and Hosseini (2010) defined a penalty number (T_p) for each link, which is equal to the amount of loss if that link fails, and based on these penalty numbers, they introduced a new weighting ratio ($P_n^{'}$) as

$$P_n^{'} = \frac{T_n}{T_{p0}} \tag{3.1}$$

where T_{p0} is the summation of penalty numbers for all links in the network. They used this weighting ratio instead of previous one in their calculations and the rest was as before. They mentioned that these penalties could be modified by other factors like the importance of demand nodes. They also showed that the suggested weighting factor behaves like a modified Boltzman's constant.

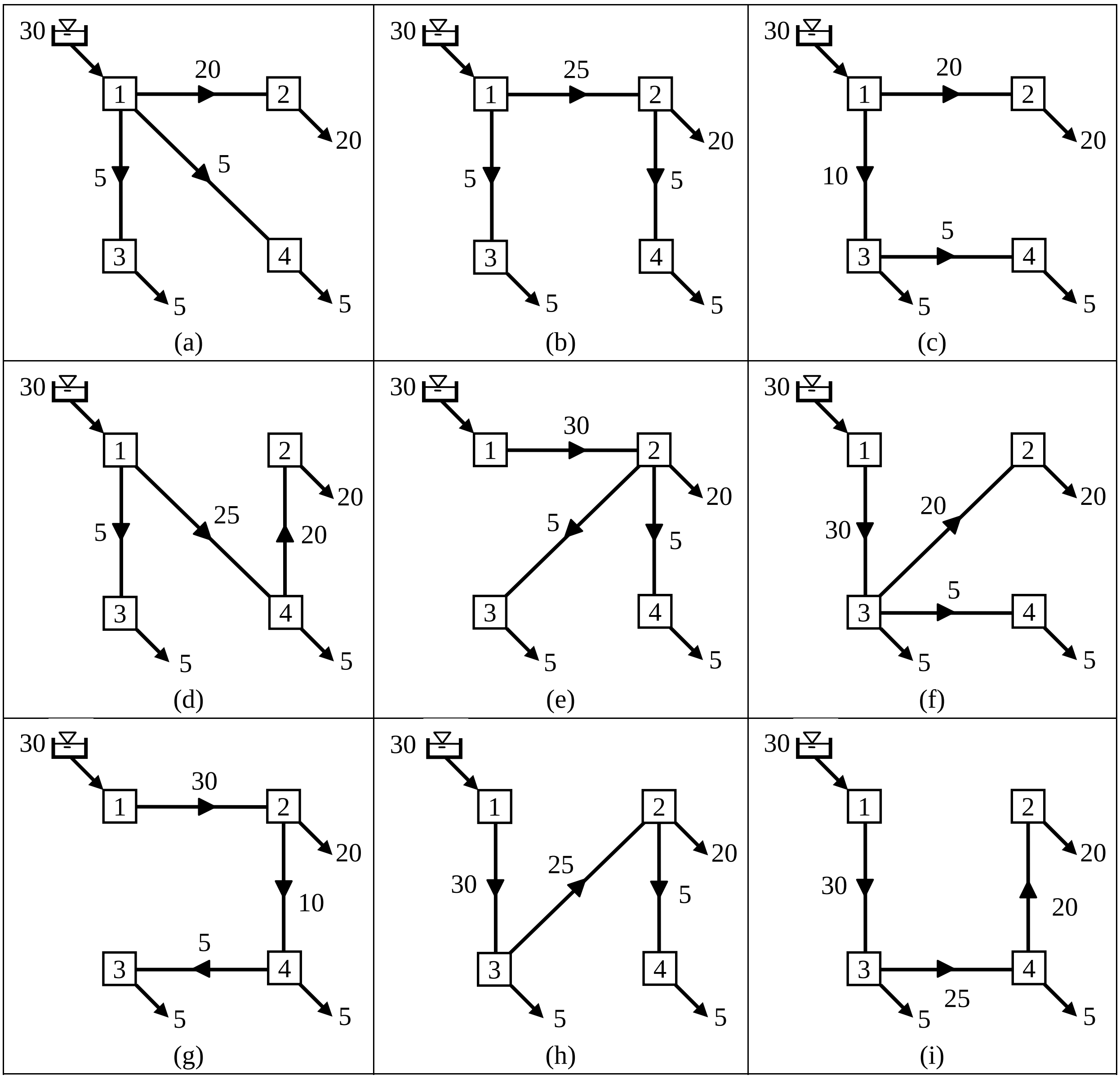
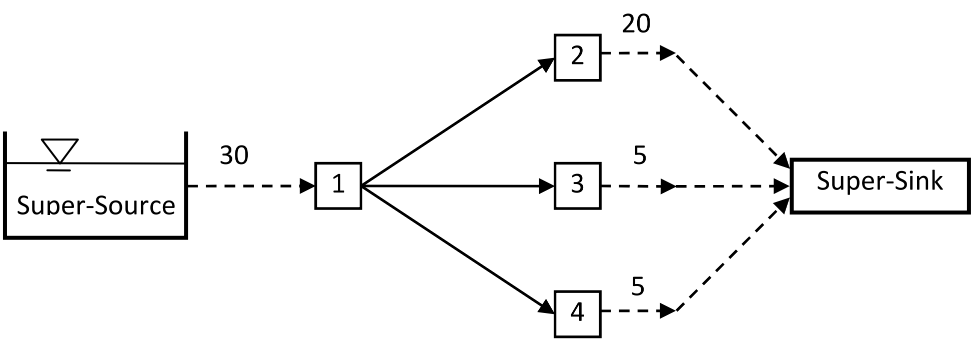


Figure 3.1. Different tree-branching networks of the sample network shown in Figure 1



$$S = -\frac{20}{30}\ln\left(\frac{20}{30}\right) - 2\times\frac{5}{30}\ln\left(\frac{5}{30}\right) = 0.8676$$

Figure 3.2. PEM diagram of the branching-tree networks

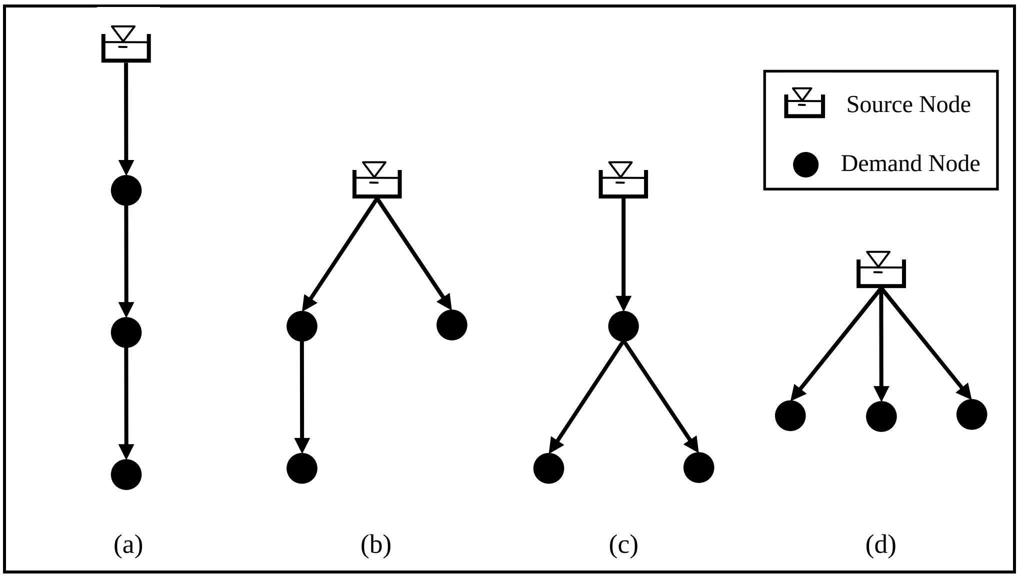


Figure 3.3. Different patterns of the sample tree-branching networks

Although the modification method proposed by Emamjomeh and Hosseini (2010) can separate networks with different patterns and same entropy values from each other, but it cannot take in to account links-failure probabilities. For example, two different networks with different link-failure probabilities but same patterns (like pattern (b) in Fig. 3.3) have identical entropy values while they do not have same reliabilities.

4. MODIFIED ENTROPY FUNCTION FOR WATER DISTRIBUTION NETWORKS

As mentioned in the preceding section, Although Tanyimboh and Templeman's entropy function for water distribution networks has its benefits and simplicity but it cannot identify different patterns as well as link-failure probability. This is while the network's risk is highly affected by both hydraulic and mechanical characteristics of a system. Thus, a new weighted entropy function is presented here which can consider both aforementioned characteristics of the system in its formulation while keeping simplicity of pervious definition. For this purpose, a penalty function is defined for different links of the network based on their probability of failure in the specified hazard scenario. This penalty function is imposed in the hydraulic entropy function of the network in an appropriate manner so that the effect of mechanical behaviour of links is considered in the network's entropy. In this manner the amount of supply loss due to absence of each link in the network is taken in to account by corresponding penalty function.

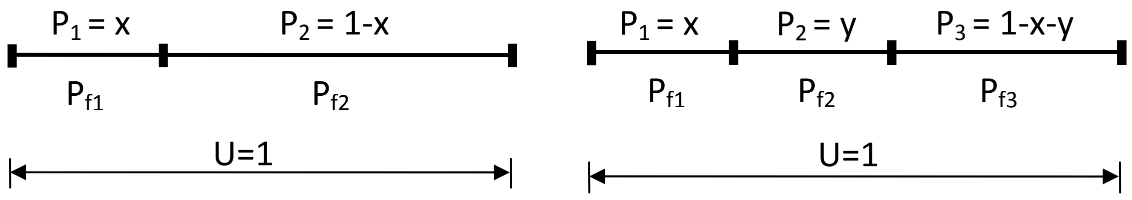
The modified entropy function for water distribution network is defined as:

$$\frac{S_N}{K} = S_N = S_0 + \sum_{n=1}^{N} P_n S_n - \ln(0.01) \tag{4.1}$$

where S_N is the new entropy value and the other parameters are defined as in Eqn. 2.1 except S_n which is calculated by the following equation

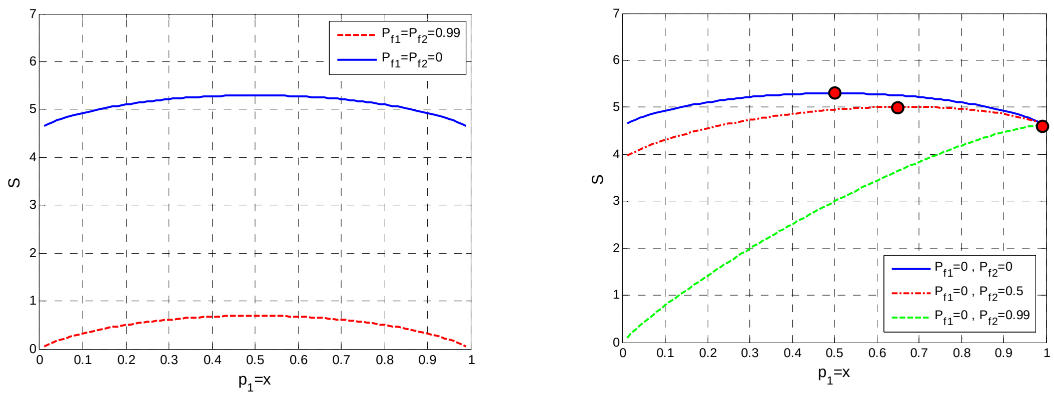
$$S_n = -\sum_{nj \in ND_n} P_{nj} \ln\left(P_{nj} / \left(1 - Pf_{nj}\right)\right) \tag{4.2}$$

where P_{nj} is obtained from Eqn. 2.6. Pf_{nj} is the failure probability of the link between node n and node j which can be obtained using analytical hazard analysis of a specified scenario or using expert judgement.



(a) (b)

Figure 4.1. Venn's diagram for ensemble space of a system with (a) 2 components, (b) 3 components



(a) (b)

Figure 4.2. New Entropy function diagram for a system with 2 components for different failure probabilities

It should be noted that a biased number (-ln 0.01) is added to the proposed entropy function to prevent negative entropy values by assuming failure probability of definitely damaged link equal to 0.99. Because entropy function is a comparative index, this biased number does not affect its concept. Moreover, in the proposed entropy function like the earlier one, it is assumed when a link is in failure state, it is completely nonoperational and no water molecule can reach the demand node from that link. In the other words, leakage state is not considered here.

In order to investigate the behaviour of proposed entropy function, a simple network is considered with one source and one demand node and two parallel links in which failure probability of links are P_{f1} and P_{f2}. In this network a water molecule has only two choices, P_1 is probability of selecting the first link and P_2 is probability of selecting the second one. So, Venn's diagram of this network will be as Fig. 4.1a and its entropy function for some different failure probabilities of links will be as Fig. 4.2. As it is seen in Fig. 4.2a, this new function is completely behaving like Tanyimboh and Templeman's function when failure probability of links are 0.99, but entropy value increases by decreasing failure probability of links. Thus, in identical failure probabilities of links, maximum entropy will occur when all links have the same chance of being selected by water molecules. Mathematically, If operational probability of links is defined as

$$Po_i = (1 - Pf_i) \tag{4.3}$$

maximum entropy of network with two parallel components and one demand node will be obtained when the amount of flow in the first pipe is

$$x_{max} = \frac{Po_1}{Po_1 + Po_2} \tag{4.4}$$

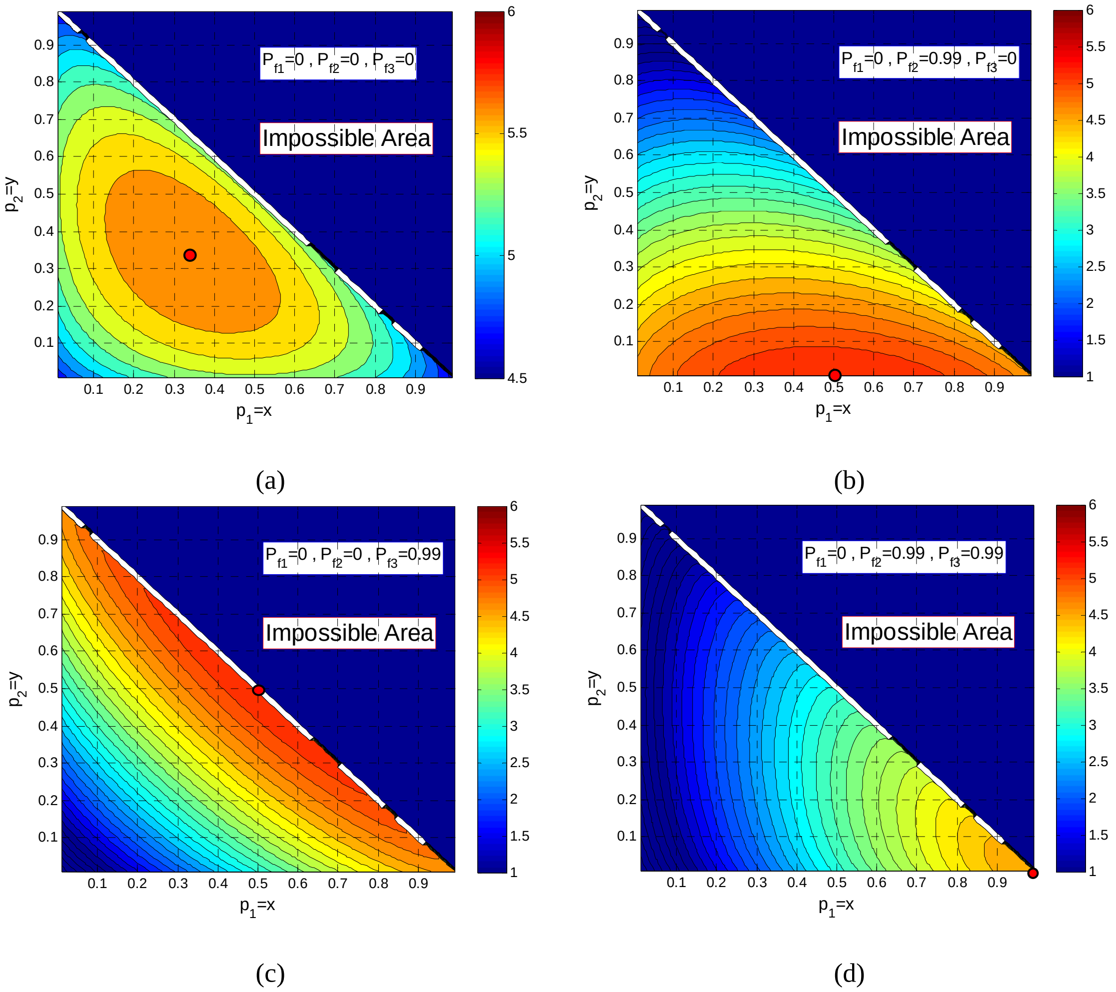


Figure 4.3. New Entropy function diagram for a system with 3 components

Eqn. 4.4 implies when failure probability of two links are the same, maximum entropy will be obtained when both of links carry the same amount of flow however less failure probability gives greater entropy value (Fig. 4.2a) and when two different links do not have the same failure probability, maximum entropy will occur when the link with greater failure probability carries lesser amount of the flow (Fig. 4.2b) so that a definitely vulnerable link must have no chance in being selected by molecules. Fig. 4.3 shows same results for a one source-one demand network with three parallel components.

With this new definition of entropy for water distribution networks, it can easily be seen that entropy value for all the tree branching networks shown in Fig. 3.1 are the same only when failure probability of all links is zero. But when failure probability of links is not zero, the network with series links has lesser entropy than parallel ones. In the other hand, failure probability of less important links in the network does not highly affect the entire network's entropy value. Therefore, the proposed entropy function can be helpful in selecting the most important links in different hazard scenarios and selecting the optimum mitigation plan.

5. CONCLUSIONS

Based on the discussions and numerical examples presented in the paper, the proposed weighted entropy-based index for calculating reliability of water distribution networks makes it possible to determine the different reliability parameters for systems with different complexity. Mechanical and hydraulic characteristics of water distribution networks can also easily be taken in to account for determining reliability of the system. The proposed definition has simplicity of previously defined entropy functions without their deficiencies. This index can easily make distinction between different networks with different mechanical probabilities of failure. So, it can be used to obtain the optimum hydraulic layout for designing a new system, or to find the best mitigation plan against different natural hazards like earthquake.

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